AP Calculus AB '21-22
Integral Test v2

Name Solution Key

Score_____

NO CALCULATOR ALLOWED

1. Find $\int_0^3 x^2\,dx$ $= \frac{x^3}{3}\Big]_0^3 = 9$

a) -18 b) -9 c) 0 (d) 9 e) 18

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2. $\frac{1}{3}\int_0^1 3x^2 \sin x^3\,dx$ $= \frac{1}{3}\int_0^1 \sin u\,du = -\frac{1}{3}\cos u\Big]_0^1 = -\frac{1}{3}\cos 1 + \frac{1}{3}\cos 0$ AND $\cos 0 = 1$

a) $-\frac{1}{3}\cos(1)$ b) $-\frac{1}{3}\cos(1)-\frac{1}{3}$

(c) $-\frac{1}{3}\cos(1)+\frac{1}{3}$ d) $\frac{1}{3}\cos(1)$

e) $\frac{1}{3}\cos(1)-\frac{1}{3}$

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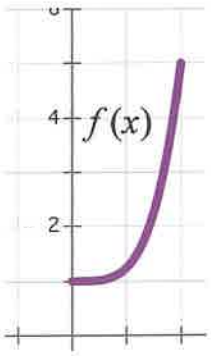
3. If $\int_0^6 f(x)\,dx = 9$, $\int_6^3 f(x)\,dx = 5$, and $\int_3^0 g(x)\,dx = 7$, then $\int_0^3 \left[\frac{1}{2}f(x) - 3g(x)\right]dx =$

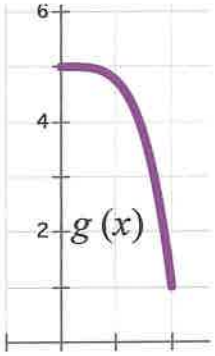
a) –28 b) –18 c) 0 d) 18 (e) 28

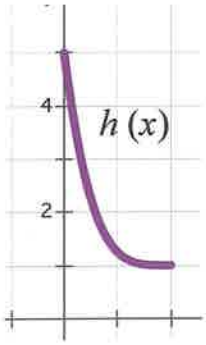
$\int_0^3 f(x) = \int_0^6 - \int_3^6 = 9 - (-5) = 14$

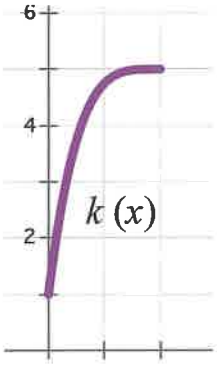
$\frac{1}{2}\int_0^3 f(x)\,dx - 3\int g(x)\,dx = \frac{1}{2}(14) - 3(-7) = 28$

4. For which of the following functions would the left-hand Riemann sum be an overestimate?









a) $f(x)$ and $g(x)$ b) $h(x)$ and $k(x)$

LEFT OVERESTIMATE = DECREASING

c) $f(x)$ and $h(x)$ d) $g(x)$ and $h(x)$

e) $f(x)$ and $k(x)$

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5. $\int_0^1 \frac{x^3 dx}{x^4+1}$ $= \frac{1}{4}\int_0^1 \frac{4x^3 dx}{x^4+1} = \frac{1}{4}\int_1^2 \frac{1}{u}du = \frac{1}{4}\ln u\Big]_1^2$

a) $-\frac{1}{8}$ b) $-\frac{1}{32}$ c) $\frac{1}{4}\ln 2$ d) $\ln 2$ e) 16

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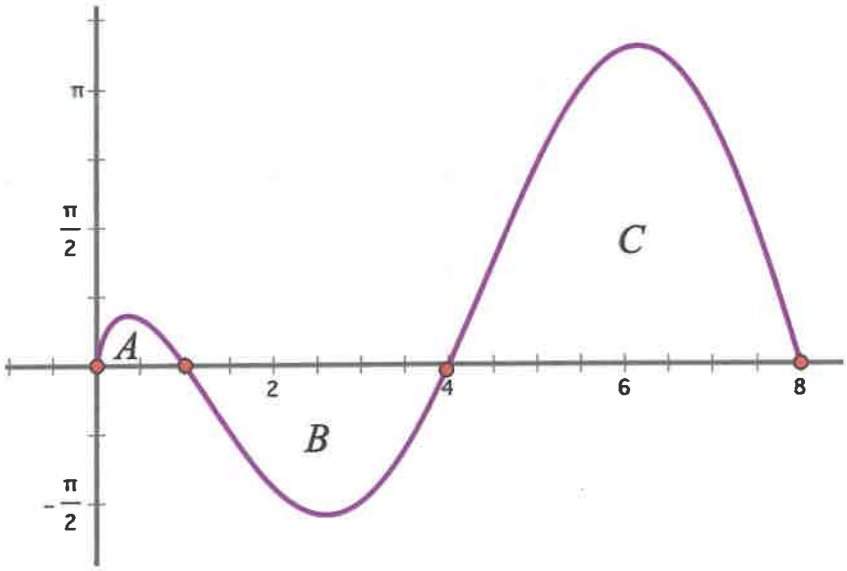
6. The basement of a house is flooding. The water pours in at a rate of $f(t)$ gallons per hour and is being pumped out at a rate of $r(t)$. When the pump is started, at time t = 0, there are 1200 gallons of water in the basement. Which of the following expresses how fast the rate of change of the number of gallons of water in the basement is changing between 0 and t hours?

a) $1200+\int_0^t [f(x)-r(x)]dx$ b) $\int_0^t [f(x)-r(x)]dx$

c) $f(t)-r(t)$ d) $\frac{1}{t}\int_0^t [f(x)-r(x)]dx$

e) $f'(t)-r'(t)$



In the figure above, A, B, and C are areas between the curve $f(x)$ and the x-axis.

7. If A=14, B=16, and C=50, what is the $\int_0^8 f(x)dx$? = 14−16+50 = 48

a) 6 b) 10 c) $\frac{40}{3}$ d) $\frac{80}{3}$ e) 48

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8. The average value of $y=1-e^{3x}$ on $x\in\left[0,\frac{1}{3}\right]$ is

a) $1-3e$ b) $\frac{1}{3}-e$ c) $2-e$ d) $\frac{e-1}{3}$ e) $-e$

$\frac{1}{1/3-0}\int_0^{1/3}1-e^{3x}\,dx \quad u=3x$

$=\frac{3}{3}\int_0^{1/3}(1-e^{3x})\,3\,dx$

$=\int_0^1 1-e^u\,du$

$u-e^u\Big]_0^1$

$(1-e)-(0-1)$

t in hours	0	12	24	36	48
$v(t)$ in km/hr	21	26.3	31.4	36.8	41.5

9. A Gravitational Slingshot Effect is sometimes used by space probes like Voyager 2 in order to increase its velocity without expending fuel. By flying close to the planet Saturn in a parabolic arc, the velocities on the table above were achieved by a probe. Which of the following is the setup for a trapezoidal sum which approximates $\int_0^{48} v(t)dt$

a) $12\left[21+26.3+31.4+36.8+41.5\right]$

b) $12\left[26.3+31.4+36.8+41.5\right]$

c) $12\left[21+26.3+31.4+36.8\right]$

d) $24\left[26.3+36.8\right]$

e) $12\left[21+2(26.3)+2(31.4)+2(36.8)+41.5\right]$

AP Calculus AB '21-22

Chapter 3 v2 FRQ

Calculator Allowed

Name Solution Key

Directions: Show all work.

1. Consider the function $f(x) = -3 + x + 3x^2 - x^3$.

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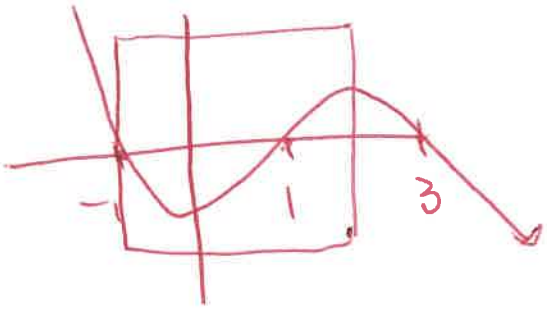
a) Draw the graph of $f(x)$ on $x \in [-1, 2]$.



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b) Prove that one zero of $f(x)$ is at $x = 1$. [Calculator evidence is not a proof.]

$f(1) = -3 + 1 + 3 - 1 = 0$

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c) Find the exact value of $\int_{-1}^{2} f(x)\,dx$. Show the antiderivative and boundary insertion steps.

$$= -3x + \frac{1}{2}x^2 + x^3 - \frac{x^4}{4}\Big]_{-1}^{2}$$

$$= (-6 + 2 + 8 - 4) - \left(-3 + \frac{1}{2} + 1 - \frac{1}{4}\right)$$

$$= \frac{9}{4}$$

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d) Find the exact area between the x-axis and $f(x)$ on $x \in [-1, 2]$. Show the antiderivative and boundary insertion steps.

$$\int_{-1}^{1} f(x)\,dx - \int_{1}^{2} f(x)\,dx$$

$$= \left[-3x + \frac{1}{2}x^2 + x^3 - \frac{1}{4}x^4\right]_{-1}^{1} - \left[(-6 + 2 + 8 - 4) - \left(-3 + \frac{1}{2} + 1 - \frac{1}{4}\right)\right]$$

$$= \left[\left(-3 + \frac{1}{2} + 1 - \frac{1}{4}\right) - \left(3 + \frac{1}{2} - 1 - \frac{1}{4}\right)\right] - \left[(-6 + 2 + 8 - 4) - \left(-3 + \frac{1}{2} + 1 - \frac{1}{4}\right)\right]$$

$$= 5{,}75$$

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2. The Peninsula Humane Society (PHS) is dedicated to the care and adoption of as many animals who they receive as possible. Since cats breed seasonally, the number of cats and kittens they receive into their facility in a given year varies roughly sinusoidally with time. The data available from 2019 shows the rate $R(t)$, measured in healthy cats per month, varies with time, measured in months after New Year's Day, according to the equation

$$R(t)=120-88\cos\left[\frac{\pi}{6}(t-2)\right].$$

The rate $A(t)$ at which adoption occur, measured in cats per month, varies with time, measured in months after New Year's Day, according to the equation

$$A(t)=125-85\cos\left[\frac{\pi}{6}(t-3)\right].$$

On New Years' Day ($t=0$), there were 131 cats in the PHS Nursery waiting to be adopted.

(a) How many cats and kittens were received at PHS in 2019?

1 PT

$\int_0^{12} R(t)dt = 1440$

(b) Find $A'(10.3)$. Using the correct units, explain the meaning of $A'(10.3)$ in context of the problem.

2 pts

$A'(10.3) = -28.008$

AT $t=10.3$ MONTHS, THE RATE AT WHICH THE CATS ARE BEING ADOPTED IS DECREASING BY 28.008 CATS/MONTH^2

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3 PTS

(c) Find the number of healthy cats and kittens predicted by the models to be in the PHS facility at the end of 2019.

$$\text{TOTAL} = 131 + \int_0^{12} R(t) - A(t)\,dt$$

$$= 71 \text{ CATS}$$

(d) Cats breed seasonally, with most births occurring between April 1st ($t=3$) and October 31st ($t=10$). Find the total change the number of healthy cats and kittens in the PHS facility during "Kitten Season." Include the units. EXPLAIN THE MEANING

UNITS

$$\int_3^{10} R(t) - A(t)\,dt = -55.780 \text{ CATS}$$

$-55,780$

DURING KITTEN SEASON, 58 ~~MORE~~ CATS WERE ADOPTED OUT THAN WERE TAKEN IN

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t days	0	1	2	3	4
$H(t)$ in mm per day	0	0.9	1.4	1.7	2.1

3. A small plant is purchased from a nursery and the change in height of the plant is measured at the end of each day for four days. The data, where $H(t)$ is measured in millimeters per day and t is measured in days, are listed above.

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(2) a. Estimate $H'(3)$. Show the work that leads to your answer. Indicate the units.

$H'(3) = \frac{2.1-1.4}{4-2} = .35 \frac{MM}{DAY^2}$

(1) (1)

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b. Explain the difference between $H(3)$ and $H'(3)$ in terms of the plant's growth.

$H(3)$ IS HOW FAST THE PLANT IS GROWING IN MM/DAY ON DAY 3

$H'(3)$ IS THE RATE OF CHANGE OF THE RATE OF GROWTH IN MM/DAY/DAY ON DAY 3

c. Use right-hand rectangles with subintervals indicated by the table to approximate $\int_0^4 H(t)\,dt$. Using correct units, explain the meaning of this value in the context of the problem.

(3)

(1)

$\int_0^4 H(t)\,dt \approx 1(.9) + 1(1.6) + 1(1.7) + 1(2.1) = 6.1$ mm (1)

THE PLANT HAS GROWN 6.1 MM IN THESE FOUR DAYS

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d. Using correct units, explain the meaning of $\frac{1}{4}\int_0^4 H(t)\,dt$ in the context of the problem.

$\frac{1}{4}\int_0^4 H(t)\,dt$ IS THE AVERAGE RATE OF GROWTH IN MM/DAY OVER THESE FOUR DAYS.