PreCalculus Acc '18-19 Name: SOLUTION KEY
Dr. Quattrin
Rational Test
CALCULATOR ALLOWED
Round to 3 decimal places. Score _____
Show all work.

1. Find the zeros and VAs of $y=\frac{x^2-2x-3}{x^3-3x^2+x-3}$ on $x\in[-4,\infty)$. Show the supporting algebraic work.

Zeros: $(-1,0)$ $y=\frac{(x-3)(x+1)}{(x^2+1)(x-3)}\approx\frac{x+1}{x^2+1}$

VAs: NONE

POE: $(3, 2/5)$

2. Find the extreme points of $y=\frac{x^2-2x-3}{x^3-3x^2+x-3}$ on $x\in[-4,\infty)$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx}=\frac{(x^2+1)(1)-(x+1)(2x)}{(x^2+1)^2}=\frac{-x^2-2x+1}{(x^2+1)^2}$$

i) $-x^2-2x+1=0 \rightarrow x=\frac{2\pm\sqrt{4+4}}{2(-1)}=-1\pm\sqrt{2}=\begin{cases}.414\\ -2.414\end{cases}$

ii) $\frac{dy}{dx}$ DNE $\rightarrow$ NONE

iii) ENDPOINTS $x=-4$

$(.414, 1.207)$

$(-2.414, -.207)$

$(-4, -.177)$

3. Find the equations of the lines tangent to and normal to $y=\frac{4x^2-13x+10}{-4x^2+x+14}$ at $x=0$? $(0, 5/7)$

Tangent: $y-5/7=\frac{-48}{48}(x-0)$

Normal: $y-5/7=\frac{49}{48}(x-0)$

$=(4x-10)(x-1)$

$\frac{dy}{dx}=\frac{(-4x^2+x+14)(8x-13)-(4x^2-13x+10)(-8x+1)}{(-4x^2+x+14)^2}$

$m=\frac{14(-13)-10(1)}{(14)^2}=\frac{-192}{196}$

4. Find the zeros, VAs, POEs and EB of $y=\frac{-4x^3+x^2+16x-4}{4x^3-x^2+36x-9}$. Show the supporting algebraic work.

$\frac{-x^2(4x-1)+4(4x-1)}{x^2(4x-1)+9(4x-1)}$

$\frac{4-x^2}{x^2+9}$

Zeros: $(\pm 2, 0)$

VAs: NONE

EB: $y=-1$

POE: $(1/4, .434)$

5. Find the extreme points of $y=\frac{4-x^2}{x^2+9}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = \frac{(x^2+9)(-2x)-(4-x^2)(2x)}{(x^2+9)^2} = \frac{-2x^3-18x+2x^3-8x}{(x^2+9)^2}$$

$$= \frac{-26x}{(x^2+9)^2}$$

i) $\frac{dy}{dx}=0 \rightarrow x=0$

$\boxed{(0, \frac{4}{9})}$

ii) $\frac{dy}{dx}$ DNE $\rightarrow$ NONE

iii) END POINTS $\rightarrow$ NONE

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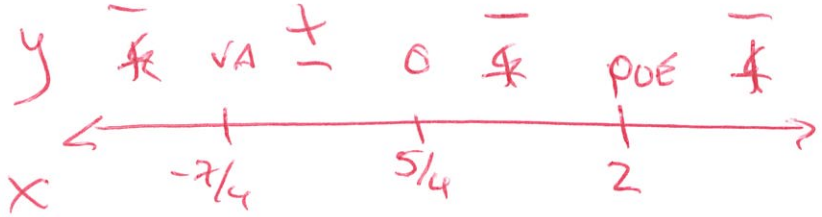
6. Write an equation of a rational function that has x–intercepts at (5, 0), VA at $x = 2$, a POE at $x = -5$, and a HA at $y = -\frac{6}{5}$.

$$y = \frac{-6(x-5)(x+5)}{5(x-2)(x+5)}$$

7. Show the sign pattern and solve $\frac{4x^2-13x+10}{-4x^2+x+14} \leq 0$.

$$\frac{(x-2)(4x-5)}{-(x-2)(4x+7)} \leq 0$$



$x \in (-\infty, -7/4) \cup [5/4, 2) \cup (2, \infty)$

~~$x \in (-7/4, 5/4]$~~

8. Find the traits and **sketch** $y=\frac{4-x^2}{x^2+9}$.

Domain: ALL REALS

Y – Intercept: $(0, 4/9)$

Zeros: $(\pm 2, 0)$

Range: $y \in (-1, 4/9]$

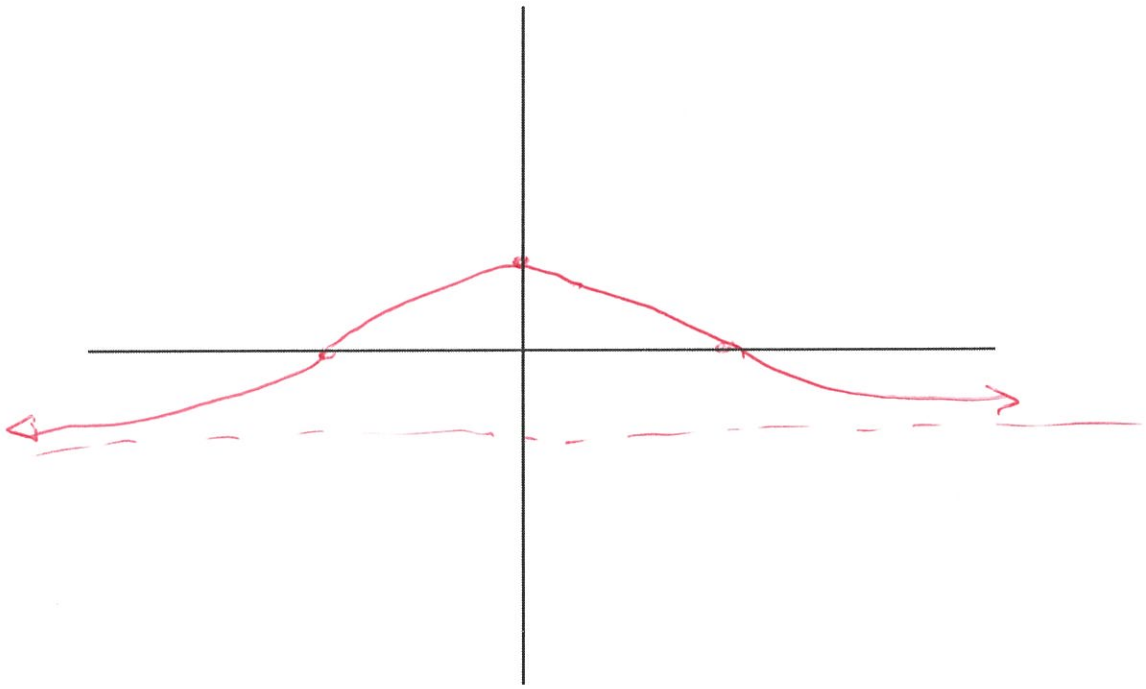
VAs: NONE

POEs: NONE

End Behavior (left): $y = -1$

End Behavior (right): $y = -1$

Extreme Points: $(0, \frac{4}{9})$



9. Find the traits and **sketch** of $y=\dfrac{x^2-2x-3}{x^3-3x^2+x-3}$ on $x\in[-4,\infty)$.

Domain: $x\in[-4,3)\cup(3,\infty)$ Y – Intercept: $(0,1)$

Zeros: $(-1,0)$ Range: $y\in[-.207,1.207]$

VAs: NONE POEs: $(5,.4)$

End Behavior (left): NONE End Behavior (right): $y=0$

Extreme Points: See #2