PreCalculus Accelerated
Dr. Quattrin
Limits and Derivatives Test
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name: Solution Key

Score ___________

1. Use the equation of the line tangent to $y = x^3 - 2x^2 + 5x - 4$ at $x = 1$ to approximate $f(1.1)$

$\cancel{f(2)} = f(1) = 1 - 2 + 5 - 4 = 0$

$f'(x) = 3x^2 - 4x + 5$

$m = f'(1) = 3 - 4 + 5 = 4$

$y - 0 = 4(x - 1)$

$f(1.1) \approx 4(1.1 - 1) = 0.4$

2. The motion of a particle is described by $x(t) = 2t^3 - 5t^2 - 56t + 8$.

a) When the particle is stopped?
b) Which direction it is moving at $t = 2$?
c) Where is it when $t = 2$?
d) Find $a(2)$.

$v(t) = 6t^2 - 10t - 56$

$a(t) = 12t - 10$

a) $v = 0 \rightarrow 6t^2 - 10t - 56$

$2(3t^2 - 5t - 28)$

$2(3t + 7)(t - 4) = 0$

$t = -7/3, 4$

b) $v(2) = 6(2)^2 - 10(2) - 56 = -52$, so LEFT

c) $x(2) = -108$

d) $a(2) = 14$

3. Evaluate the following limits:

a) $\lim_{x \to 4} \frac{x^2 - 16}{x^2 - x - 12} =$ $\lim_{x \to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}(x+3)}$

$= \frac{4+4}{4+3}$

$= \frac{8}{7}$

b) $\lim_{x \to 3} \frac{x^3 - 27}{4x^3 - 12x^2 - x + 3}$ $= \lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{4x^2(x-3) - 1(x-3)}$

$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}(4x^2-1)}$

$= \frac{9+9+9}{4(3)^2 - 1} = \frac{27}{35}$

c) $\lim_{x \to -5} \frac{x^3 + 5x^2 - 9x - 45}{2x^3 + 11x^2 - x - 30} =$ $\lim_{x \to -5} \frac{\cancel{(x+5)}(x^2-9)}{\cancel{(x+5)}(2x^2+x-6)}$

$= \lim_{x \to -5} \frac{x^2-9}{2x^2+x-6}$

$= \frac{(-5)^2 - 9}{2(-5)^2 + (-5) - 6} = \frac{16}{39}$

$$\begin{array}{r|rrrr} -5 & 1 & 5 & -9 & -45 \\ & & -5 & 0 & 45 \\ \hline & 1 & 0 & -9 & \emptyset \end{array}$$

$$\begin{array}{r|rrrr} -5 & 2 & 11 & -1 & -30 \\ & & -10 & -5 & 30 \\ \hline & 2 & 1 & -6 & \emptyset \end{array}$$

PreCalculus Accelerated
Dr. Quattrin
Limits and Derivatives Test
NO CALCULATOR ALLOWED

Name: Solution Key

4. Set up, but do not solve, the limit definition of the derivative for $y = 12x^7 - 11x^4 + x^2 - 31x$.

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\left[12(x+h)^7 - 11(x+h)^4 + (x+h)^2 - 31(x+h)\right] - \left[12x^7 - 11x^4 + x^2 - 31x\right]}{h}$$

5. A particle's position at time t is described by $x(t) = t^3 - 4t^2 + 5t - 20$. When is the particle moving left? (Show the sign pattern of the velocity.)

$$v(t) = 3t^2 - 8t + 5 = (3t-5)(t-1) = 0$$

$$t = 5/3,\ 1$$

v	+	0	−	0	+
t		1		5/3	

$$t \in (1, 5/3)$$

6. Use the Power Rule to find:

a) Find $\dfrac{dy}{dx}$ if $y = 5x^3 - 3x^2 + 2x + 73$

$$\frac{dy}{dx} = 15x^2 - 6x + 2$$

b) Find $f'(x)$ if $f(x) = 13x^{12} - 2x^3 + 7x + 3 + \dfrac{1}{5x}$ $= 13x^{12} - 2x^3 + 7x + 3 + \frac{1}{5}x^{-1}$

$$f'(x) = 156x^{11} - 6x^2 + 7 - \frac{1}{5}x^{-1}$$

c) $D_x\left[\sqrt[3]{x^5} - \dfrac{2}{\sqrt{x^3}} - \sqrt[5]{x} + \pi^2 + 2x\right] = D_x\left[x^{5/3} - 2x^{-3/2} - x^{1/5} + \pi^2 + 2x\right]$

$$= \frac{5}{3}x^{2/3} + 3x^{-5/2} + 0 + 2 - \frac{1}{5}x^{-4/5}$$

$$= \frac{5}{3}x^{2/3} + 3x^{-5/2} + 2 - \frac{1}{5}x^{-4/5}$$