PreCalculus Accelerated '17-18 Name: Solution Key

Dr. Quattrin

Limits and Derivatives Practice Test

CALCULATOR ALLOWED Score ___________

Round to 3 decimal places. Show all work.

1. Use the equation of the line tangent to $y=6x^3-3x^2+5x-4$ at $x=1$ to approximate $f(.9)$

$y(1)=6-3+5-4=4$

$\frac{dy}{dx}=18x^2-6x+5$

$m=18(1)^2-6(1)+5=17$

$y-4=17(x-1)$

$f(.9)\approx y=17(.9-1)+4=2.3$

2. The motion of a particle is described by $y(t)=2t^3+5t^2-4t+3$.

a) When the particle is stopped?

b) Which direction it is moving at $t=4$?

c) Where is it when $t=4$?

d) Find $a(4)$.

$V=6t^2+10t-4$

$a=12t+10$

a) $V=0 \Rightarrow 6t^2+10t-4=0$

$3t^2+5t-2=0 \rightarrow t=\frac{-5\pm\sqrt{5^2-4(3)(-2)}}{2(3)}=\frac{1}{3}$ or -2

$(3t-1)(t+2)=0$

$t=-2, 1/3$

b) $v(4)=6(4)^2+10(4)-4>0 \therefore$ RIGHT

c) $x(4)=2(4)^3+5(4)^2-4(4)+3=195$

d) $a(4)=12(4)+(10)=58$

3. Evaluate the following limits:

a) $\lim\limits_{x\to 1} \frac{x^2+2x-3}{x^2-1} =$ $\lim\limits_{x \to 1} \frac{(x+3)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

$= \frac{4}{2} = 2$

b) $\lim\limits_{x\to 3} \frac{x^3+27}{x^4-14x^2+45}$ $= \cancel{\lim\limits_{x\to 3}} = \frac{54}{0} = DNE$

$\lim\limits_{x \to -3} \frac{x^3+27}{x^4-14x^2+45}$

$= \lim\limits_{x \to -3} \frac{\cancel{(x+3)}(x^2+3x+9)}{(x^2-9)(x^2-5)} = \frac{27}{(-6)(31)} = \frac{-27}{186}$

$(x-3)\cancel{(x+3)}$

c) $\lim\limits_{x\to -1} \frac{3x^2+2x-1}{x^3+x^2+4x+4} =$ $\lim\limits_{x \to -1} \frac{(3x-1)(x+1)}{x^2(x+1)+4(x+1)}$

$= \lim\limits_{x\to -1} \frac{3x-1}{x^2+4} = \frac{-4}{5}$

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4. Set up, but do not solve, the limit definition of the derivative for $y = 5x^4 - x^3 + 7x^2 + 3^4$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\left(5(x+h)^4 - (x+h)^3 + 7(x+h)^2 + 3^4\right) - \left(5x^4 - x^3 + 7x^2 + 3^4\right)}{h}$$

5. Use the Power Rule to find:

a. $\frac{dy}{dx}$ if $y = 6x^7 - 19x^4 + 3x^2 - 12x - 13$

$$\frac{dy}{dx} = 42x^6 - 76x^3 - 6x - 12$$

b. $D_x\left[\sqrt[4]{x^7} - \frac{6}{x^5} - \sqrt[3]{x} + \pi^2 - x\right] = D_x\left[x^{7/4} - 6x^{-5} - x^{1/3} + \pi^2 - x\right]$

$$= \frac{7}{4}x^{3/4} + 30x^{-6} - \frac{1}{3}x^{-2/3} - 1$$

c. $\frac{d}{dx}\left[x^7 - 4\sqrt[8]{x^7} + 7^3 - \frac{1}{\sqrt[7]{x^4}} + \frac{1}{5x}\right] = D_x\left[x^7 - 4x^{7/8} + 7^3 - x^{-4/7} + \frac{1}{5}x^{-1}\right]$

$$= 7x^6 - \frac{7}{2}x^{-1/8} + 0 + \frac{4}{7}x^{-11/7} - \frac{1}{5}x^{-2}$$

6. Set up, but do not solve, the limit definition of the derivative of $y = \sqrt[4]{x^7} - \frac{6}{x^5} - \sqrt[3]{x} + \pi^2 x$

$$\lim_{h \to 0} \frac{\left[\sqrt[4]{(x+h)^7} - \frac{6}{(x+h)^5} - \sqrt[3]{x+h} + \pi^2(x+h)\right] - \left[\sqrt[4]{x^7} - \frac{6}{x^5} - \sqrt[3]{x} + \pi^2 x\right]}{h}$$

7. A particle's position at time t is described by $x(t) = t^4 - 4t^2 - 5$. When is the particle moving left? (Show the sign pattern of the velocity.)

$V = 4t^3 - 8t = 0$

$\cancel{4t}\ 4t(t^2 - 2) = 0$

$t = 0, \pm\sqrt{2}$

V	$-$	0	$+$	0	$-$	0	$+$
t		$-\sqrt{2}$		0		$\sqrt{2}$	

$t \in (-\infty, -\sqrt{2}) \cup (0, \sqrt{2})$