

PreCalc ACC '18
Spring Final – Part 1
Calculator Allowed

Name: SOLUTION KEY
score _____

Show all work. Round to 3 decimals.

1. Find the following derivatives:

a. $\frac{d}{dx}(\sec 4x^2) = \sec 4x^2 \tan 4x^2 (8x) = 8x \sec 4x^2 \tan 4x^2$

b. $\frac{d}{dx}(\ln(x^3 + 4x)) = \frac{1}{x^3 + 4x} (3x^2 + 4) = \frac{3x^2 + 4}{x^3 + 4x}$

c. $\frac{d}{dx}(e^{-2x} \cos x) = e^{-2x} (-\sin x) + \cos x e^{-2x} (-2)$
 $= -e^{-2x} (\sin x + 2 \cos x)$

d. $\frac{d}{dx}\left(\frac{e^{3x}}{16+x^2}\right) = \frac{(16+x^2)e^{3x}(3) - e^{3x}(2x)}{(16+x^2)^2}$
 $= \frac{e^{3x} [3x^2 - 2x + 48]}{(16+x^2)^2}$

You must do problems #3, #4, and #5.

3. Find domain and zeros of $f(x) = x^3 - x^2 - 16x + 16$ on $x \in [-5, 6]$.

$$x^2(x-1) - 16(x-1)$$

$$\text{Zeros: } (\pm 4, 0) (1, 0)$$

$$\text{Domain } x \in [-5, 6]$$

4. Find the extreme points of $f(x) = x^3 - x^2 - 16x + 16$ on $x \in [-5, 6]$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = 3x^2 - 2x - 16$$

$$i) \frac{dy}{dx} = 0 \rightarrow x = \frac{2 \pm \sqrt{4 - 4(3)(-16)}}{2(3)} = \frac{8}{3}, -2 \quad (-2, 36)$$

$$\left(\frac{8}{3}, -14.815\right)$$

$$ii) \frac{dy}{dx} \text{ DNE} \rightarrow \text{NONE}$$

$$(-5, -54)$$

$$(6, 100)$$

$$iii) \text{ ENDPOINTS: } x = -5, 6$$

5. Find the Point of Inflection for $f(x) = x^3 - x^2 - 16x + 16$. Show the algebraic work to support the result.

$$f''(x) = 6x - 2 = 0$$

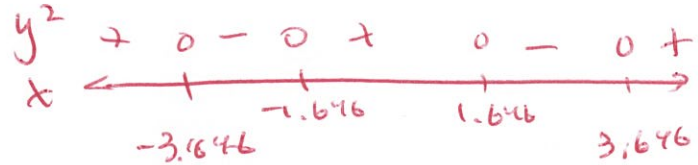
$$\left(\frac{1}{3}, 10.593\right)$$

$$x = \frac{1}{3}$$

You may skip either # 6 and #7, or #8 and #9, or #10 and #11.

6. Find domain and zeros of $y = \sqrt{x^4 - 16x^2 + 36}$.

$$\begin{aligned} & (\pm 3.646, 0) \\ & (\pm 1.646, 0) \end{aligned}$$



$$x \in (-\infty, -3.646] \cup [-1.646, 1.646] \cup [3.646, \infty)$$

7. Find the extreme points of $y = \sqrt{x^4 - 16x^2 + 36}$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = \frac{1}{2} (x^4 - 16x^2 + 36)^{-1/2} (4x^3 - 32x) = \frac{2x^3 - 16x}{(x^4 - 16x^2 + 36)^{1/2}}$$

$$i) \frac{dy}{dx} \Rightarrow x = 0, \pm 1.646 \quad (0, 6)$$

$$ii) \frac{dy}{dx} \Rightarrow x = \pm 3.646, \pm 1.646 \quad (\pm 1.646, 0) \quad (\pm 3.646, 0)$$

iii) ENDPOINTS: NONE

8. Find domain, VAs, POEs, and zeros of $g(x) = \frac{2x^2 - 9x - 5}{x^2 - 25}$.

$$\frac{(2x+1)(x-5)}{(x-5)(x+5)}$$

Domain $x \neq \pm 5$

VA: $x = 5$

POE $(-5, 9/10)$

Zero: $(-1/2, 0)$

9. Find the extreme points of $g(x) = \frac{2x^2 - 9x - 5}{x^2 - 25}$. Show the algebraic work to support the critical values.

$$\approx \frac{2x+1}{x+5}$$

$$\frac{dy}{dx} = \frac{(x+5)(2) - (2x+1)(1)}{(x+5)^2} = \frac{9}{(x+5)^2}$$

i) $\frac{dy}{dx} = 0 \rightarrow$ NONE

ii) $\frac{dy}{dx}$ DNE $\rightarrow x = -5$ BUT THIS IS A VA ON $g(x)$

iii) ENDPOINTS: NONE

10. Find domain, VAs, and zeros of $y = (x^2 - 6x)e^{x+1}$.

Domain: All Real

Zeros: $(0, 0)$ $(6, 0)$

VAs: NONE

11. Find the extreme points of $y = (x^2 - 6x)e^{x+1}$. Show the algebraic work to support the critical values.

$$\begin{aligned} \frac{dy}{dx} &= (x^2 - 6x)e^{x+1} + e^{x+1}(2x - 6) \\ &= e^{x+1}(x^2 - 4x - 6) \end{aligned}$$

$$i) \frac{dy}{dx} = 0 \rightarrow x = \frac{+4 \pm \sqrt{16 + 24}}{2(1)} = \begin{cases} -1.162 \\ \cancel{+1.162} \\ 5.162 \\ \cancel{-5.162} \end{cases} \begin{matrix} (-1.162, \overset{7.078}{\cancel{-42.845}}) \\ (+5.162, \overset{-2052.034}{\cancel{-2052.034}}) \end{matrix}$$

ii) $\frac{dy}{dx}$ NONE NONE

iii) $\frac{dy}{dx}$ ENDPOINTS: NONE

PreCalc ACC '18
 Spring Final – Part 2
 NO Calculator Allowed

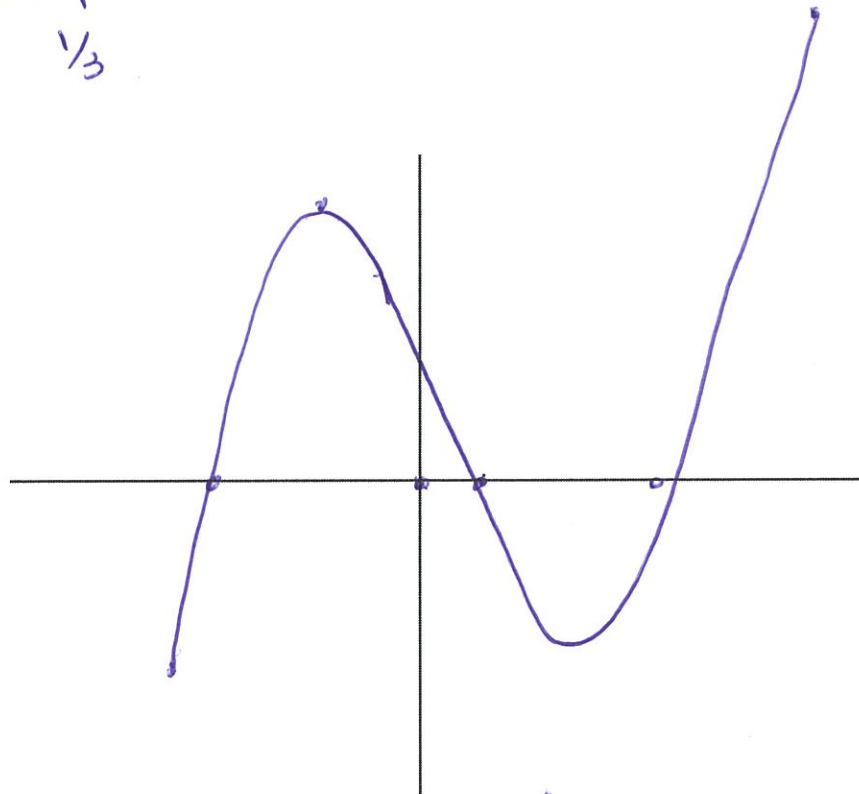
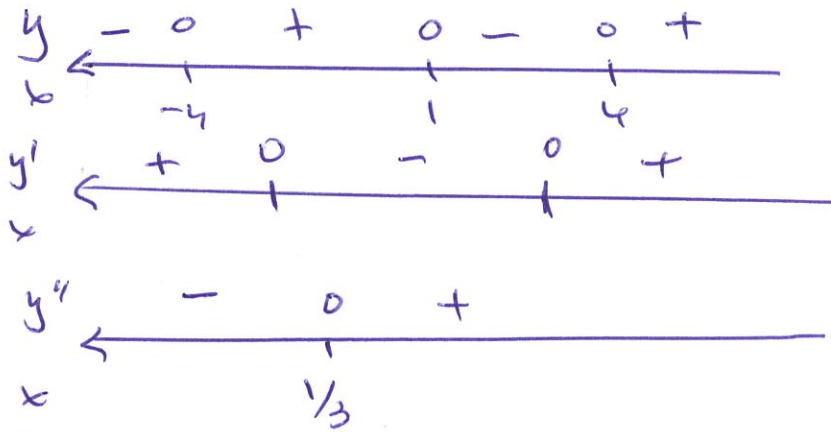
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Show all work. Round to 3 decimals.

You must do #12.

12. Find the traits and sketch $f(x) = x^3 - x^2 - 16x + 16$ on $x \in [-5, 6]$.

Show the sign patterns for $f(x)$, $f'(x)$, and $f''(x)$.



You may skip either #13, #14, or #15.

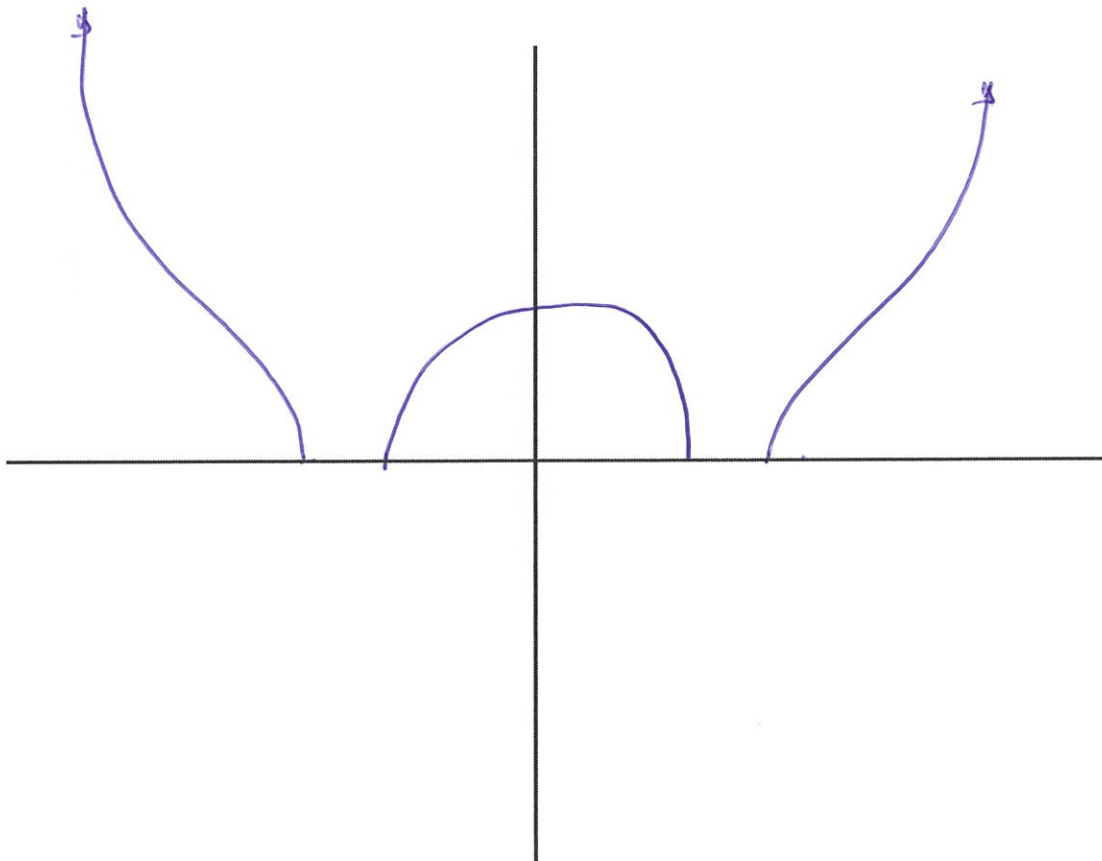
13. Find the traits and sketch of $y = \sqrt{x^4 - 16x^2 + 32}$.

Y-intercept: $(0, 4\sqrt{2})$

Range: $y \in [0, \infty)$

End Behavior (Left): \downarrow

End Behavior (Right): \uparrow



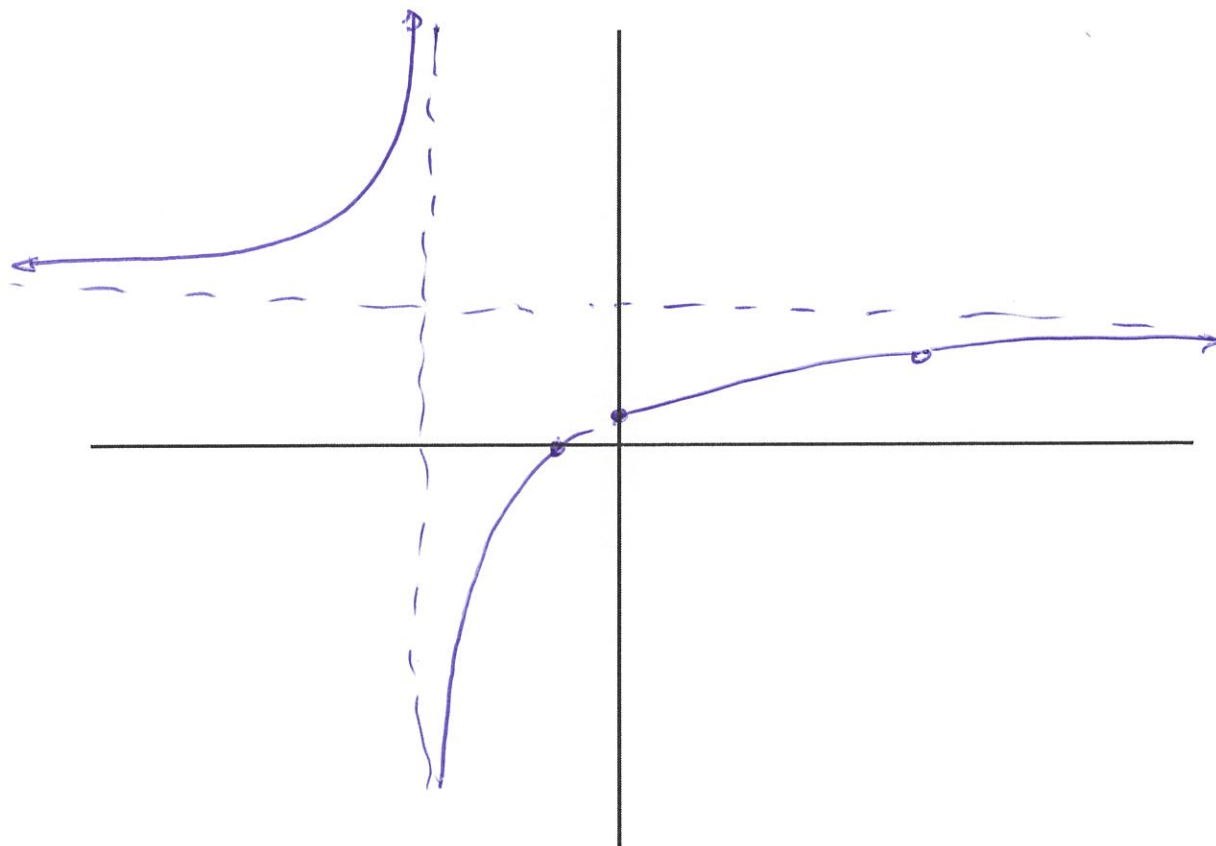
14. Find the traits and **Sketch** $g(x) = \frac{2x^2 - 9x - 5}{x^2 - 25}$.

Y-intercept: $(0, 1/5)$

Range: $y \in (-\infty, 9/10) \cup (9/10, 2) \cup (2, \infty)$

End Behavior (Left): $y = 2$

End Behavior (Right): $y = 2$



15. Find the Traits and **Sketch** of $y = (x^2 - 6x)e^{x+1}$.

Y-intercept: $(0, 0)$

Range: $y \in [-2052.034, \infty)$

End Behavior (Left): $y = 0$

End Behavior (Right): UP

