**PreCalc ACC '18-19** **Name:** Solution Key
**Spring Final**
**Calculator allowed throughout.** **score _____**
**Show all work. Round to 3 decimals.**

1. Find the following derivatives:

a. $\frac{d}{dx}\left(\tan 4x^2\right)$ $= \sec^2 4x^2 (8x)$

b. $\frac{d}{dx}\left(\ln\left(x^2+7x\right)\right)$ $= \frac{2x+7}{x^2+7x}$

c. $\frac{d}{dx}\left(e^{\frac{1}{2}x}\csc x\right)$ $= e^{-1/2x}(-\csc x \cot x) + \csc x\left(e^{-1/2x}(-1/2)\right)$

$= -e^{-1/2x}\csc x\left(\cot x + 1/2\right)$

d. $\frac{d}{dx}\left(\frac{\sin 5x}{25+x^2}\right)$ $= \frac{(25+x^2)(\cos 5x)(5) - \sin 5x\,(2x)}{(25+x^2)^2}$

2. Find the end behavior of each of the following functions. Show the limits that lead to your conclusions.

a) $y=(4x^2-16x)e^{-0.25x}$

Left end: $\lim_{x\to-\infty} e^{-.25x} = \infty \therefore$ UP

Right End: $\lim_{x\to+\infty} e^{-.25x} = 0 \therefore y=0$

b) $y=\ln(-x^3-6x^2+5x+30)$

Left end: $\lim_{x\to-\infty} y = +\infty \therefore$ UP

Right End: $\lim_{x\to\infty} y =$ DNE $\therefore$ NONE

c) $y=-\sqrt{\frac{16x}{x^2+4}}$.

Left end: $\lim_{x\to-\infty} y$ DNE $\therefore$ NONE

Right End:

$\lim_{x\to\infty} -\sqrt{\frac{16x}{x^2+4}} = 0 \therefore y=0$

**You must do problems #3, #4, and #5.**

3. Find domain and zeros of $f(x)=-x^4+6x^2-8$ on $x\in[-1.5,3]$.

DOMAIN: $x\in[-1.5,3]$

ZEROS: $-(x^2-4)(x^2-2)=0 \rightarrow x=\pm 2, \pm\sqrt{2}$ BUT $-2$ NOT IN DOMAIN

$(2,0)\ (\pm\sqrt{2},0)$

4. Find the extreme points of $f(x)=-x^4+6x^2-8$ on $x\in[-1.5,3]$. Show the algebraic work to support the critical values.

i) $\frac{dy}{dx}=-4x^3+12x=-4x(x^2-3)=0$

$x=0, \pm\sqrt{3}$ BUT $-\sqrt{3}$ IS NOT IN DOMAIN

ii) NONE

iii) $x=-1.5, 3$

$(0,-8)$

$(\sqrt{3},1)$

$(-1.5,.4375)$

$(3,-35)$

5. Find the Point of Inflection for $f(x)=-x^4+6x^2-8$ on $x\in[-1.5,3]$. Show the algebraic work to support the result.

$\frac{d^2y}{dx^2}=-12x^2+12=0 \rightarrow x=\pm 1$

$(1,-3)$

$(-1,-3)$

**You may skip either # 6 and #7, or #8 and #9, or #10 and #11.**

6. Find domain and zeros of $y=(4x^2-16x)e^{-0.25x}$.

Domain: All Reals $\quad 4x(x-4)$

Zeros $(0,0)(4,0)$

7. Find the extreme points of $y=(4x^2-16x)e^{-0.25x}$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = (4x^2-16x)e^{-.25x}(-1/4) + e^{-.25x}(8x-16)$$

$$= (-x^2+4x+8x-16)\,e^{-.25x}$$

$$= -e^{-.25}(x^2-12x+16)$$

i) $\frac{dy}{dx}=0 \Rightarrow x = \frac{12\pm\sqrt{12^2-4(1)(16)}}{2} = \begin{cases} 10.472 \\ 1.528 \end{cases}$

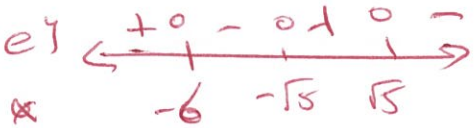
ii) NONE $\qquad (1.528, -10.312)$

iii) NONE $\qquad (10.472, 19.776)$

8. Find domain, VAs, and zeros of $y=\ln\left(-x^3-6x^2+5x+30\right)$ on $x\in[-7,5]$.

Domain $x\in[-7,-6)\cup(-\sqrt{5},\sqrt{5})$ $(-x^2+5)(x+6)$

VAs: $x=-6,\pm\sqrt{5}$

eg 


Zeros: $(-6.032,0)$

$(-2.177,0)$

$(2.209,0)$

9. Find the extreme points of $y=\ln\left(-x^3-6x^2+5x+30\right)$ on $x\in[-7,5]$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx}=\frac{-3x^2-12x+5}{-x^3-6x^2+5x+30}$$

i) $-3x^2-12x+5=0 \rightarrow x=$ $\begin{cases}-3.828\\-.472\end{cases}$ $(-.472, 3.274)$

ii) $-x^3-6x^2+5x+30=0 \rightarrow x=-6,\pm\sqrt{5}$ NOT IN DOMAIN

iii) $x=-7$ $(-7, 3.784)$

10. Find domain, VAs, and zeros of $y=-\sqrt{\frac{16x}{x^2+4}}$.

DOMAIN: $x \in [0, \infty)$

ZEROS: $(0, 0)$

VAS: NONE

11. Find the extreme points of $y=-\sqrt{\frac{16x}{x^2+4}}$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{16x}{x^2+4}\right)^{-1/2}\left[\frac{(x^2+4)(16)-16x(2x)}{(x^2+4)^2}\right]$$

$$= -\frac{1}{2}\frac{(x^2+4)^{1/2}}{(16x)^{1/2}}\frac{16x^2+64-32x^2}{(x^2+4)^2}$$

$$= \frac{8x^2-32}{(16x)^{1/2}(x^2+4)^{3/2}}$$

i) $\frac{dy}{dx}=0 \rightarrow 8x^2-32=0 \rightarrow x=\pm 2$ $(2,-2)$

$-2$ NOT IN DOMAIN

ii) $16x=0 \rightarrow x=0$ $(0,0)$

$x^2+4=0 \rightarrow$ NO SOLUTION

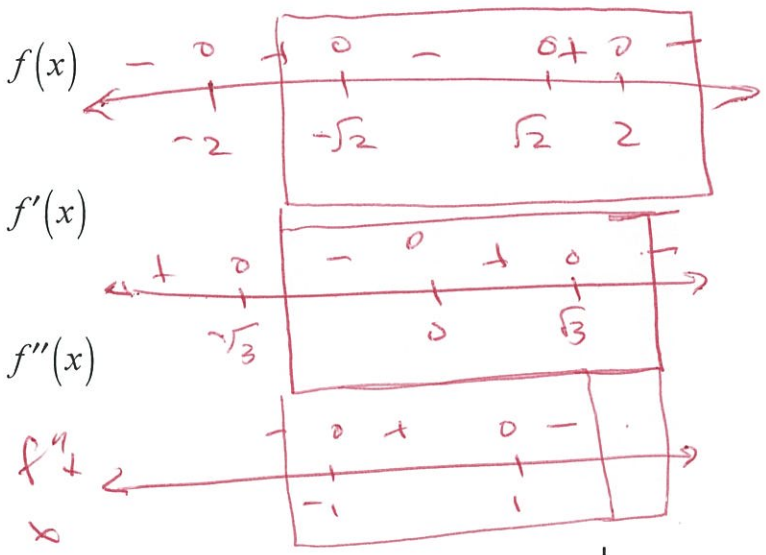
iii) NO RESTRICTION

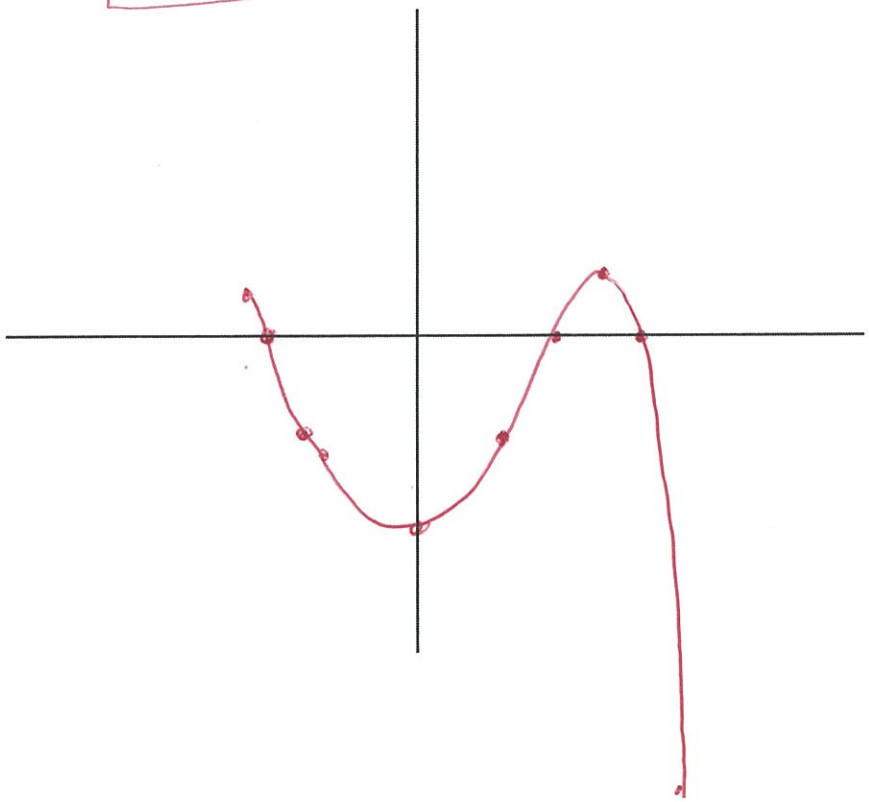
You must do #12.

12. Show the sign patterns for $f(x)$, $f'(x)$, and $f'(x)\,f'(x)$, where

$$f(x)=-x^4+6x^2-8 \text{ on } x\in[-1.5,3]$$

Then sketch $f(x)$.



**You may skip either #13, #14, or #15.**

13. Find the traits and **sketch** of $y=(4x^2-16x)e^{-0.25x}$.

Domain: All Reals

Range: $y \in [-10.312, \infty)$

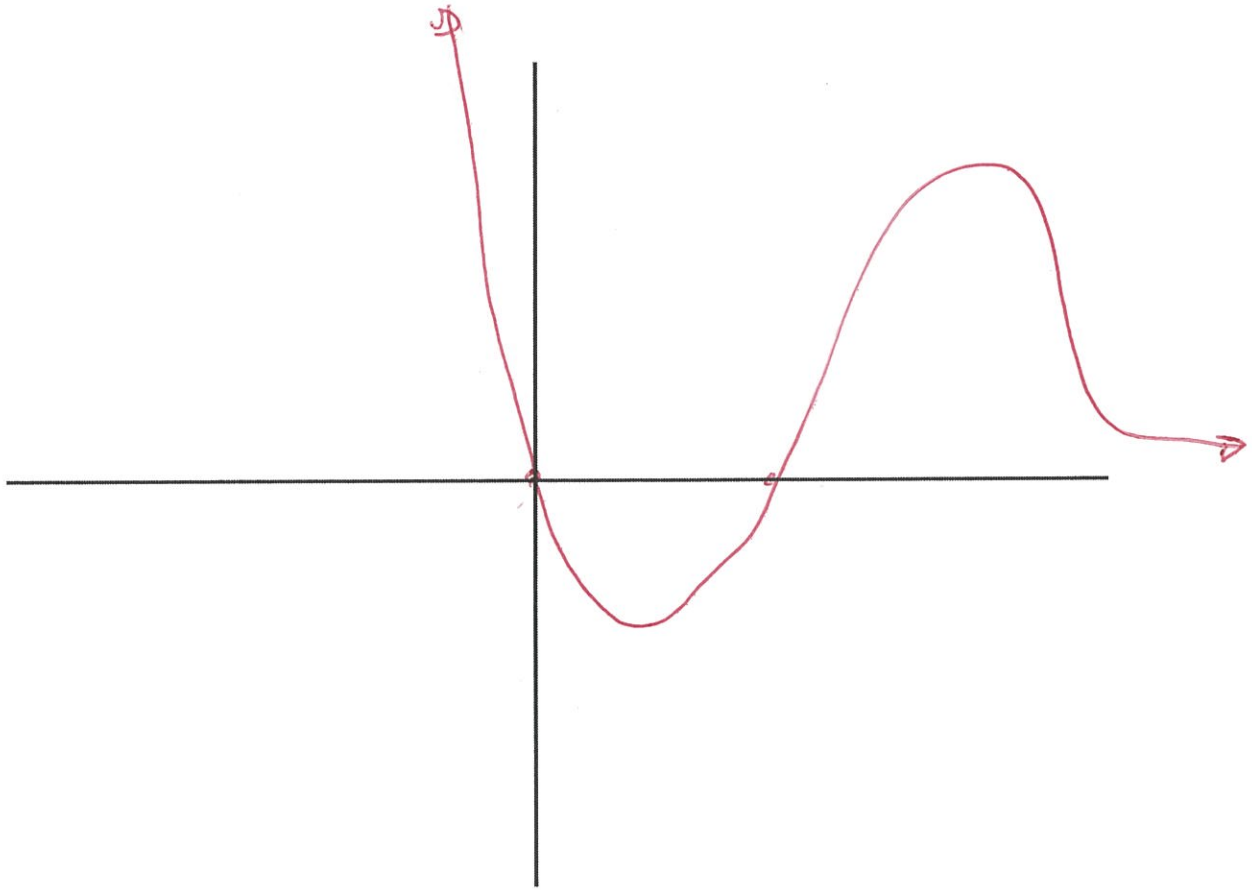
$x$ – intercepts: $(0,0)$ $(4,0)$

$y$ – intercept: $(0,0)$

Extreme Points: $(1.528, -10.312)$ $(10.472, 19.776)$

End Behavior (Left): UP

End Behavior (Right): $y=0$



14. Find the traits and **sketch** $y=\ln\left(-x^3-6x^2+5x+30\right)$ on $x\in[-7,5]$.

Actual Domain: SEE #8

Range: $y\in(-\infty, 3.784]$

$x$ – intercepts: SEE #8

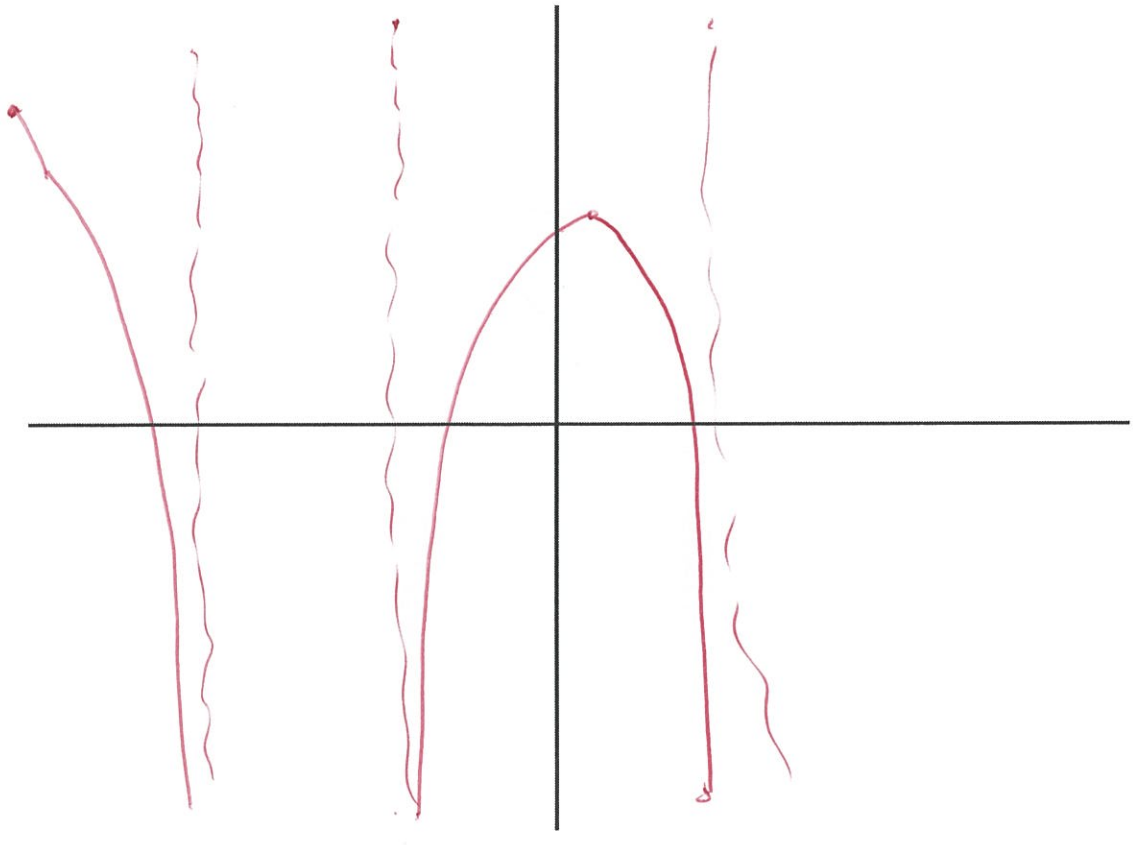
$y$ – intercept: $(0, \ln 30)$

VAs: $x=-6, \pm\sqrt{5}$

Extreme Points: SEE #9

End Behavior (Left): NONE

End Behavior (Right): NONE



15. Find the Traits and **sketch** of $y=-\sqrt{\frac{16x}{x^2+4}}$.

Domain: $x \in [0, \infty)$

Range: $y \in [-2, 0]$

$x$ – intercepts: $(0,0)$

$y$ – intercept: $(0,0)$

Extreme Points: $(0,0)$ $(2,-2)$

End Behavior (Left): NONE

End Behavior (Right): $y=0$