Precalculus Accelerated
Fall Final

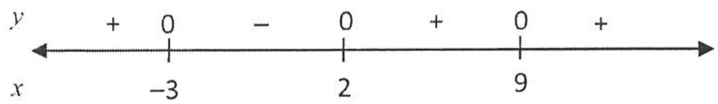
Name: Solution Key
12/20/17 Period:__________

A.M.D.G.

Instructions: Round all answers at three decimal places when displaying decimal values. "Exact value" answers should contain radicals and fractions, not decimals. All problems are 10 points each.

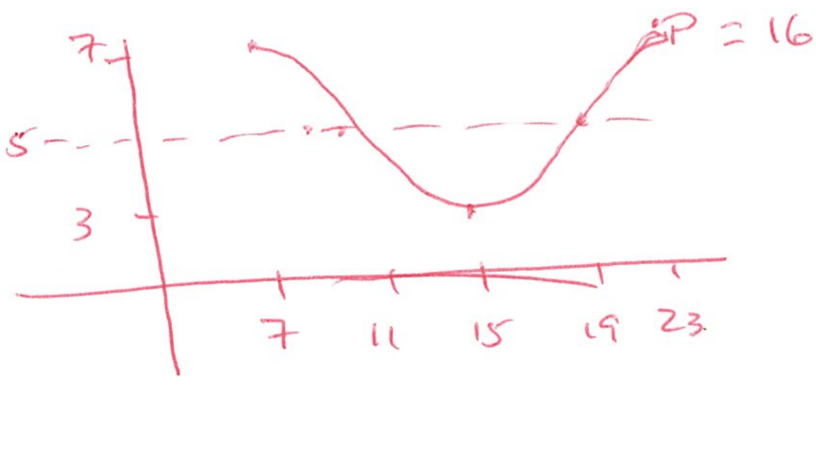
1) Find an inequality that has this sign pattern and solution:

$x \in (-\infty, -3] \cup [2, \infty)$



$(x+3)(x-2)(x-9)^2 \geq 0$

2) Sketch the primary cycle of $y = 5 + 2\cos\left[\frac{\pi}{8}(x-7)\right]$. Label the x- and y-axes appropriately.



3) Given the inequality $x^3 + 3x^2 - 4x - 12 \geq 0$,

a. Factor to find the zeros algebraically.

$x^2(x+3) - 4(x+3)$

$(x^2-4)(x+3)$

$(x-2)(x+2)(x+3)$

zeros: $(2,0)$ $(-2,0)$ $(-3,0)$

b. Show the sign pattern and solve.

y: − 0 + 0 − 0 +

x: −3, −2, 2

$x \in [-3,-2] \cup [2,\infty)$

4) Sketch the complete graph of $y = 2x^4 - 8x^3 - 6x^2 + 24x + 5$. Find the zeroes, critical values, y-intercept, and maximum/minimum value(s). State the window you used, and determine the range of this function.

Window: $x \in [-4.7, 4.7]$

$y \in [-50, 50]$

Zeroes: $(-1.653, 0)$ $(-.201, 0)$ $(1.904, 0)$ $(3.950, 0)$

y-intercept: $(0, 5)$

Critical Values: $x = -3.2, .889, -1.063$

Extreme Values: $y = -32.094, 17.223, 15.129$

Range: $y \in [-32.094, \infty)$

5) Evaluate each of the following limits:

a) $\lim_{h\to 0}\frac{3x^2h+3xh^2}{h}$

b) $\lim_{x\to 5}\frac{x^2-25}{x^2-8x+15} = \lim_{x\to 5}\frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x-3)}$

$= \frac{10}{2} = 5$

$= \lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh)}{\cancel{h}}$

$= \lim_{h\to 0} 3x^2+3xh = 3x^2$

6) Prove the following trig identity:

$\sin^2 A\csc A+\sin A\tan^2 A=\sin A\sec^2 A$

$\sin^2 A\left(\frac{1}{\sin A}\right)+\sin A\frac{\sin^2 A}{\cos^2 x} = \frac{\sin A}{\cos^2 A}$

$\sin A+\frac{\sin^3 A}{\cos^2 A}$

$\frac{\sin A\cos^2 A+\sin^3 A}{\cos^2 A}$

$\frac{\sin A(\cos^2 A+\sin^2 A)}{\cos^2 A} = \frac{\sin A(1)}{\cos^2 A}$

6) Use the power rule to find the derivatives of the following functions

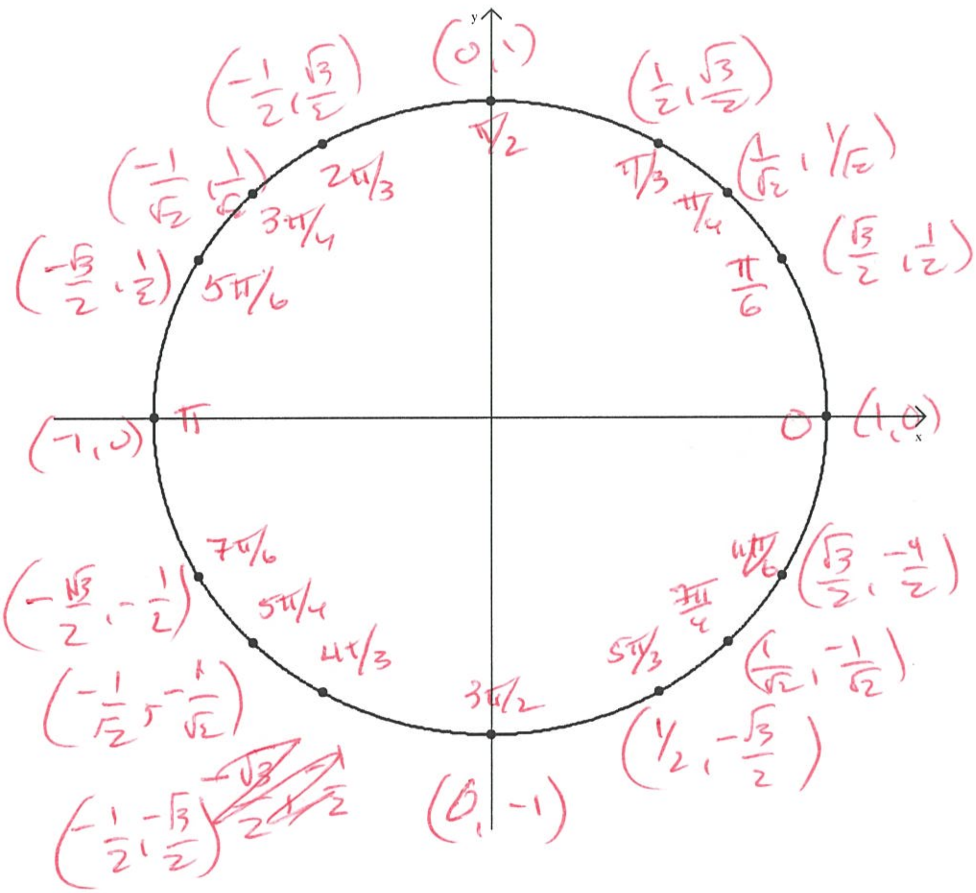
a) $f(x)=x^4-5x^2+15x+75$

$f'(x)=4x^3-10x+15$

b) $g(x)=10x^{4/5}-11x^{-3}+15x^3+19$

$g'(x= 8x^{-1/5}+33x^{-4}+45x^2$

7) Fill in the coordinates and the radian measures for the angles on the unit circle provided below.



8) Use the values on the unit circle above to evaluate each of the following. You must provide the exact value for each expression.

a) $12\cos\left(\frac{3\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)$

$= 12(0)(1) = 0$

b) $\tan^2\left(\frac{5\pi}{6}\right) - \sec^2\left(\frac{5\pi}{6}\right)$

$\left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{1}{3} - \frac{4}{3}$

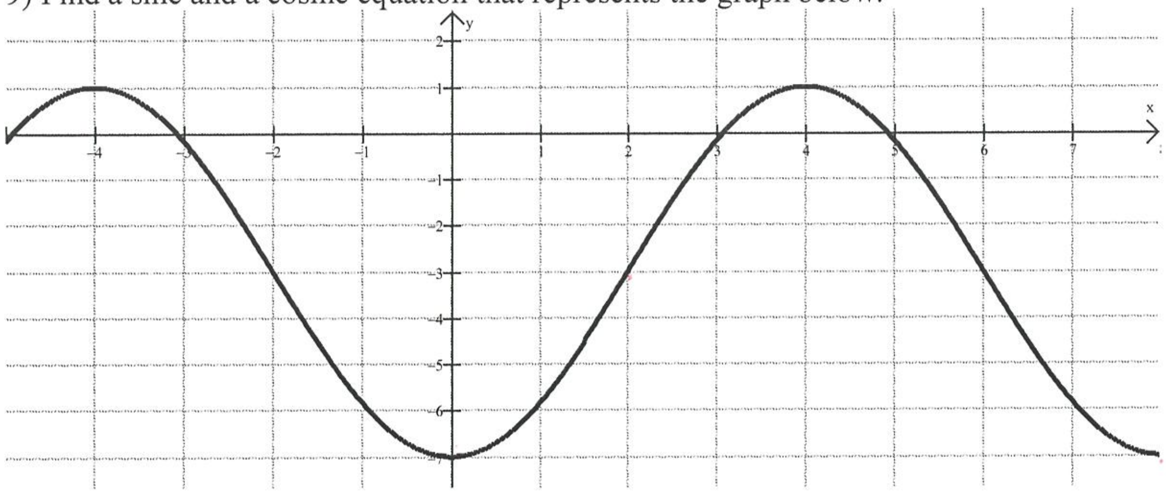
$= -1$

c) $\sin\left(\frac{3\pi}{4}\right) - \cos\left(\frac{5\pi}{4}\right)$

$+\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

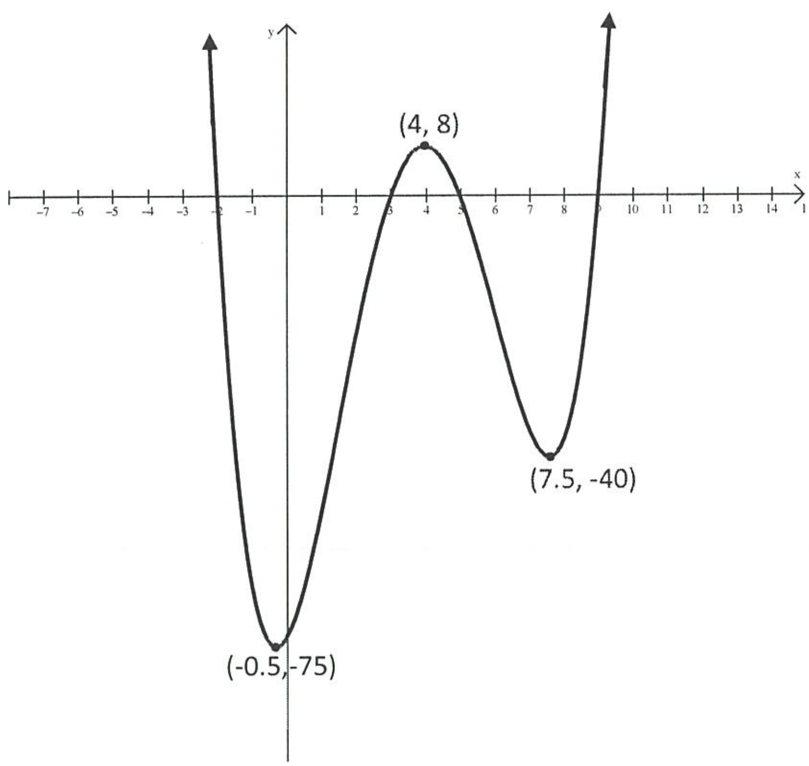
9) Find a sine and a cosine equation that represents the graph below.



$y = -3 + 4\sin\frac{\pi}{4}(x-2)$

$y = -3 + 4\cos\frac{\pi}{4}(x-4)$

10) For the graph illustrated below, list each of the following.



a) Critical Values: $x = -.5, 4, 7.5$

b)Maximum Value(s): 8

c) Minimum Value(s): -75 & -40

d) The Absolute Extreme Point: $(-.5, -7.5)$

e) x-intercepts: $(2,0)$, $(5,0)$ $(3,0)$ $(9,0)$

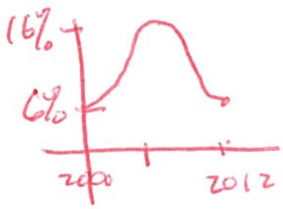
f) Range: $y \in [-75, \infty)$

11) Sunspot activity (changes in the magnetic field of the sun) is known to vary sinusoidally with time. Suppose that in the year 2000 (what we will call $t = 0$), the sunspot activity is at its minimum value of 6 percent coverage. 6 years later, sunspot activity is at its maximum value of 16 percent coverage.

a) Sketch the graph of at least one cycle of this function.

b) Find the equation of a function that represents this situation

c) Find the sunspot activity you would predict for 2018 (that is, $t = 18$), and 2020 (that is, $t = 20$)



b) $S = 11 - 5\cos\frac{\pi}{6}t$

c) $S(18) = 16\%$

$S(20) = 13.5\%$

12) If $(-5,-12)$ is on the terminal side of angle Q, find all six trig functions of Q and find the value of Q in radians.

$\sin Q = -12/13$ $\qquad$ $\csc Q = -13/12$

$\cos Q = -5/13$ $\qquad$ $\sec Q = -13/5$

$\tan Q = 12/5$ $\qquad$ $\cot Q = 5/12$

Q (in radians) = $-1.965 \pm 2\pi n$