PreCalculus Acc 17'-18'   Name: Solution Key

Dr. Quattrin

Limits and Derivatives Test

CALCULATOR ALLOWED   Score _____

Round to 3 decimal places.   Show all work.

1.   Find the zeros of $y = 3x^3 - x^2 - 12x + 4$. Show the algebraic work to support the zeros.

$$y = x^2(3x-1) - 4(3x-1) = 0$$

$$= (x^2 - 4)(3x - 1) = 0$$

$$x = \pm 2, \frac{1}{3}$$

$$(\pm 2, 0) \quad (\frac{1}{3}, 0)$$

2.   Find the extreme points of $y = 3x^3 - x^2 - 12x + 4$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = 9x^2 - 2x - 12 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4(9)(-12)}}{2(9)} = \begin{cases} 1.271 \\ -1.049 \end{cases}$$

$$(1.271, -6.708)$$

$$(-1.049, 12.025)$$

3.   Find the zeros of $y = x^4 - 29x^2 + 100$. Show the algebraic work to support the zeros.

$$(x^2 - 25)(x^2 - 4) = 0$$

$$x = \pm 5, \pm 2$$

$$(\pm 5, 0)$$

$$(\pm 2, 0)$$

4.   Find the extreme points of $y = x^4 - 29x^2 + 100$. Show the derivative and algebra to support the critical values.

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$$\frac{dy}{dx} = 4x^3 - 58x = 0$$

$$2x(2x^2 - 29) = 0$$

$$x = 0, \pm 3.807$$

$(0, 100)$

$(\pm 3.807, -110.25)$

5. At SI, there was an outbreak of the norovirus. The number of members of the school community as well as the numbers of siblings and members of other infected people outside of SI is given by the function $N(t) = \frac{1}{6}t^3 - 20t^2 + 580t + 1$ on $t \in [0,48]$, where N is the number of people infected, and t is the number of hours since the outbreak started.

   a) Find all times at which you have a maximum or minimum number of people infected.
   b) Find the absolute maximum number of people infected.
   c) Find the interval of time when the number of people infected with the norovirus is increasing.

a) $N' = \frac{1}{2}t^2 - 40t + 580 = 0$

$$t = \frac{40 \pm \sqrt{40^2 - 4(\frac{1}{2})580}}{2(\frac{1}{2})} = \begin{cases} 19.029 \\ \cancel{60.976} \text{ NOT IN DOMAIN} \end{cases}$$

DON'T FORGET THE ENDPOINTS : $t = 0, 48$

N' : $+$ 0 $-$ 0 $+$

t : 0 ——— 19.029 ——— 60.976 ——— 48
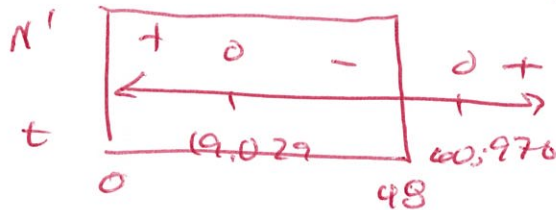
b) ONLY MAX IS AT $t = 19.029$

MAX # $= N(19.029) = 4944.173$

c) $t \in [0, 19.029)$

6.　　　The sign pattern for the derivative of $H(x)$ is given. (a) Is $x=-6$ at a maximum, a minimum, or neither? Why? (b) Is $x=3$ at a maximum, a minimum, or neither? Why?

$$G'(x) \quad \xleftarrow{\quad -\ 0\ -\ 0\ +\ 0\ -\quad}$$
$$x \qquad\qquad -6 \quad \tfrac{1}{4} \quad 3$$

a) $x=-6$ IS NEITHER, BECAUSE THE SIGN of G' DOES NOT CHANGE

b) $x=3$ IS AT A MAX BECAUSE G' SWITCHES $+$ TO $-$

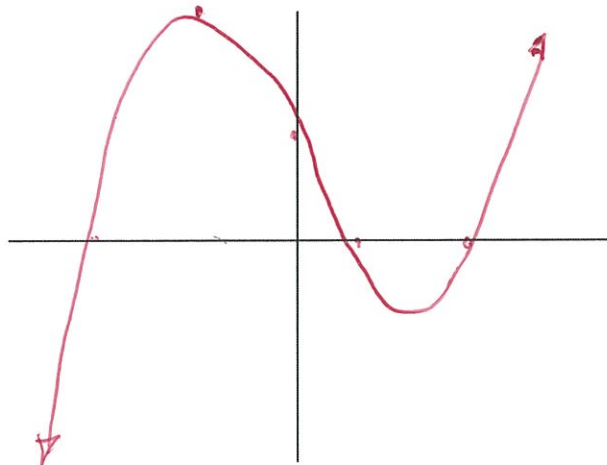7.　　　Find the traits and **sketch** $y = 3x^3 - x^2 - 12x + 4$.

Domain: $x \in$ ALL REALS 　　　　　　Range: ALL REALS

$Y-$ Int: $(0,4)$ 　　　　　　　　　　End Behavior: LEFT DOWN $/$ RT UP)

Zeros: $(\pm 2, 0)$ $(\tfrac{1}{3}, 0)$ 　　　　　Extreme Points: SEE #2

8. Find the traits and **sketch** of $y = x^4 - 29x^2 + 100$.

Domain: ALL REAL

Range: $x \in [-140.25, \infty)$

Y – Int: $(0, 100)$

End Behavior: BOTH ENDS UP

Zeros: SEE #3

Extreme Points: SEE #4