Honors Precalc
Sample Spring Final – Part 1
Calculator Allowed
Show all work. Round to 3 decimals.

Name: KEY
Date:
Period:

Multiple Choice (3 pts. each)

1. $\lim\limits_{x\to\infty}\dfrac{\sin\left(\frac{3}{x}\right)}{\frac{1}{x}} = \frac{0}{0} \overset{L.H.}{=} \lim\limits_{x\to\infty}\dfrac{\cos\left(\frac{3}{x}\right)\cdot\frac{-3}{x^2}}{\frac{-1}{x^2}} = \lim\limits_{x\to\infty}\cos\left(\frac{3}{x}\right)\cdot\frac{-3}{x^2}\cdot\frac{-x^2}{1}$

(a) -3
(b) 3
(c) 1
(d) 0
(e) DNE

$= \lim\limits_{x\to\infty}\cos\left(\frac{3}{x}\right)\cdot 3 = \cos 0\cdot 3 = 1\cdot 3 = 3$

2. If f is a continuous function defined by $f(x)=\begin{cases} x^2+bx & x\le 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x>5 \end{cases}$, then $b=$

(a) -6
(b) -5
(c) -4
(d) 4
(e) 5

$f(5) = \lim\limits_{x\to 5} f(x)$ ✓

If $\lim\limits_{x\to 5} f(x)$ exists, then

$\lim\limits_{x\to 5^+} f(x) = \lim\limits_{x\to 5^-} f(x)$

$\lim\limits_{x\to 5^+} f(x) = \lim\limits_{x\to 5} 5\sin\left(\frac{5\pi}{2}\right) = 5$

$\lim\limits_{x\to 5^-} f(x) = \lim\limits_{x\to 5} x^2+bx = 25+5b$

$5 = 25+5b$
$-20 = 5b$
$b = -4$

3. If $y=\sin^{-1}(5x)$, then $\frac{dy}{dx}=$

(a) $\dfrac{1}{1+25x^2}$

(b) $\dfrac{5}{1+25x^2}$

(c) $\dfrac{-5}{\sqrt{1-25x^2}}$

(d) $\dfrac{1}{\sqrt{1-25x^2}}$

(e) $\dfrac{5}{\sqrt{1-25x^2}}$

$y' = \dfrac{1}{\sqrt{1-(5x)^2}}\cdot 5 = \dfrac{5}{\sqrt{1-25x^2}}$

$\boxed{\text{pt.}}$

$(\frac{\pi}{4}, 0)$

4. An equation of the line tangent to the graph of $y=\cos(2x)$ at $x=\pi/4$ is

(a) $y-1=-(x-\pi/4)$

(b) $y-1=-2(x-\pi/4)$

(c) $y=2(x-\pi/4)$

(d) $y=-(x-\pi/4)$

(e) $y=-2(x-\pi/4)$

$\boxed{\text{slope}}$

$y' = -\sin(2x)\cdot 2$

$y'\big|_{x=\frac{\pi}{4}} = -\sin(2\cdot\frac{\pi}{4})\cdot 2$

$= -2$

$y-0=-2(x-\frac{\pi}{4})$

5. If the derivative of f is given by $f'(x)=e^x-3x^2$, at which of the following values of x does f have a relative maximum value?

(a) -0.46

(b) 0.20

(c) 0.91

(d) 0.95

(e) 3.73

$f'(x)=0 \Rightarrow x=-0.459,\ 0.910,\ 3.733$

f'	$-$	0	$+$	0	$-$	0	$+$
x		-0.459		0.910		3.733	
		min		max		min.	

Free Response (10 pts. each)

1. Find the domain and extreme points of $y=\frac{2x^3+x^2-2x-1}{x^2-1}$. $= \frac{x^2(2x+1)-(2x+1)}{(x-1)(x+1)}$

$y = \frac{(x^2-1)(2x+1)}{(x-1)(x+1)} = \frac{(x-1)(x+1)(2x+1)}{(x-1)(x+1)}$

Domain: $x \in (-\infty,-1)\cup(-1,1)\cup(1,\infty)$

$y \approx 2x+1$

Extreme Points: none

$y' = 2$ $\begin{array}{l} =0 \\ =\text{DNE} \\ =\text{EoaASD} \end{array}$

c.v.: $x=1,-1$

no e.v.s

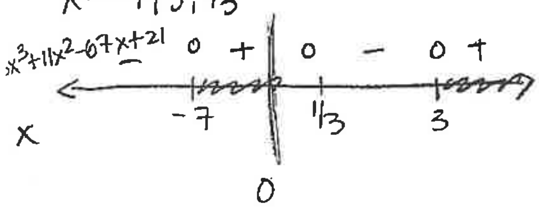
2. Find the domain, y - intercept, and extreme points of

$$f(x)=\begin{cases} x^2e^{-x} & x\le 0 \\ -\sqrt{3x^3+11x^2-67x+21} & x>0 \end{cases}.$$

$3x^3+11x^2-67x+21=0$

$(x+7)(x-3)(3x-1)=0$

$x=-7, 3, 1/3$



Domain: $x\in(-\infty,\frac{1}{3}]\cup[3,\infty)$

y - intercept: $(0,0)$

Extreme Points: $(0,0)$, $(\frac{1}{3},0)$, $(3,0)$

$-x^2e^{-x}+2xe^{-x}$

$$f'(x)=\begin{cases} x^2\cdot e^{-x}\cdot -1+e^{-x}\cdot 2x & x<0 \\ -\frac{1}{2}(3x^3+11x^2-67x+21)^{-1/2}\cdot(9x^2+22x-67) & x>0 \end{cases}$$

$$=\begin{cases} -xe^{-x}(x-2) & \text{if } x<0 \\ -\dfrac{9x^2+22x-67}{2(3x^3+11x^2-67x+21)^{1/2}} & \text{if } x>0 \end{cases}$$

$-xe^{-x}(x-2) \begin{array}{l} =0 \\ =\text{DNE} \\ \doteq \text{EOA ASD} \end{array}$

$-\dfrac{9x^2+22x-67}{2(3x^3+11x^2-67x+21)^{1/2}} \begin{array}{l} =0 \\ =\text{DNE} \\ \doteq \text{EOA ASD} \end{array}$

c.v.: $x=\cancel{0},\cancel{2}$

c.v.: $x=-4.\cancel{212}, 1.\cancel{767}, \cancel{-7}, 1/3, 3$ ✓ ✓

e.v.: $y=$ 0, 0

Note: $f'(x)=\text{DNE}$ @ $x=0 \Rightarrow (0,0)$ is an E.P.

3. Find the domain, zeros, and extreme points of $y = \ln(2x^2 - 2x - 12)$.

Domain ·

$2x^2 - 2x - 12 = 0$

$2(x^2 - x - 6) = 0$

$2(x-3)(x+2) = 0$

Domain: $x \in (-\infty, -2) \cup (3, \infty)$

Zeros: $(-2.098, 0)$, $(3.098, 0)$

Extreme Points: none

POIs: none

Zeros: $\ln(2x^2 - 2x - 12) = 0$

$e^{\ln(2x^2-2x-12)} = e^0$

$2x^2 - 2x - 12 = 1$

$2x^2 - 2x - 13 = 0$

$x = -2.098,\ 3.098$

EPs: $y' = \dfrac{4x-2}{2x^2-2x-12} = \dfrac{2(2x-1)}{2(x-3)(x+2)}$ $= 0$ $=$ DNE $=$ ~~EoaASD~~ –

c.v: $x = \cancel{\tfrac{1}{2}},\ \cancel{3},\ \cancel{-2}$

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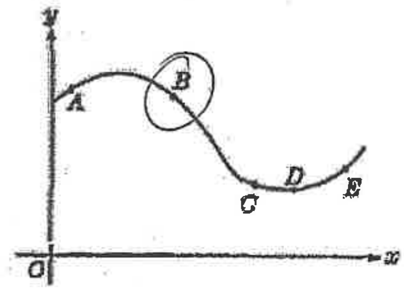
Honors Precalc
Sample Spring Final – Part 2
NO Calculator Allowed (10 pts. each)
Show all work. Round to 3 decimals.

Multiple Choice (3 pts. each)

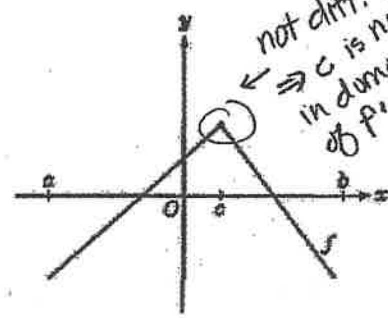
6. At which of the five points on the graph in the figure at right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?



decr. CD

(a) A
(b) B
(c) C
(d) D
(e) E

7. The function f, whose graph consists of two line segments, is shown at right. Which of the following are true for f on the open interval (a, b)?



not diff. ⇒ c is not in domain of f'

I. The domain of the derivative of f is the open interval (a, b).
II. f is continuous on the open interval (a, b). ✓
III. The derivative of f is positive on the open interval (a, c). ✓
incr. ✓

(a) I only
(b) II only
(c) III only
(d) II and III only
(e) I, II, and III

8. If r is positive and increasing, for what value of r is the rate of increase of r^3 twelve times that of r?

(a) $\sqrt[3]{4}$
(b) 2
(c) $\sqrt[3]{12}$
(d) $2\sqrt{3}$
(e) 6

$$\frac{d}{dt}(r^3) = 12\frac{dr}{dt}$$

$$3r^2\frac{dr}{dt} = 12\frac{dr}{dt}$$

$$3r^2 = 12$$

$$r^2 = 4$$

$$r = \pm 2 \Rightarrow r = 2$$

9. The slope of the tangent to the curve $y^3x+y^2x^2=6$ at $(2, 1)$ is

(a) $-\frac{3}{2}$

(b) -1

(c) $\frac{5}{14}$

(d) $-\frac{3}{14}$

(e) 0

$$\frac{d}{dx}\left(y^3x+y^2x^2=6\right)$$

$$y^3\cdot\frac{dx}{dx}+x\cdot 3y^2\frac{dy}{dx}+y^2\cdot 2x\cdot\frac{dx}{dx}+x^2\cdot 2y\frac{dy}{dx}=0$$

$$1^3\cdot 1+2\cdot 3\cdot 1^2\cdot\frac{dy}{dx}+1^2\cdot 2\cdot 2+2^2\cdot 2\cdot 1\frac{dy}{dx}=0$$

$$1+6\frac{dy}{dx}+4+8\frac{dy}{dx}=0$$

$$14\frac{dy}{dx}=-5$$

$$\frac{dy}{dx}=\frac{-5}{14}$$

10. If $f(x)$ and $g(x)$ are differentiable functions with values as given in the chart below, and $k(x)=f\left(g\left(x^2\right)\right)$, what is $k'(2)$?

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	4	$2/3$	$-5/2$
2	4	2	$4/3$	$-3/2$
4	8	1	$8/3$	$1/2$

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{16}{3}$

(e) None of the above

$$k'(x)=f'(g(x^2))\cdot g'(x^2)\cdot 2x$$

$$k'(2)=f'(g(4))\cdot g'(4)\cdot 4$$

$$=f'(1)\cdot\frac{1}{2}\cdot 4$$

$$=\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\cdot 4=\frac{4}{3}$$

Free Response (10 pts. each)

4. Find the traits and sketch $f(x)=\begin{cases} x^2e^{-x} & x\le 0 \\ -\sqrt{3x^3+11x^2-67x+21} & x>0 \end{cases}$.

Domain: $x \in (-\infty, \frac{1}{3}] \cup [3, \infty)$

y - intercept: $(0,0)$

$$\lim_{x\to\infty} -\sqrt{3x^3+11x^2-67x+21} = -\infty$$

Zeros: $(0,0)$, $(\frac{1}{3},0)$, $(3,0)$

$$\lim_{x\to-\infty} x^2e^{-x} = \infty\cdot e^{\infty} = \infty$$

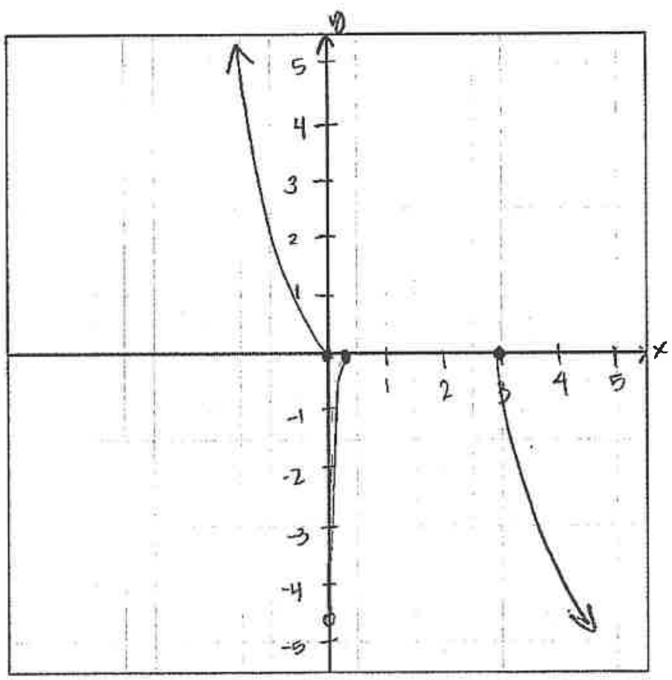
POEs: none

VAs: none

End Behavior: ↖ ↘

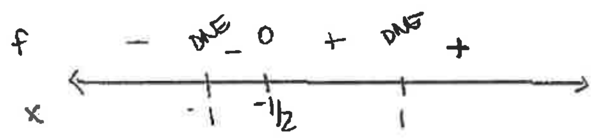
Extreme Points: $(0,0)$, $(1/3,0)$, $(3,0)$

Range: $y \in (-\infty, \infty)$

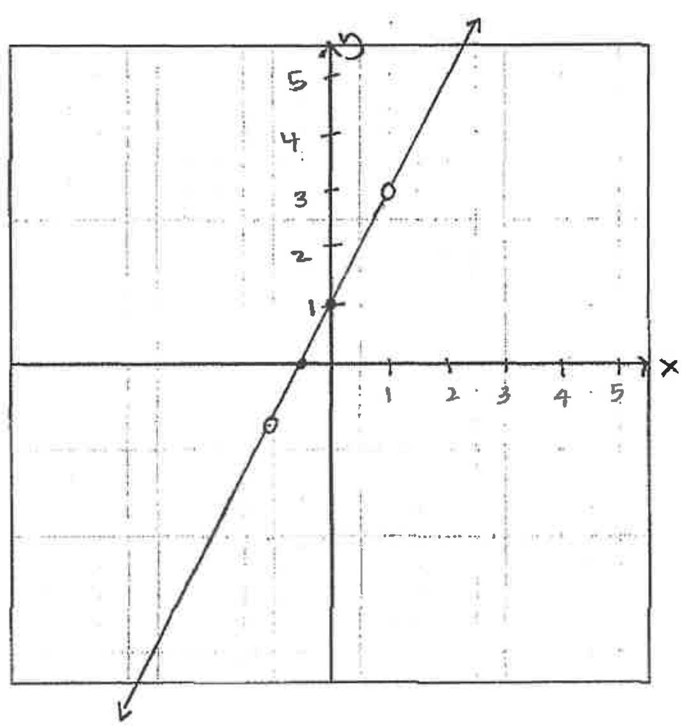


5. Show the sign patterns of f and f' for $y = \dfrac{2x^3 + x^2 - 2x - 1}{x^2 - 1}$ and use them to sketch the curve.

Sign Pattern for f:



Sign Pattern for f':



6. Show the sign patterns for the derivative, list all traits of $y = \ln(2x^2 - 2x - 12)$, and use them to sketch the curve.

Domain: $x \in (-\infty, -2) \cup (3, \infty)$

y - intercept: none

Zeros: $(-2.098, 0)$, $(3.098, 0)$

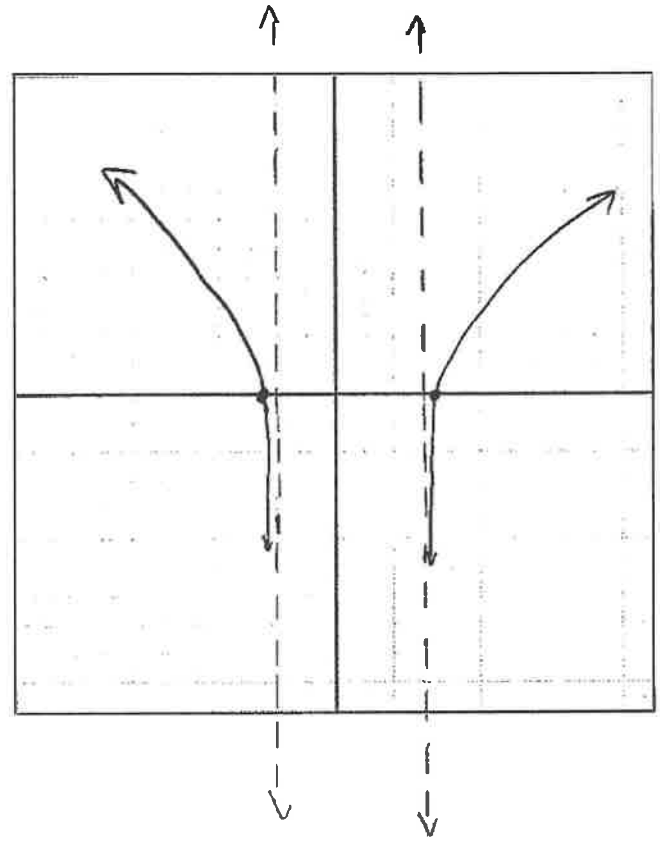
POEs: none

VAs: $x = -2$, $x = 3$

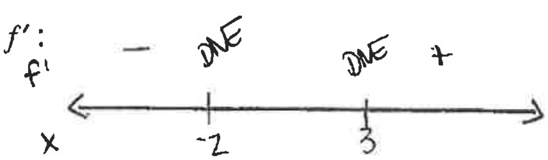
End Behavior: $\nwarrow$ $\nearrow$

Extreme Points: none

Range: $y \in (-\infty, \infty)$

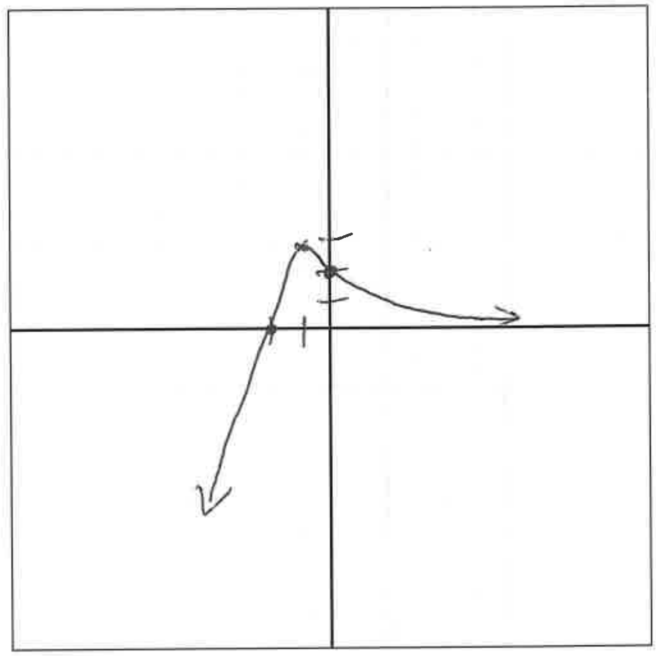


Sign Pattern for f':



$$y = e^{-x}(x+2)$$

7. Find the traits and sketch the graph of $y = xe^{-x} + 2e^{-x}$.



Domain: $x \in (-\infty, \infty)$

Zeros: $e^{-x}(x+2) = 0$

$x+2=0 \quad (-2,0)$

$x=-2$

Y-int: $e^{-0}(0+2) = 1 \cdot 2 = 2$

$(0,2)$

VAs: none

POEs: none

EB: $\lim\limits_{x \to \infty} e^{-x}(x+2) = 0$

$y=0$ on right

$\lim\limits_{x \to -\infty} e^{-x}(x+2) = e^{\infty}(-\infty) = -\infty$

down on left

Extreme Values:

$y' = e^{-x}(1) + (x+2)e^{-x}(-1)$

$= -e^{-x}[-1 + x + 2]$

$= -e^{-x}(x+1) \longrightarrow$

$-e^{-x}(x+1) = 0$

$x+1=0$

$x=-1$

$e^{-(-1)}(-1+2)$

$e(1) = e$

$(-1, e)$

2.718

Range:

$y \in (-\infty, e]$

8. Find $\frac{d}{dx}\left[\ln(\csc(2x^7))\right]$

$$\frac{d}{dx}\left[\ln(\csc(2x^7))\right] = \frac{1}{\csc(2x^7)}\left(-\csc 2x^7 \cot 2x^7\right) \cdot (14x^6)$$

$$= -14x^6 \cot 2x^7$$