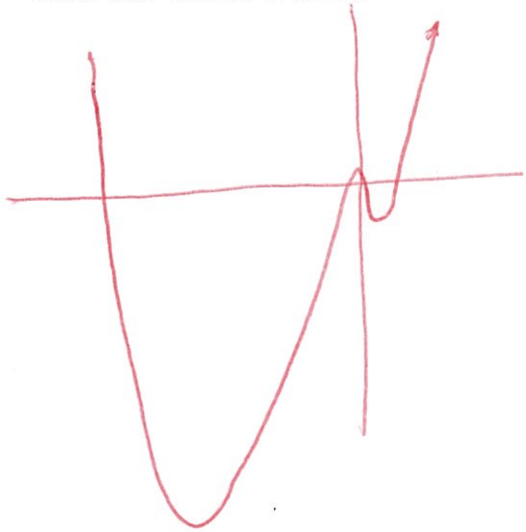
PreCalculus Acc 2018-19
Fall Final
Dr. Quattrin

Name SOLUTION KEY

score 150

1. Given $f(x) = 5x^4 + 122x^3 - 655x^2 + 486x - 72$, sketch the complete graph and state the window used.



$x \in [-40, 5]$

$y \in [-500{,}000, 10000]$

2. Find the Zeros and Extreme Points of $f(x) = 5x^4 + 122x^3 - 655x^2 + 486x - 72$

ZEROS: $(-29.629, 0)$ $(3.766, 0)$
$(.162, 0)$
$(.701, 0)$

ETREME POINTS $(-21.412, -457432.1)$
$(.422, 35.774)$
$(2.690, -857.757)$

3. The motion of a particle is described by $y(t)=t^3-t^2-11t+12$.

a) When the particle is stopped?

$v(t) = 3t^2 - 2t - 11 = 0$

$t = 2.263, -1.596$

b) Which direction is the particle moving at $t=7$?

$v(7) = 3(49) - 14 - 11 > 0 \therefore$ UP

c) Where is the particle when $t=7$?

$y(7) = 229$

d) Find the acceleration when $v(t)=-11$.

$v(t) = 3t^2 - 2t - 11 = -11$

$3t^2 - 2t = 0$

$t(3t-2) = 0$

$t = 0, 2/3$

$a(t) = 6t - 2$

$a(0) = -2$

$a(2/3) = 2$

e) Where is the particle when the acceleration is zero?

$a(t) = 6t - 2 = 0$

$t = 1/3$

$x(1/3) = 8.259$

4. Find:

a. $\lim_{x\to\frac{1}{3}} \dfrac{6x^2+x-1}{3x^2-16x+5}$ $= \lim_{x\to 1/3} \dfrac{\cancel{(3x-1)}(2x+1)}{\cancel{(3x-1)}(x-5)}$

$= \dfrac{2(1/3)+1}{(1/3-5)} = \dfrac{5/3}{-14/3} = \boxed{-\dfrac{5}{14}}$

b. $\lim_{x\to -4} \dfrac{x^4-14x^2+3x-20}{x^3+6x^2-2x-40}$ $= \lim_{x\to -4} \dfrac{\cancel{(x+4)}(x^3-4x^2+2x-5)}{\cancel{(x+4)}(x^2+2x-10)} = \dfrac{(-4)^3-4(-4)^2+(-8)-5}{(-4)^2-8-10}$

$-4)$ 1 0 −14 3 −20
−4 16 −8 20
1 −4 2 −5 0

$-4)$ 1 6 −2 −40
−4 −8 40
1 2 −10 0

$= \dfrac{~~-110~~ -141}{-2}$

$= \boxed{\dfrac{141}{2}}$

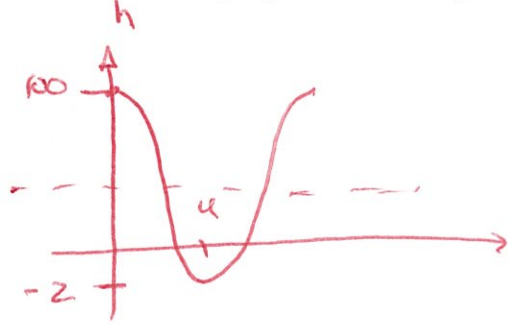
c. $\lim_{x\to -1} \dfrac{x^4+2x^3+x^2}{x^3-7x^2-6x}$ $= \lim_{x\to -1} \dfrac{x^2(x^2+2x+1)}{x(x^2-7x-6)} = \dfrac{0}{-2} = \boxed{0}$

−11

$= \cancel{\lim_{x\to -1} \dfrac{x^2(x+1)^2}{x(x-6)(x+1)}}$

5. In episode 218 of the Mythbusters, Jamie Hyneman bungee-jumped from a 100-foot-high lift to bob for apples in a pool. Like the Bouncing Spring Problem, Jamie's height from the water's surface varies sinusoidally with time. It took four seconds for Jamie to fall from the tower to the water and two feet under the surface.

a. Sketch the graph of his drop.



b. Create an equation that models Jamie's height *h* in terms of time *t*.

$h = 49 + 50 \cos \frac{\pi}{4} t$

c. How far above the water is Jamie at 1.4 seconds?

$h(1.4) = 72.154$ FT

d. At what time did Jamie come back out of the water?

$0 = 49 + 51 \cos \frac{\pi}{4} t$

$-.961 = \cos \frac{\pi}{4} t$

$\left.\begin{array}{l} 2.861 \pm 2\pi n \\ -2.861 \pm 2\pi n \end{array}\right\} = \frac{\pi}{4} t$

$t = \begin{array}{l} 3.642 \pm 8n \\ -3.642 \pm 8n \end{array}$

$t_2 = 4.358$ SEC

6. $\csc B = \dfrac{29}{20}$ in Quadrant II. Find the other five exact trig values and the approximate value of B: $r=29$ $y=20$ $x=-21$

$\sin B =$ $20/29$ $\cos B =$ $-21/29$ $\tan B =$ $-20/21$

$\cot B =$ $-21/20$ $\csc B = \dfrac{29}{20}$ $\sec B =$ $-29/21$

$B =$ $\boxed{2.381 \pm 2\pi n}$

$136.397 \pm 360n$ IN DEGREES

7. Use the equation of the line tangent to $f(x) = 5x^3 + 3x^2 - 4x$ at $x = -1$ to approximate $f(-0.9)$.

$f(-1) = -5 + 3 + 4 = 2$

$f'(x) = 15x^2 + 6x - 4$

$m = f'(-1) = 15 - 6 - \cancel{4} = 5$

$y - 2 = 5(x+1)$

$f(-0.9) \approx y = 5(-.1) + 2 = 1.5$

8. Use synthetic substitution to find all the exact zeros algebraically of $f(x)=-4x^4+19x^3+41x^2-171x-45$.

$$
\begin{array}{r|rrrrr}
3 & -4 & 19 & 41 & -171 & -45 \\
 & & -12 & 21 & 186 & 45 \\
\hline
 & -4 & 7 & 62 & +15 & \emptyset
\end{array}
$$

$$
\begin{array}{r|rrrr}
-3 & -4 & 7 & 62 & 15 \\
 & & 12 & -57 & -15 \\
\hline
 & -4 & 19 & 5 & \emptyset
\end{array}
$$

$(x-3)(x+3)(-4x^2+19x+5)$

$(x-3)(x+3)(-4x-1)(x+5)$

$(\pm 3, 0)$

$(-1/4, 0)$

$(-5, 0)$

9. Solve $-4x^4+19x^3+41x^2-171x-45>0$. Be sure to show the sign pattern.

$$\begin{array}{c|ccccccccc} y & - & 0 & + & 0 & - & 0 & + & 0 & - \\ x & & -5 & & -3 & & -1/4 & & 3 & \end{array}$$

$x \in (-5,-3) \cup (-1/4, 3)$

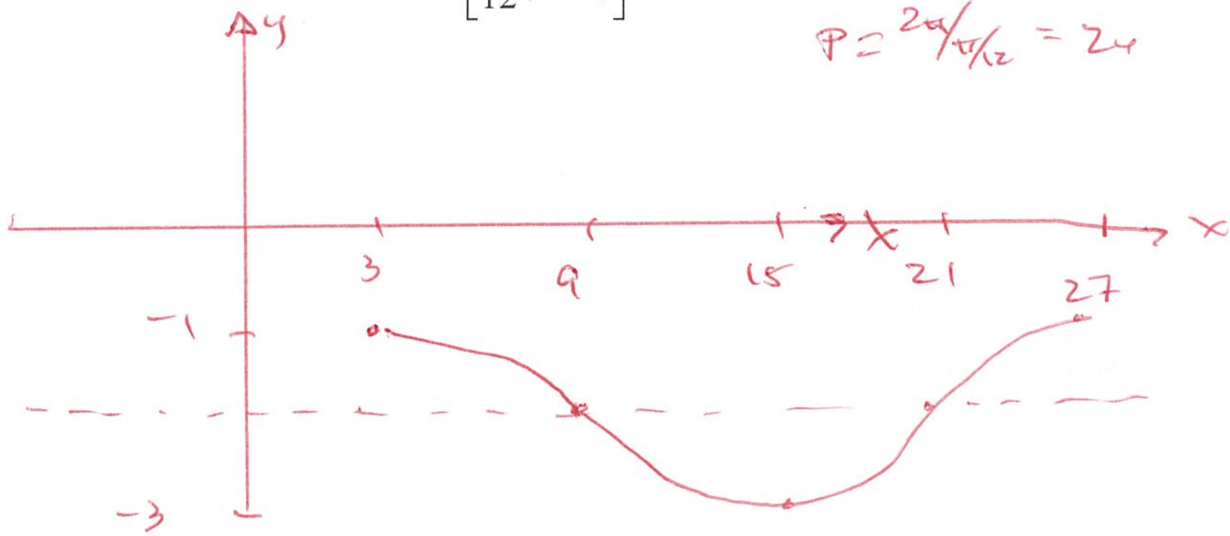
10. Find an inequality that has this sign pattern and solution:

$$\begin{array}{c|ccccccc} y & - & 0 & + & 0 & - & 0 & + \\ x & & -3 & & 1 & & 5 & \end{array}$$ and $x \in (-3,1) \cup (5,\infty)$

$(x+3)(x-1)(x-5) > 0$

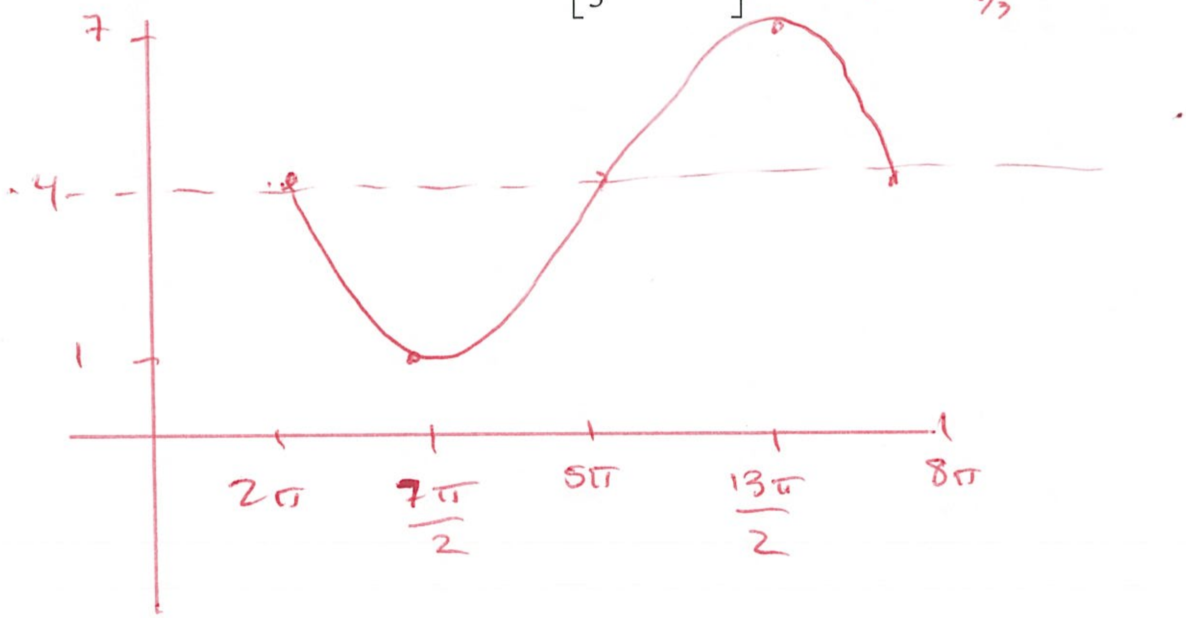
11. Graph on cycle of $y=-2+\cos\left[\frac{\pi}{12}(x-3)\right]$.

$P = 2\pi / \frac{\pi}{12} = 24$



12. Graph one cycle of $y=4-3\sin\left[\frac{1}{3}(x-2\pi)\right]$.

$P = \frac{2\pi}{1/3} = 6\pi$



13. Prove the identity: $\dfrac{\tan^2 w - \sec w - 5}{\tan^2 w + 3\sec w + 3} = \dfrac{\sec w - 3}{\sec w + 1}$

$$\frac{\sec^2 w - 1 - \sec w - 5}{\sec^2 w - 1 + 3\sec w + 3}$$

$$\frac{\sec^2 w - \sec w - 6}{\sec^2 w + 3\sec w + 2}$$

$$\frac{(\sec w - 3)(\sec w + 2)}{(\sec w + 1)(\sec w + 2)}$$

$$\frac{\sec w - 3}{\sec w - 1}$$

14. Use the Power Rule to find:

a. $\dfrac{dy}{dx}$ if $y = 12x^7 - 31x^4 + 25x^2 - 7x + 5$

$$\frac{dy}{dx} = 84x^6 - 124x^3 + 50x - 7$$

b. $f'(x)$ if $f(x) = x^5 - 7\sqrt[5]{x^7} + 7^2 - \dfrac{1}{\sqrt[3]{x^{10}}} + \dfrac{5}{6x}$ $= x^5 - 7x^{7/5} + 7^2 - x^{-10/3} + \frac{5}{6}x^{-1}$

$$f'(x) = 5x^4 - \frac{49}{5}x^{2/5} + \frac{10}{3}x^{-13/3} - \frac{5}{6}x^{-2}$$

c. $\dfrac{d}{dx}\left[\dfrac{12}{x^4} - 8\sqrt[4]{x^3} + \sqrt{x^7} - \pi x^3\right]$ $= D_x\left[12x^{-4} - 8x^{3/4} + x^{7/2} - \pi x^3\right]$

$$= -48x^{-5} - 6x^{-1/4} + \frac{7}{2}x^{5/2} - 3\pi x^2$$