

## DIFFERENTIATION FORMULAS:

$$\frac{d}{dx} \cos u = -\sin u \, Du$$

$$\frac{d}{dx} e^u = e^u \, Du$$

$$\frac{d}{dx} (u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{d}{dx} a = 0$$

$$\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} \, Du$$

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \, Du - u \, Dv}{v^2}$$

$$\frac{d}{dx} \sin u = \cos u \, Du$$

$$\frac{d}{dx} a^u = a \, Du$$

$$\frac{d}{dx} \tan u = \sec^2 u \, Du$$

$$\frac{d}{dx} \cot^{-1} u = \frac{-1}{1+u^2} \, Du$$

$$\frac{d}{dx} \sec^{-1} u = \frac{1}{u\sqrt{u^2-1}} \, Du$$

$$\frac{d}{dx} \csc u = -\csc u \cot u \, Du$$

$$\frac{d}{dx} (uv) = u \, Dv + v \, Du$$

$$\frac{d}{dx} u^n = n u^{n-1} \, Du$$

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \, Du$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \, Du$$

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \, Du$$

$$\frac{d}{dx} \cot u = -\csc^2 u \, Du$$

$$\frac{d}{dx} \sec u = \sec u \tan u \, Du$$

$$\frac{d}{dx} \csc^{-1} u = \frac{-1}{u\sqrt{u^2-1}} \, Du$$

PreCalculus ACC '18-19

Name: SOLUTION KEY

Chapter 10 Test

CALCULATOR ALLOWED

Score \_\_\_\_\_

Round to 3 decimal places. Show all work.

Find the derivative of each of the following functions.

1a.  $f(x) = e^{5x^2} + x^4 = e^{5x^2}(10x) + 4x^3$

1b.  $g(x) = \cos^4 x = 4 \cos^3 x (-\sin x)$   
 $= -4 \cos^3 x \sin x$

1c.  $h(x) = \cot^4 x = (-\csc^2 x^4)(4x^3)$   
 $= -4x^3 \csc^2 x^4$

1d.  $p(x) = x^4 \sec x = x^4 (\sec x \tan x) + \sec x (4x^3)$   
 $= x^3 \sec x (x \tan x + 4)$

2. Find domain and  $x$ -intercepts of  $y = (2-x)\sqrt{16-x^2}$ .

$x$ -INTS:  $(2, 0), (\pm 4, 0)$

DOMAIN  $x \in [-4, 4]$

3. Find the extreme points of  $y = (2-x)\sqrt{16-x^2}$ . Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = (2-x) \left( \frac{-x}{(16-x^2)^{1/2}} \right) + (16-x^2)^{1/2} (-1)$$

$$\begin{aligned} u &= 2-x \\ du &= -1 \\ v &= (16-x^2)^{1/2} \\ Dv &= \frac{1}{2} (16-x^2)^{-1/2} (-2x) \end{aligned}$$

$$= \frac{x^2 - 2x}{(16-x^2)^{1/2}} + (-1)(16-x^2)^{1/2}$$

$$= \frac{x^2 - 2x - 16 + 16x^2}{(16-x^2)^{1/2}} = \frac{17x^2 - 2x - 16}{(16-x^2)^{1/2}}$$

i)  $17x^2 - 2x - 16 = 0$

$$x = \frac{2 \pm \sqrt{2^2 - 4(17)(-16)}}{2(17)} = \begin{cases} \cancel{2.361} \\ \cancel{-1.694} \\ 3.372 \\ -2.372 \end{cases}$$

~~$(2.361, -1.165)$~~

~~$(-1.694, 13.386)$~~

$(3.372, -2.952)$

$(\pm 4, 0)$   $(-2.372, 14.081)$

ii)  $16-x^2=0 \Rightarrow x = \pm 4$

iii) NO RESTRICTION

4. Find domain and  $x$ -intercepts of  $y = (x^2 - 2x - 3)e^{\frac{1}{2}x}$ .

Domain: All Real Numbers

Zeros  $(x-3)(x+1)$

$(3, 0)$   $(-1, 0)$

5. Find the extreme points of  $y = (x^2 - 2x - 3)e^{\frac{1}{2}x}$ . Show the algebraic work to support the critical values.

$$u = x^2 - 2x - 3 \quad v = e^{\frac{1}{2}x}$$

$$D_u = 2x - 2 \quad D_v = \frac{1}{2}e^{\frac{1}{2}x}$$

$$\begin{aligned} \frac{dy}{dx} &= (x^2 - 2x - 3) \left( \frac{1}{2}e^{\frac{1}{2}x} \right) + (2x - 2) e^{\frac{1}{2}x} \\ &= e^{\frac{1}{2}x} \left[ \frac{1}{2}x^2 - x - \frac{3}{2} + 2x - 2 \right] = e^{\frac{1}{2}x} \left( \frac{1}{2}x^2 + x - \frac{7}{2} \right) \end{aligned}$$

$$i) \frac{1}{2}x^2 + x - \frac{7}{2} = 0 \rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4(\frac{1}{2})(-\frac{7}{2})}}{1} = \begin{cases} 1.828 \\ -3.828 \end{cases}$$

ii) none

iii) none

$$(1.828, -8.267)$$

$$(-3.828, 2.848)$$

6. Find domain, VAs, and  $x$ -intercepts of  $y = \ln(x^3 + 4x^2 - 9x - 36)$ .

Domain:  $x \in (-4, -3) \cup (3, \infty)$

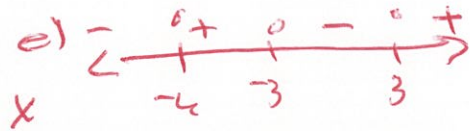
VA:  $x = \pm 3, -4$

$x$ -int:  $(-3.822, 0)$

$(-3.202, 0)$

$(3.202, 0)$

$x^2(x+4) - 9(x+4)$



zeros:  $x^3 + 4x^2 - 9x - 36 = 0$

7. Find the extreme points of  $y = \ln(x^3 + 4x^2 - 9x - 36)$  on  $x \in (-4, 4]$ . Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = \frac{3x^2 + 8x - 9}{x^3 + 4x^2 - 9x - 36}$$

$$i) 3x^2 + 8x - 9 = 0 \rightarrow x = \frac{-8 \pm \sqrt{8^2 - 4(3)(-9)}}{2(3)} = \begin{cases} 0.852 \\ -3.519 \end{cases}$$

ii)  $x^3 + 4x^2 - 9x - 36 = 0$   
 ~~$x = \pm 3, -4$~~

$(-3.519, 0.487)$   
 $(4, 4.025)$

iii)  ~~$x = -4$~~   $x = 4$

PreCalculus ACC '18-19  
Chapter 10 Test  
NO CALCULATOR ALLOWED

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8. Find the traits and **sketch**  $y = (2-x)\sqrt{16-x^2}$ .

Domain:  $x \in [-4, 4]$

Range:  $y \in [-2.952, 14.081]$

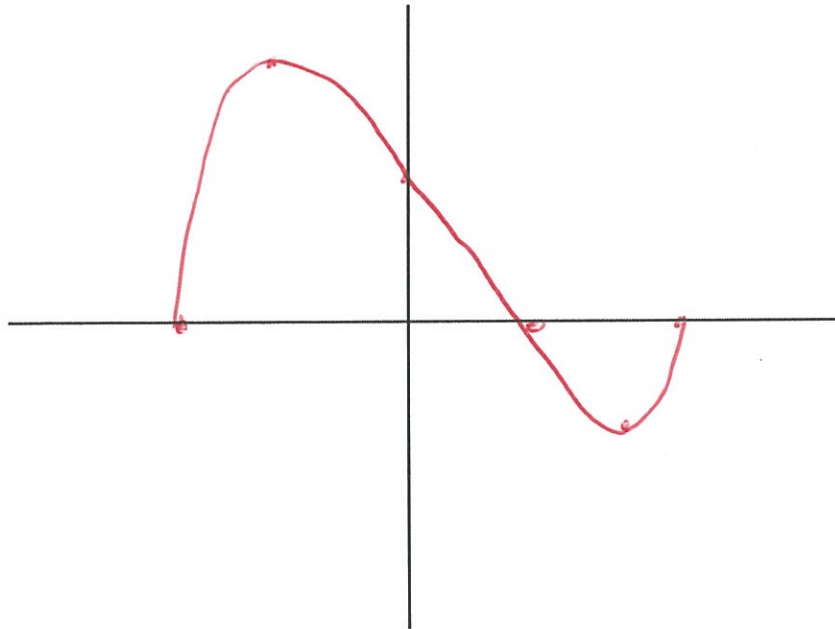
$x$  - intercepts:  $(-4, 0), (2, 0)$

$y$  - intercept:  $(0, 8)$

Extreme Points: *SEE #3*

End Behavior (Left): *NONE*

End Behavior (Right): *NONE*



9. Find the traits and **sketch** of  $y = (x^2 - 2x - 3)e^{\frac{1}{2}x}$ .

Domain: *All reals*

Range:  $y \in [-8.2767, \infty)$

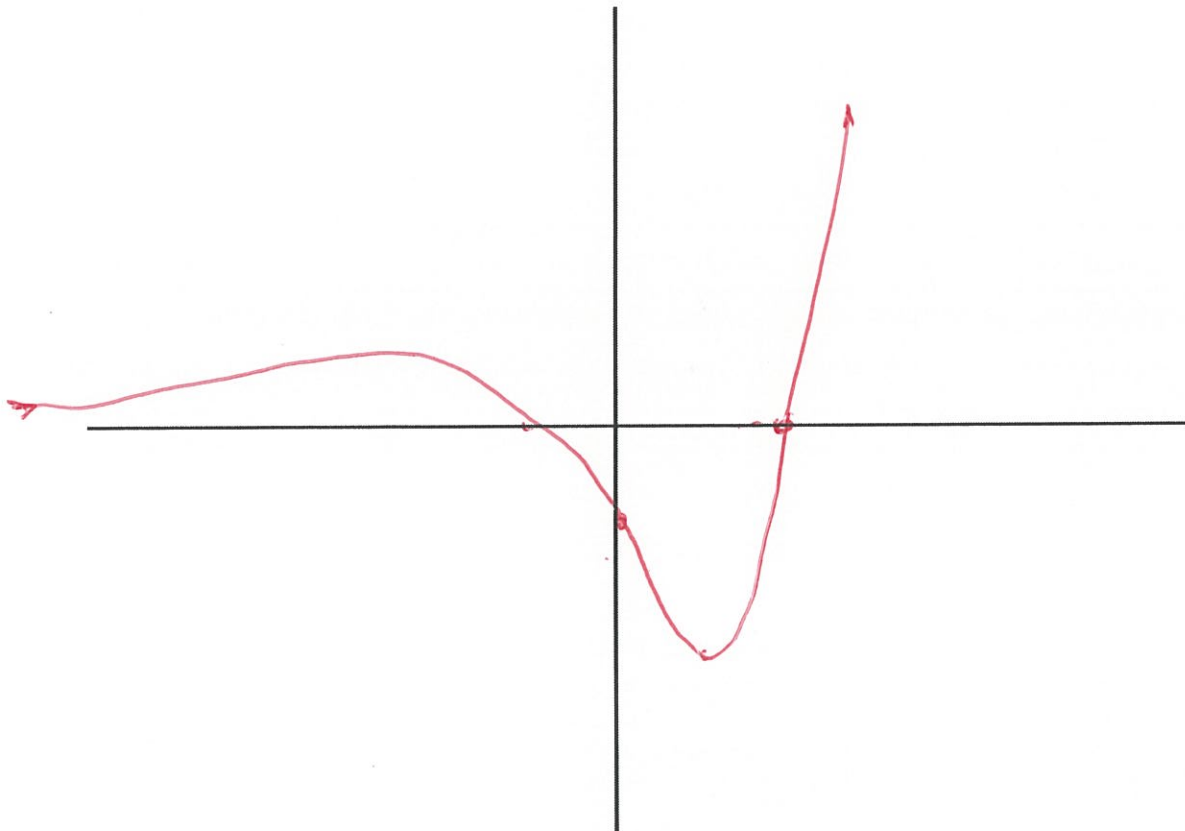
x - intercepts:  $(3, 0)$   $(-1, 0)$

y - intercept:  $(0, -3)$

Extreme Points: *SEE #2*

End Behavior (Left):  $y = 0$

End Behavior (Right): *UP*



10. Find the traits and **sketch** of  $y = \ln(x^3 + 4x^2 - 9x - 36)$  on  $x \in (-4, 4]$ .

Domain:  $x \in (-4, -3) \cup (3, 4]$       Range:  $y \in (-\infty, 4.025]$

VAs:  $x = \pm 3, -4$        $y$ -intercept: **NONE**

Extreme Points: **SEE #7**

End Behavior (Left): **NONE**

End Behavior (Right): **NONE**

