

PreCalculus Accelerated

Dr. Quattrin

Limits and Derivatives Test

CALCULATOR ALLOWED

Round to 3 decimal places. Show all work.

Name: SOLUTION KEY

Score \_\_\_\_\_

1. Use the equation of the line tangent to  $y = x^3 - 2x^2 + 5x - 4$  at  $x = 1$  to approximate  $f(1.1)$

$$f(x) = f(1) = 1 - 2 + 5 - 4 = 0$$

$$f'(x) = 3x^2 - 4x + 5$$

$$m = f'(1) = 3 - 4 + 5 = 4$$

$$y - 0 = 4(x - 1)$$

$$f(1.1) \approx 4(1.1 - 1) = 0.4$$

2. The motion of a particle is described by  $x(t) = 2t^3 - 5t^2 - 56t + 8$ .

a) When the particle is stopped?

$$v(t) = 6t^2 - 10t - 56$$

b) Which direction it is moving at  $t = 2$ ?

$$a(t) = 12t - 10$$

c) Where is it when  $t = 2$ ?

d) Find  $a(2)$ .

a)  $v = 0 \rightarrow 6t^2 - 10t - 56$

$$2(3t^2 - 5t - 28)$$

$$2(3t+7)(t-4) = 0$$

$$t = -\frac{7}{3}, 4$$

b)  $v(2) = 6(2)^2 - 10(2) - 56 = -52$ , so LEFT

c)  $x(2) = -108$

d)  $a(2) = 14$

3. Evaluate the following limits:

a)  $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 - x - 12} = \lim_{x \rightarrow 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}(x+3)}$

$$= \frac{4+4}{4+3}$$

$$= \frac{8}{7}$$

b)  $\lim_{x \rightarrow 3} \frac{x^3 - 27}{4x^3 - 12x^2 - x + 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{4x^2(x-3) - 1(x-3)}$

$$= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{(x-3)}(4x^2 - 1)}$$

$$= \frac{9+9+9}{4(3)^2 - 1} = \frac{27}{35}$$

c)  $\lim_{x \rightarrow -5} \frac{x^3 + 5x^2 - 9x - 45}{2x^3 + 11x^2 - x - 30} = \lim_{x \rightarrow -5} \frac{\cancel{(x+5)}(x^2 - 9)}{\cancel{(x+5)}(2x^2 + x - 6)}$

$$\begin{array}{r} -5 \\[-1ex] 1 \ 5 \ -9 \ -45 \\[-1ex] -5 \ 0 \quad 45 \\[-1ex] 1 \ 0 \ -9 \ \emptyset \end{array}$$

$$= \lim_{x \rightarrow -5} \frac{x^2 - 9}{2x^2 + x - 6}$$

$$= \frac{(-5)^2 - 9}{2(-5)^2 + (-5) - 6} = \frac{16}{39}$$

$$\begin{array}{r} -5 \\[-1ex] 2 \ 11 \ -1 \ -30 \\[-1ex] -10 \ -5 \ 30 \\[-1ex] 2 \ 1 \ -6 \ \emptyset \end{array}$$

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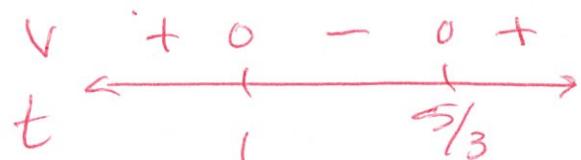
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4. Set up, but do not solve, the limit definition of the derivative for  $y = 12x^7 - 11x^4 + x^2 - 31x$ .

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{[12(x+h)^7 - 11(x+h)^4 + (x+h)^2 - 31(x+h)] - [12x^7 - 11x^4 + x^2 - 31x]}{h}$$

5. A particle's position at time  $t$  is described by  $x(t) = t^3 - 4t^2 + 5t - 20$ . When is the particle moving left? (Show the sign pattern of the velocity.)

$$V(t) = 3t^2 - 8t + 5 = (3t-5)(t-1) = 0$$
$$t = \frac{5}{3}, 1$$



$$t \in (1, \frac{5}{3})$$

6. Use the Power Rule to find:

a) Find  $\frac{dy}{dx}$  if  $y = 5x^3 - 3x^2 + 2x + 73$

$$\frac{dy}{dx} = 15x^2 - 6x + 2$$

b) Find  $f'(x)$  if  $f(x) = 13x^{12} - 2x^3 + 7x + 3 + \frac{1}{5x}$   $= 13x^{12} - 2x^3 + 7x + 3 + \frac{1}{5}x^{-1}$

$$f'(x) = 156x^{11} - 6x^2 + 7 - \frac{1}{5}x^{-2}$$

c)  $D_x \left[ \sqrt[3]{x^5} - \frac{2}{\sqrt{x^3}} - \sqrt[5]{x} + \pi^2 + 2x \right] = D_x \left[ x^{5/3} - 2x^{-3/2} + \pi^2 + 2x \right]$

$$= \frac{5}{3}x^{2/3} + 3x^{-5/2} + 0 + 2 - \frac{1}{5}x^{-4/5}$$

$$= \frac{5}{3}x^{2/3} + 3x^{-5/2} + 2 - \frac{1}{5}x^{-4/5}$$