Calculus '21-22
Dr. Quattrin
Derivative Practice Test

Name Solution Key

score ____________

1a. $\frac{d}{dx}\left[-3x^7+13^x-\frac{7}{5}\sqrt[4]{x^7}-\frac{3}{\sqrt[5]{x^6}}-\frac{1}{2x^3}\right]$

$$=\frac{d}{dx}\left[-3x^7+13^x-\frac{7}{5}x^{7/4}-3x^{-6/5}-\frac{1}{2}x^{-3}\right]$$

$$=-21x^6+13^x\ln 13-\frac{49}{20}x^{3/4}+\frac{18}{5}x^{-11/5}+\frac{3}{2}x^{-4}$$

1b. $\frac{d}{dx}\left(\cos\left(x^3\right)\right)=\left(-\sin x^3\right)\left(3x^2\right)$

$$=-3x^2\sin x^3$$

1c. $\frac{d}{dx}\left[\tan^{-1}\left(5x^2\right)\right]=\frac{1}{1+\left(5x^2\right)^2}\left(10x\right)$

$$=\frac{10x}{1+25x^4}$$

2a. $\frac{d}{dx}\left(15x^4e^{-5x}\right)$

$u \qquad Dv \quad + \quad v \quad Du$

$= 15x^4(-5e^{-5x}) + e^{-5x}(60x^3)$

$u = 15x^4$

$Du = 60x^3$

$v = e^{-5x}$

$Dv = e^{-5x}(-5)$

$= -5e^{-5x}$

$= -75x^4e^{-5x} + 60x^3e^{-5x}$

$= 15x^3e^{-5x}[-5x+4]$

2b. $\frac{d}{dx}\left(\frac{3x}{5+x^2}\right) = \frac{(5+x^2)(3) - 3x(2x)}{(5+x^2)^2}$

$= \frac{15 + 3x^2 - 6x^2}{(5+x^2)^2}$

$= \frac{15 - 3x^2}{(5+x^2)^2}$

$= \frac{-3(x^2-5)}{(5+x^2)^2}$

3a. $\frac{d}{dx}\left(\ln^4\left(1-x^2\right)\right)$

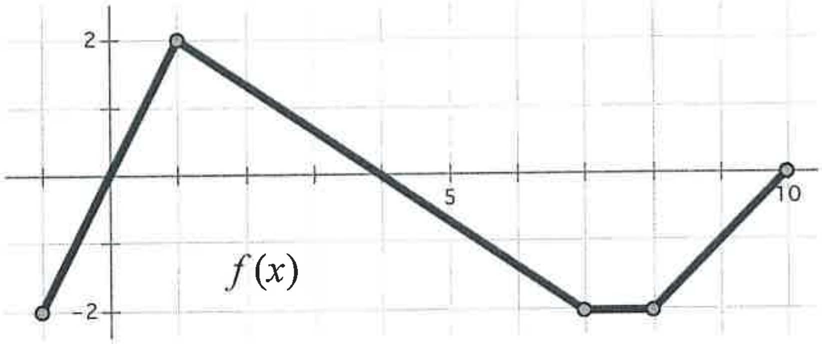
$$= \left(4\ln^3(1-x^2)\right) \cdot \left(\frac{1}{1-x^2}\right)(-2x)$$

$$= \frac{-8x\ln^3(1-x^2)}{1-x^2}$$

3b. If $y=\sin^3 5x$, find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = (3\sin^2 5x)(\cos 5x)(5)$$

$$= 15\sin^2 5x\cos 5x$$



x	$g(x)$	$g'(x)$
0	-1	1
2	1	3
4	3	6
6	6	12
8	4	8

4. Let $f(x)$ be the function whose graph is given above and let $g(x)$ be a differentiable function with selected values for $g(x)$ and $g'(x)$ given on the table above.

a) Find the equation of the line tangent to $f(x)$ at $x = 4$.

$f(4) = 0 \quad f'(4) = \frac{-2}{3}$

$y - 0 = \frac{-2}{3}(x-4)$

b) Let K be the function defined by $K(x) = f(g(x))$. Find $K'(4)$.

$K'(x) = f'(g(x)) \cdot g'(x)$

$K'(4) = f'(g(4)) \cdot g'(4)$

$= f'(3) \cdot g'(4)$

$= \left(-\frac{2}{3}\right)(6) = -4$

c) Let M be the function defined by $M(x) = \overset{u}{g(x)} \cdot \overset{v}{f(x)}$. Find $M'(4)$.

$$M'(x) = g(x) \cdot f'(x) + f(x) \cdot g'(x)$$

$$M'(4) = g(4) \cdot f'(4) + f(4)\, g'(4)$$

$$(3)\left(-\frac{2}{3}\right) + 0 \cdot (6)$$

$$= -2$$

d) Let J be the function defined by $J(x) = \dfrac{f(2x)}{g(x)}$. Find $J'(2)$.

$$J' = \frac{g(x) \cdot f'(2x) \cdot 2 - f(2x)\, g'(x)}{[g(x)]^2}$$

$$J(2) = \frac{g(2) \cdot f'(4) \cdot 2 - f(4)\, g'(2)}{[g(2)]^2}$$

$$= \frac{1\left(-2/3\right)(2) - 0(3)}{1^2} = \frac{-4}{3}$$

5. If $h(t) = \ln(t^2+1)$, find $h''(x)$

$$h'(t) = \frac{1}{t^2+1}(2t) = \frac{2t}{t^2+1}$$

$$h''(t) = \frac{(t^2+1)(2) - (2t)(2t)}{(t^2+1)^2}$$

$$= \frac{2t^2+2-4t^2}{(t^2+1)^2} = \frac{-2t^2+2}{(t^2+1)^2}$$

$$= \frac{-2(t^2-1)}{(t^2+1)^2}$$

6. If $y = \sin e^x$, then $\frac{d^2y}{dx^2} =$

$$\frac{dy}{dx} = (\cos e^x)(e^x)$$

$$\frac{d^2y}{dx^2} = \cos e^x (e^x) + e^x(-\sin e^x)(e^x)$$

$$= e^x(\cos e^x - e^x \sin e^x)$$

7a. Write the equation of the line tangent and normal to $f(x)=\sqrt[3]{x+3}$ at $x=5$. Show all work.

$f(x) = (x+3)^{1/3}$

$f(5) = 8^{1/3} = 2$

$f'(x) = \frac{1}{3}(x+3)^{-2/3} = \frac{1}{3(x+3)^{2/3}}$

$m = f'(5) = \frac{1}{3(8)^{2/3}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

TANGENT $y - 2 = \frac{1}{12}(x-5)$

NORMAL $y - 2 = -12(x-5)$

7b. Use the tangent line found in 7a to approximate $f(4.9)$

$f(4.9) \approx y - 2 = \frac{1}{12}(4.9-5)$.

$y - 2 = -.008$

$y = 1.992$