

PreCalculus Accelerated
Dr. Quattrin
Limits and Derivatives Test
CALCULATOR ALLOWED

Name: Solution Key

Score _____

Round to 3 decimal places. Show all work.

1. Use the equation of the line tangent to $y = 2x^3 - x^2 - 7x + 4$ at $x = 1$ to approximate $f(1.1)$

$$y_1 = y(1) = 2 - 1 - 7 + 4 = -2$$
$$\frac{dy}{dx} = 6x^2 - 2x - 7$$
$$m = \left. \frac{dy}{dx} \right|_{x=1} = 6 - 2 - 7 = -3$$

$$y + 2 = -3(x - 1)$$

$$f(1.1) \approx y = -3(1.1 + 1) - 2 = -2.3$$

2. The motion of a particle is described by $x(t) = 8t^3 - 57t^2 + 72t + 8$.

- a) When the particle is stopped?
b) Which direction it is moving at $t = -1$?
c) Where is it when $t = -1$?
d) Find $a(2)$.

$$v(t) = 24t^2 - 114t + 72$$
$$a(t) = 48t - 114$$

$$a) v(t) = 0 = 24t^2 - 114t + 72$$
$$= 6(t-4)(3t-3)$$
$$t = 4, 3/4$$

$$b) v(-1) = 24 + 114 + 72 > 0 \therefore \text{RIGHT}$$

$$c) x(-1) = -8 - 57 - 72 + 8 = -129$$

$$d) a(2) = 48(2) - 114 = 18$$

3. Evaluate the following limits:

$$\begin{aligned}
 \text{a) } \lim_{x \rightarrow 5} \frac{x^2 - 6x + 5}{x^2 - 25} &= \lim_{x \rightarrow 5} \frac{(x-5)(x-1)}{(x-5)(x+5)} \\
 &= \frac{5-1}{5+5} = \frac{4}{10} = \frac{2}{5}
 \end{aligned}$$

$$\text{b) } \lim_{x \rightarrow -2} \frac{x^3 + 8}{3x^3 + 6x^2 + x + 2} = \lim_{x \rightarrow -2} \frac{(x+2)(x^2 - 2x + 4)}{3x^2(x+2) + 1(x+2)}$$

$$\begin{array}{r}
 -2 \) \ 3 \ 6 \\
 \hline
 \end{array}$$

$$= \lim_{x \rightarrow -2} \frac{x^2 - 2x + 4}{3x^2 + 1}$$

$$= \frac{12}{13}$$

$$\text{c) } \lim_{x \rightarrow 7} \frac{x^3 - 7x^2 - 3x + 21}{3x^3 - 20x^2 - 11x + 28} = \lim_{x \rightarrow 7} \frac{(x-7)(x^2 - 3)}{(x-7)(3x^2 + x - 4)}$$

$$\begin{array}{r}
 7 \) \ 1 \ -7 \ -3 \ 21 \\
 \underline{7 \ 0 \ -21} \\
 1 \ 0 \ -3 \ 0
 \end{array}$$

$$= \frac{49 - 3}{3(49) + 7 - 4} = \frac{46}{150} = \frac{23}{75}$$

$$\begin{array}{r}
 7 \) \ 3 \ -20 \ -11 \ 28 \\
 \underline{21 \ 7 \ -28} \\
 3 \ +1 \ -4 \ 0
 \end{array}$$

PreCalculus Accelerated
Dr. Quattrin
Limits and Derivatives Test
NO CALCULATOR ALLOWED

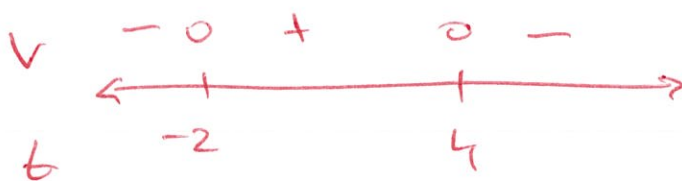
Name: SOLUTION KEY

4. Set up, but do not solve, the limit definition of the derivative for $y = 11x^5 + 7x^4 + 4x^2 - 12x - 3$.

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{[11(x+h)^5 + 7(x+h)^4 + 4(x+h)^2 - 12(x+h) - 3] - [11x^5 + 7x^4 + 4x^2 - 12x - 3]}{h}$$

5. A particle's position at time t is described by $x(t) = -t^3 + 3t^2 + 24t - 5$. When is the particle moving left? (Show the sign pattern of the velocity.)

$$\begin{aligned} v(t) &= -3t^2 + 6t + 24 \\ &= -3(t^2 - 2t - 8) \\ &= -3(t-4)(t+2) \end{aligned}$$



$$t \in (-\infty, -2) \cup (4, \infty)$$

6. Use the Power Rule to find:

a) Find $\frac{dy}{dx}$ if $y = -4x^3 - 5x^2 + 17x + 37$

$$\frac{dy}{dx} = -12x^2 - 10x + 17$$

b) Find $f'(x)$ if $f(x) = 12x^4 - 6x^3 + 4x + 7 + \frac{1}{3x}$

$$f'(x) = 48x^3 - 18x^2 + 4 - \frac{1}{3}x^{-2}$$

c) $D_x \left[\sqrt[3]{x^4} - \frac{3}{\sqrt{x^3}} - \sqrt[4]{x} + 8x - 6 \right] = D_x \left[x^{4/3} - 3x^{-3/2} - x^{1/4} + 8x - 6 \right]$

$$= \frac{4}{3}x^{-3/3} + \frac{9}{2}x^{-5/2} - \frac{1}{4}x^{-3/4} + 8$$