PreCalculus ACC '18-19
Trig Identities Group Test
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Name Solution Key

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For Problem 1-6, use:

(20, -21) is on the terminal side of C and
$\sec Q = -\frac{13}{12}$ in QIII

to find the exact values of:

$\cos C = \frac{20}{29}$ $\sin C = \frac{-21}{29}$ $\tan C = \frac{-21}{20}$

$\sin Q = \frac{-5}{13}$ $\cos Q = -\frac{12}{13}$ $\tan Q = \frac{5}{12}$

1. sin (C+Q)

$= \sin C \cos Q + \cos C \sin Q$

$= \left(\frac{-21}{29}\right)\left(\frac{-12}{13}\right) + \left(\frac{20}{29}\right)\left(\frac{-5}{13}\right)$

$= \frac{152}{377}$

2. cos (C-Q)

$= \cos C \cos Q + \sin C \sin Q$

$= \frac{20}{29}\left(\frac{-12}{13}\right) + \left(\frac{-21}{29}\right)\left(\frac{-5}{13}\right)$

$= \frac{-135}{377}$

3. $\tan(C+Q)$

$= \frac{\tan C + \tan Q}{1 - \tan C \tan Q}$

$= \frac{\frac{-21}{20} + \frac{5}{12}}{1 - \left(\frac{-21}{20}\right)\left(\frac{5}{12}\right)} = \frac{-152}{240} \cdot \frac{240}{345}$

$= -\frac{152}{345}$

4. sec (2C) $= \frac{1}{\cos^2 C - \sin^2 C}$

$= \frac{1}{\left(\frac{20}{29}\right)^2 - \left(\frac{21}{29}\right)^2} = \frac{841}{-41}$

5. cot (2C) $= \frac{1 - \left(-21/20\right)^2}{2\left(-21/20\right)} =$

$= \frac{-41}{400} \cdot \frac{10}{-21} = \frac{41}{840}$

6. $\csc 2C$ $= \frac{1}{2 \sin C \cos C}$

$= \frac{1}{2\left(\frac{-21}{29}\right)\left(\frac{20}{29}\right)}$

$= -\frac{841}{840}$

7. Prove:

$$\frac{1-\csc\alpha}{2\csc\alpha+1} = \frac{\sin^2\alpha+\cos^2\alpha-\csc^2\alpha}{2\cot^2\alpha+3\csc\alpha+3}$$

$$= \frac{1-\csc^2\alpha}{2(\csc^2\alpha-1)+3\csc\alpha+3}$$

$$= \frac{(1-\csc\alpha)(1+\csc\alpha)}{2\csc^2\alpha+3\csc\alpha+1}$$

$$= \frac{(1-\csc\alpha)\cancel{(1+\csc\alpha)}}{(2\csc\alpha+1)\cancel{(\csc\alpha+1)}}$$

9. Solve for $x \in [0, 60^\circ]$:

$$\frac{\cos x\cos 20^\circ - \sin x\sin 20^\circ}{\sin x\cos 20^\circ + \cos x\sin 20^\circ} = \sqrt{3}$$

$$\frac{\cos(x+20)}{\sin(x+20)}$$

$\cancel{\tan}$ COT $(x+20) = \sqrt{3}$

$\tan(x+20) = 1/\sqrt{3}$

$x+20 = \begin{cases} 30 \pm 360n \\ 210 \pm 360n \end{cases}$

$x = \begin{cases} 10 \pm 360n \\ 220 \pm 360n \end{cases}$

$\boxed{x = \cancel{40}}$ $\boxed{x = 10}$

8. Solve for $\theta \in [0^\circ, 360^\circ]$: $\cos 2\theta = \cos\theta$

$2\cos^2\theta - 1 - \cos\theta = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -1/2$ $\quad$ $\cos\theta = 1$

$\theta = \pm 120 \pm 360n$ $\quad$ $\theta = 0 \pm 360n$

$\cancel{\theta = \{60, 300, 90, 27}$ 360

$\cancel{\theta:}\ \{0, 360, 120, 240\}$

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For Problem 1-6, use:

(20, -21) is on the terminal side of C and
$\sec Q = -\frac{13}{12}$ in QIII
to find the exact values of:

1. $\sin(2Q) = 2\sin Q\cos Q$

$= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$

$= \frac{120}{169}$

2. $\cos(2Q) = \cos^2 Q - \sin^2 Q$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2$

$= \frac{119}{169}$

3. $\tan(2Q) = \frac{2\tan Q}{1-\tan^2 Q}$

$= \frac{2\left(5/12\right)}{1-\left(5/12\right)^2} = \frac{5/6}{1-25/144}$

$= \frac{5}{6}\cdot\left(\frac{144}{119}\right)$

$= \cancel{-}\frac{120}{119}$

4. $\sec(C+Q) = \frac{1}{\cos C\cos Q \mp \sin C\sin Q}$

$= \frac{1}{\left(\frac{20}{29}\right)\left(\frac{-12}{13}\right) - \left(\frac{-21}{29}\right)\left(\frac{-5}{13}\right)}$

$= \frac{1}{\frac{-240-105}{377}} = -\frac{377}{345}$

5. $\cot(C-Q) = \frac{1 \pm \tan C\tan Q}{\tan C - \tan Q}$

$= \frac{1+\left(\frac{-21}{20}\right)\left(\frac{5}{12}\right)}{\frac{-21}{20}-\frac{5}{12}} = \frac{\frac{\cancel{225}\,-135}{240}}{\frac{-352}{240}}$

$= -\frac{\cancel{225}\,-135}{352}$

6. $\csc(Q+C) = \frac{1}{\sin Q\cos C + \cos Q\sin C}$

$= \frac{1}{\left(\frac{-21}{29}\right)\left(\frac{-12}{13}\right) + \left(\frac{20}{29}\right)\left(\frac{-5}{13}\right)}$

$= \frac{377}{152}$

7. Prove: $\dfrac{\sec^2\lambda+3\csc\lambda\sin\lambda-5}{\tan^2\lambda-4\tan\lambda+3}=\dfrac{1+\tan\lambda}{\tan\lambda-3}$

$$\frac{(\tan^2\lambda+1)+3-5}{\tan^2\lambda-4\tan\lambda+3}$$

$$\frac{\tan^2\lambda-1}{\tan^2\lambda-4\tan\lambda+3}$$

$$\frac{\cancel{(\tan\lambda-1)}(\tan\lambda+1)}{\cancel{(\tan\lambda-1)}(\tan\lambda-3)}$$

9. Solve for A: $\cos^2(2A)-\sin^2(2A)=\dfrac{\sqrt{3}}{2}$

$$\cos 4A=\frac{\sqrt{3}}{2}$$

$$4A=\begin{cases} .524 \pm 2\pi n \\ -.524 \pm 2\pi n \end{cases}$$

$$A= \pm .131 \pm \frac{\pi}{2}n$$

8. Solve exactly for $x\in[-120, 120^\circ]$:
$\sec^2(3x)+\tan(3x)=1$

$$\tan^2 3x+1+\tan 3x=1$$

$$\tan 3x(\tan 3x+1)=0$$

$$\tan 3x=0 \qquad \tan 3x=-1$$

$$3x=\begin{cases} 0^\circ \pm 360^\circ n \\ 180^\circ \pm 360^\circ n \end{cases} \qquad 3x=\begin{cases} \cancel{135}^\circ \pm 360^\circ n \\ \cancel{+405}^\circ \pm 360^\circ n \end{cases} = \begin{cases} -45 \pm 360n \\ 135 \pm 360n \end{cases}$$

$$x=\begin{cases} 0 \pm 120^\circ n \\ 60 \pm 120^\circ n \end{cases} \qquad x=\begin{cases} -15^\circ \pm 120n \\ 45^\circ \pm 120n \end{cases} = \begin{cases} -15 \pm 120n \\ 45 \pm 120n \end{cases}$$

$$x=\{\cancel{0, +120, -120, 60, -60, 15, -105^\circ, 75, -45^\circ}\}$$

105

$$\{-45, 0, 60, 120^\circ, -120^\circ, \cancel{15}, -60\}$$

45 -75

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For Problem 1-6, use:

(20, -21) is on the terminal side of C and
$\sec Q = -\frac{13}{12}$ in QIII
to find the exact values of:

1. sin (2C) $= 2\sin C\cos C$

$= 2\left(\frac{-21}{29}\right)\left(\frac{20}{29}\right)$

$= -\frac{840}{841}$

2. cos (2C)

$= \cos^2 C - \sin^2 C$

$= \left(\frac{20}{29}\right)^2 - \left(\frac{-21}{29}\right)^2$

$= \frac{-41}{841}$

3. $\tan(2C)$ $= \frac{2\tan C}{1-\tan^2 C}$

$= \frac{2(-21/20)}{1-(-21/20)^2} = \frac{-21}{10}\cdot\frac{400}{-41}$

$= \frac{840}{41}$

4. sec (Q-C) $= \frac{1}{\cos Q\cos C + \sin Q\sin C}$

$= \frac{1}{\left(\frac{-12}{13}\right)\left(\frac{20}{29}\right) + \left(\frac{-5}{13}\right)\left(\frac{-21}{29}\right)}$

$= \frac{377}{-135}$

5. cot (Q-C) $= \frac{1+\cancel{\cot}\tan Q\,\cancel{\cot}\tan C}{\cancel{\cot}\tan Q - \cancel{\cot}\tan C}$

$= \frac{1+\left(\frac{5}{12}\right)\left(\frac{-21}{20}\right)}{\frac{5}{12}-\left(\frac{-21}{20}\right)} = \frac{\frac{-135}{240}}{\frac{352}{240}}$

$= +\frac{135}{352}$

6. $\csc(Q-C)$ $= \frac{1}{\sin Q\cos C - \cos Q\sin C}$

$= \frac{1}{\left(\frac{-5}{13}\right)\left(\frac{20}{29}\right) - \left(\frac{-21}{29}\right)\left(\frac{-12}{29}\right)}$

$= \frac{377}{-352}$

7. Prove: $1-\frac{5}{2}\csc\theta=\frac{6\sin^2\theta-19\sin\theta+10}{6\sin^2\theta-4\sin\theta}$

$$=\frac{(2\sin\theta-5)\cancel{(3\sin\theta-2)}}{2\sin\theta\,\cancel{(3\sin\theta-2)}}$$

$$=\frac{2\sin\theta}{2\sin\theta}-\frac{5}{2\sin\theta}$$

$$=1-\frac{5}{2}\csc\theta$$

8. Solve for $\theta\in[-180°,\ 180°]$:

$\sin\theta\cos\ 20°+\cos\theta\sin 20°=\frac{1}{2}$

$$\sin(\theta+20)=\tfrac{1}{2}$$

$$\theta+20°=\begin{cases}30\pm 360n\\150\pm 360n\end{cases}$$

$$\theta=\begin{cases}10\pm 360n\\130\pm 360n\end{cases}$$

$$\theta=10,\ \cancel{-153}\ 130°$$

9. Solve for $x\in[0,\ \pi)$:

$\csc\left(4x-\frac{\pi}{3}\right)=2+2\csc\left(4x-\frac{\pi}{3}\right)$

$$\csc\left(4x-\tfrac{\pi}{3}\right)=-2$$

$$\sin\left(4x-\tfrac{\pi}{3}\right)=\frac{-1}{2}$$

$$4x-\frac{\pi}{3}=\begin{cases}-\pi/6\pm 2\pi n\\7\pi/6\pm 2\pi n\end{cases}$$

$$4x=\begin{cases}\pi/6\pm 2\pi n\\\frac{3\pi}{2}\pm 2\pi n\end{cases}$$

$$x=\begin{cases}\pi/24\pm \pi/2\, n\\3\pi/8\pm \pi/2\, n\end{cases}$$

$$=\frac{\pi}{24},\ \frac{5\pi}{24},\ \frac{3\pi}{8},\ \frac{7\pi}{8}$$

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For Problem 1-6, use:

(20, -21) is on the terminal side of C and
$\sec Q = -\frac{13}{12}$ in QIII
to find the exact values of:

1. sin (C-Q) $= \sin C \cos Q - \cos C \sin Q$

$= \left(\frac{-21}{29}\right)\left(\frac{-12}{13}\right) - \left(\frac{20}{29}\right)\left(\frac{-5}{13}\right)$

$= \frac{352}{377}$

2. cos (Q-C) $= \cos Q \cos C + \sin Q \sin C$

$= \left(\frac{-12}{13}\right)\left(\frac{20}{29}\right) + \left(\frac{-5}{13}\right)\left(\frac{-21}{29}\right)$

$= \frac{-135}{377}$

3. $\tan(C-Q)$ $= \frac{\tan C - \tan Q}{1 + \tan C \tan Q}$

$= \frac{-21/20 - 5/12}{1 + \left(\frac{-21}{20}\right)\left(\frac{5}{12}\right)} = \frac{-352/240}{135/240}$

$= \frac{-352}{135}$

4. sec (2Q) $= \frac{1}{\cos^2 Q - \sin^2 Q}$

$= \frac{1}{\left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2}$

$= \frac{169}{119}$

5. cot (2Q) $= \frac{1 - \tan^2 Q}{2\tan Q}$

$= \frac{1 - (5/12)^2}{2(5/12)} = \frac{119/144}{5/6}$

$= \frac{119}{120}$

6. $\csc(2Q)$ $= \frac{1}{2\sin Q \cos Q}$

$= \frac{1}{2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)}$

$= \frac{169}{120}$

7. Prove: $\dfrac{\cos^4 x-\sin^4 x}{\cos^3 x-\sin^3 x}=-\dfrac{\cos x+\sin x}{1+\sin x\cos x}$

$$\frac{(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)}{(\cos x-\sin x)(\cos^2 x+\sin x\cos x+\sin^2 x)}$$

$$\frac{\cancel{(\cos x-\sin x)}(\cos x+\sin x)(1)}{\cancel{(\cos x-\sin x)}(1+\sin x\cos x)}$$

9. Solve for $\theta\in[0°,\ 360°]$: $\cos\theta\sin\theta=\dfrac{\sqrt{3}}{4}$

$2\sin\theta\cos\theta=\dfrac{\sqrt{3}}{2}$

$\sin 2\theta=\sqrt{3}/2$

$2\theta=\begin{cases}60\pm 360n\\120\pm 360n\end{cases}$

$\theta=\begin{cases}30\pm 180n\\60\pm 180n\end{cases}$

$\theta=\{30, 210, 60, 240\}$

8. Solve for $x\in[-360°,\ 360°]$:
$3-3\sin x-2\cos^2 x=0$

$3-3\sin x-2(1-\sin^2 x)$

$2\sin^2 x-3\sin x+1=0$

$(2\sin x-1)(\sin x-1)=0$

$\sin x=\frac{1}{2}$ $\quad\sin x=1$

$x=\begin{cases}30\pm 360n\\150\pm 360n\end{cases}$ $\quad x=90\pm 360n$

$x=\{30, -330, 150, -210, 90, -270\}$