

1. Find the zeros and VAs of $y = \frac{4x^2 - 16x}{x^3 - 4x^2 + x - 4}$. Show the supporting algebraic work.

Zeros: $(0, 0)$

$$\frac{4x(x-4)}{x^2(x-4) + (x-4)} = \frac{4x(x-4)}{(x^2+1)(x-4)}$$

VAs: NONE

POE: $(4, \frac{16}{17})$

2. Find the extreme points of $y = \frac{4x^2 - 16x}{x^3 - 4x^2 + x - 4}$. Show the derivative and algebra to support the critical values.

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left[\frac{4x}{x^2+1} \right] = \frac{(x^2+1)(4) - 4x(2x)}{(x^2+1)^2} \\ &= \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2} \\ &= \frac{-4x^2 + 4}{(x^2+1)^2} \end{aligned}$$

i) $-4x^2 + 4 = 0$

$$\begin{aligned} x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

$$\begin{aligned} (1, 2) \\ (-1, -2) \end{aligned}$$

ii) $x^2 + 1 = 0 \rightarrow \text{NEVER}$

iii) END POINTS $\rightarrow \text{NONE}$ Given

3. Find the equations of the lines tangent to and normal to $y = \frac{2x^2 - x - 3}{3 + 2x - x^2}$ at $x = 0$?

Tangent: $y + 1 = \frac{1}{3}(x - 0)$

Normal: $y + 1 = 3(x - 0)$

$$\approx \frac{(2x-3)(x+1)}{-(x-3)(x+1)}$$

$$\approx \frac{2x-3}{3-x}$$

$$\frac{dy}{dx} = \frac{(3-x)(2) - (2x-3)(-1)}{(3-x)^2}$$

$$= \frac{6-2x+2x-3}{(3-x)^2}$$

$$= \frac{3}{(3-x)^2}$$

$$m_{\text{tan}} = \frac{1}{3}$$

4. Find the zeros, VAs, POEs and HA of $y = \frac{16-x^2}{x^2-25}$. Show the supporting algebraic work.

Zeros: $(\pm 4, 0)$

$$y = \frac{(4-x)(4+x)}{(x-5)(x+5)}$$

VAs: $x = \pm 5$

HA: $y = -1$

POE: NONE

5. Find the extreme points of $y = \frac{16-x^2}{x^2-25}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = \frac{(x^2-25)(-2x) - (16-x^2)(2x)}{(x^2-25)^2}$$

$$= \frac{-2x^3 + 50x + 2x^3 - 32x}{(x^2-25)^2}$$

$$= \frac{16x}{(x^2-25)^2}$$

i) $\frac{dy}{dx} = 0 \rightarrow 16x = 0 \Rightarrow x = 0 \quad (0, -\frac{16}{25})$

ii) $\frac{dy}{dx}$ DNE $\rightarrow x^2-25=0 \rightarrow x = \pm 5$ BUT THESE
ARE VAS

iii) ENDPOINTS : NONE GIVEN

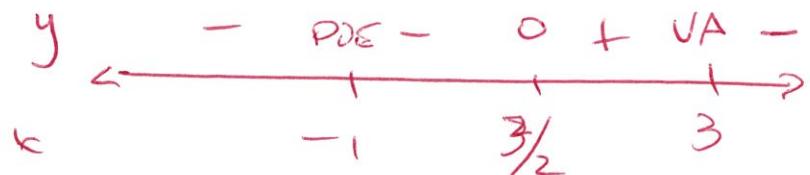
$\therefore (0, -\frac{16}{25})$

6. Write an equation of a rational function that has x -intercepts at $(-2, 0)$, VA at $x = 4$, a POE at $x = -3$, and a HA at $y = \frac{3}{7}$.

$$y = \frac{3(x+2)(x+3)}{7(x-4)(x+3)}$$

7. Show the sign pattern and solve $\frac{2x^2-x-3}{3+2x-x^2} \leq 0$.

$$\frac{(2x-3)(x+1)}{-(x-3)(x+1)}$$



$$x \in (-\infty, -1) \cup (-1, \frac{3}{2}] \cup (3, \infty)$$

8. Find the traits and sketch $y = \frac{16-x^2}{x^2-25}$.

Domain: $x \neq \pm 5$

Y-Intercept: $(0, -\frac{16}{25})$

Zeros: $(\pm 4, 0)$

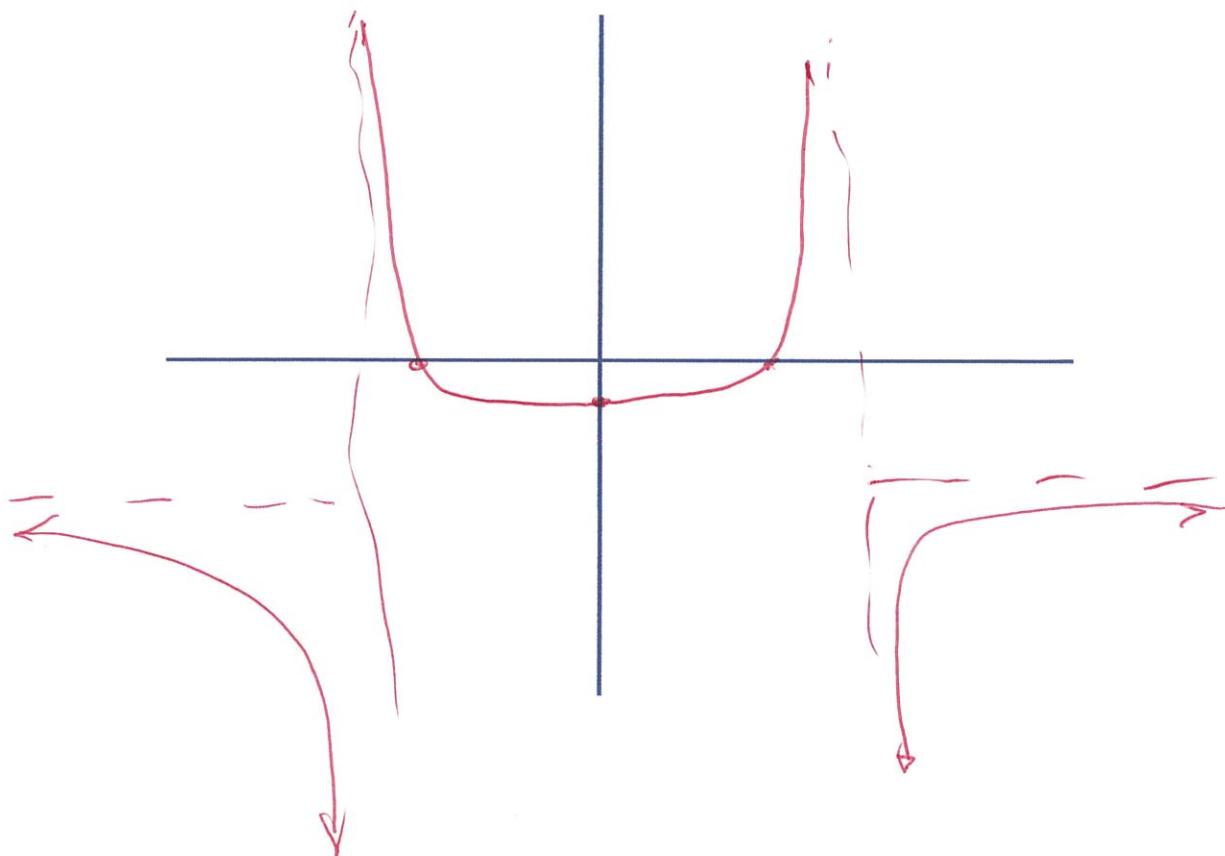
VAs: $x = \pm 5$

Range: $y \in (-\infty, -1) \cup \left[-\frac{16}{25}, \infty\right)$

End Behavior: $y = -1$

Extreme Points: $(0, -\frac{16}{25})$

POE: NONE



9. Find the traits and sketch of $y = \frac{4x^2 - 16x}{x^3 - 4x^2 + x - 4}$.

Domain: $x \neq 4$

Y-Intercept: $(0, 0)$

Zeros: $(0, 0)$

Range: $y \in [-2, \frac{16}{17}) \cup (\frac{16}{17}, 2]$

VAs: none

End Behavior: $y = 0$

POEs: $(4, \frac{16}{17})$

Extreme Points: $(1, 2) (-1, -2)$

