PreCalculus Accelerated
Dr. Quattrin
Limits and Derivatives Test
CALCULATOR ALLOWED
Round to 3 decimal places. Show all work.

Name: Solution Key

Score ___________

1. Use the equation of the line tangent to $y = 2x^3 - x^2 - 7x + 4$ at $x = 1$ to approximate $f(1.1)$

$\frac{dy}{dx} = 6x^2 - 2x - 7$

$y_1 = y(1) = 2 - 1 - 7 + 4 = -2$

$m = \left.\frac{dy}{dx}\right|_{x=1} = 6 - 2 - 7 = -3$

$y + 2 = -3(x - 1)$

$f(1.1) \approx y = -3(1.1 - 1) - 2 = -2.3$

2. The motion of a particle is described by $x(t) = 8t^3 - 57t^2 + 72t + 8$.

a) When the particle is stopped?
b) Which direction it is moving at $t = -1$?
c) Where is it when $t = -1$?
d) Find $a(2)$.

$v(t) = 24t^2 - 114t + 72$
$a(t) = 48t - 114$

a) $v(t) = 0 = 24t^2 - 114t + 72$
$= 6(t-4)(\cancel{3t}\ldots(4t-3)$
$t = 4, 3/4$

b) $v(-1) = 24 + 114 + 72 > 0 \therefore$ RIGHT

c) $x(-1) = -8 - 57 - 72 + 8 = -129$

d) $a(2) = 48(2) - 114 = 18$

3. Evaluate the following limits:

a) $\lim_{x\to 5}\frac{x^2-6x+5}{x^2-25} = \lim_{x\to 5}\frac{(x-5)(x-1)}{(x-5)(x+5)}$

$= \frac{5-1}{5+5} = \frac{4}{10} = \frac{2}{5}$

b) $\lim_{x\to -2}\frac{x^3+8}{3x^3+6x^2+x+2} = \lim_{x\to -2}\frac{\cancel{(x+2)}(x^2 - 2x+4)}{3x^2\cancel{(x+2)}+1\cancel{(x+2)}}$

$-2)\ 3 \quad 6$

$= \lim_{x\to -2}\frac{x^2-2x+4}{3x^2+1}$

$= \frac{12}{13}$

c) $\lim_{x\to 7}\frac{x^3-7x^2-3x+21}{3x^3-20x^2-11x+28} = \lim_{x\to 7}\frac{\cancel{(x-7)}(x^2-3)}{\cancel{(x-7)}(3x^2+x-4)}$

$$\begin{array}{r|rrrr} 7 & 1 & -7 & -3 & 21 \\ & & 7 & 0 & -21 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$= \frac{49-3}{3(49)+7-4} = \frac{46}{150} = \frac{23}{75}$

$$\begin{array}{r|rrrr} 7 & 3 & -20 & -11 & 28 \\ & & 21 & 7 & -28 \\ \hline & 3 & +1 & -4 & 0 \end{array}$$

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4. Set up, but do not solve, the limit definition of the derivative for $y = 11x^5 + 7x^4 + 4x^2 - 12x - 3$.

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\left[11(x+h)^5 + 7(x+h)^4 + 4(x+h)^2 - 12(x+h) - 3\right] - \left[11x^5 + 7x^4 + 4x^2 - 12x - 3\right]}{h}$$

5. A particle's position at time t is described by $x(t) = -t^3 + 3t^2 + 24t - 5$. When is the particle moving left? (Show the sign pattern of the velocity.)

$$v(t) = -3t^2 + 6t + 24$$
$$= -3(t^2 - 2t - 8)$$
$$= -3(t-4)(t+2)$$

v	−	0	+	0	−
t		−2		4	

$$t \in (-\infty, -2) \cup (4, \infty)$$

6. Use the Power Rule to find:

a) Find $\frac{dy}{dx}$ if $y = -4x^3 - 5x^2 + 17x + 37$

$$\frac{dy}{dx} = -12x^2 - 10x + 17$$

b) Find $f'(x)$ if $f(x) = 12x^4 - 6x^3 + 4x + 7 + \frac{1}{3x}$ $= 12x^4 - 6x^3 + 4x + 7 + \frac{1}{3}x^{-1}$

$$f'(x) = 48x^3 - 18x^2 + 4\cancel{x} - \frac{1}{3}x^{-2}$$

c) $D_x\left[\sqrt[7]{x^4} - \frac{3}{\sqrt{x^3}} - \sqrt[4]{x} + 8x - 6\right] = D_x\left[x^{4/7} - 3x^{-3/2} - x^{1/4} + 8x - 6\right]$

$$= \frac{4}{7}x^{-3/7} + \frac{9}{2}x^{-5/2} - \frac{1}{4}x^{-3/4} + 8$$