

PreCalculus ACC '19-20
Chapter 10 Test – Form A
CALCULATOR ALLOWED

Name: SOLUTION KEY

Score _____

Round to 3 decimal places. Show all work.

Find the derivative of each of the following functions.

1a. $f(x) = e^{5x^2+x^4}$

$$f' = e^{5x^2+x^4} (10x+4x^3)$$

1b. $g(x) = \tan^4 x$

$$g'(x) = 4 \tan^3 x \sec^2 x$$

1c. $h(x) = \sin x^4$

$$h' = \cos x^4 (4x^3)$$

1d. $p(x) = x \csc x$

$$p' = x(-\csc x \cot x) + \csc x (1)$$

$$= \csc x (1 - x \cot x)$$

2. Find domain and x -intercepts of $y = (x+1)\sqrt{9-x^2}$.

$$\begin{aligned} &(-1, 0) \\ &(\pm 3, 0) \\ &x \in [-3, 3] \end{aligned}$$

3. Find the extreme points of $y = (x+1)\sqrt{9-x^2}$. Show the algebraic work to support the critical values.

$$\begin{aligned} \frac{dy}{dx} &= (x+1) \frac{1}{2} (9-x^2)^{-1/2} (-2x) + (9-x^2)^{1/2} (1) \\ &= \frac{-x(x+1)}{(9-x^2)^{1/2}} + \frac{(9-x^2)}{(9-x^2)^{1/2}} = \frac{-2x^2 - x + 9}{(9-x^2)^{1/2}} \end{aligned}$$

$$\text{i) } \frac{dy}{dx} = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+72}}{-4} = \begin{cases} -2.386 \\ 1.806 \end{cases}$$

$$\text{ii) } \frac{dy}{dx} \text{ DNE} \Rightarrow x = \pm 3$$

iii) NO END POINTS GIVEN

$$\begin{aligned} &(\pm 3, 0) \quad (-2.386, -2.520) \\ &\quad \quad \quad (1.806, 6.733) \end{aligned}$$

4. Find domain and x -intercepts of $y = (x^2 - 3x - 10)e^{\frac{1}{2}x}$.

$$(5, 0) \quad (-2, 0) \quad (x-5)(x+2)$$

DOMAIN ALL REALS

5. Find the extreme points of $y = (x^2 - 3x - 10)e^{\frac{1}{2}x}$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = (x^2 - 3x - 10)e^{-\frac{1}{2}x} \left(-\frac{1}{2}\right) + e^{-\frac{1}{2}x} (2x - 3)$$

$$= -\frac{1}{2}e^{-\frac{1}{2}x} [x^2 - 3x - 10 + 4x + 6]$$

$$= -\frac{1}{2}e^{-\frac{1}{2}x} (x^2 - 7x - 4)$$

$$\frac{dy}{dx} = 0 \rightarrow x = \frac{7 \pm \sqrt{63}}{2} = \begin{cases} \text{---} & -0.531 \\ \text{---} & 7.531 \end{cases}$$

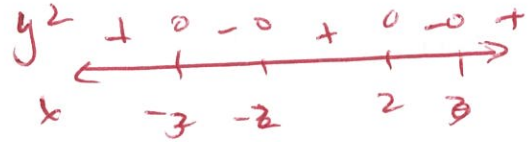
$$\left(\text{---}, \text{---}\right), \quad (-0.531, -10.595) \\ (7.531, 558)$$

6. Find domain, VAs, and x -intercepts of $f(x) = \ln(x^4 - 13x^2 + 36)$ on $x \in [-4, 5]$.

$$(x^2 - 9)(x^2 - 4)$$

VAs @ $x = \pm 3, \pm 2$

zeros: $(\pm 1.951, 0)$
 $(\pm 3.032, 0)$



Domain $y \in [-4, -3) \cup (-2, 2) \cup (3, 5]$

7. Find the extreme points of $f(x) = \ln(x^4 - 13x^2 + 36)$ on $x \in [-4, 5]$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = \frac{4x^3 - 26x}{(x^2 - 9)^2 (x^2 - 4)^2}$$

i) $\frac{dy}{dx} = 0 \rightarrow x = 0, \pm \sqrt{\frac{13}{2}}$ $(0, \ln 36)$

ii) $\frac{dy}{dx} \text{ DNE} \rightarrow x = \pm 3, \pm 2$

NOT EXTREMES

iii) END POINTS $x = -4, 5$

$(4, 4.431)$

$(5, 5.817)$

8. Find the traits and **sketch** $y = (x+1)\sqrt{9-x^2}$.

Domain: $x \in [-3, 3]$

Range: $y \in [-2.520, 6.733]$

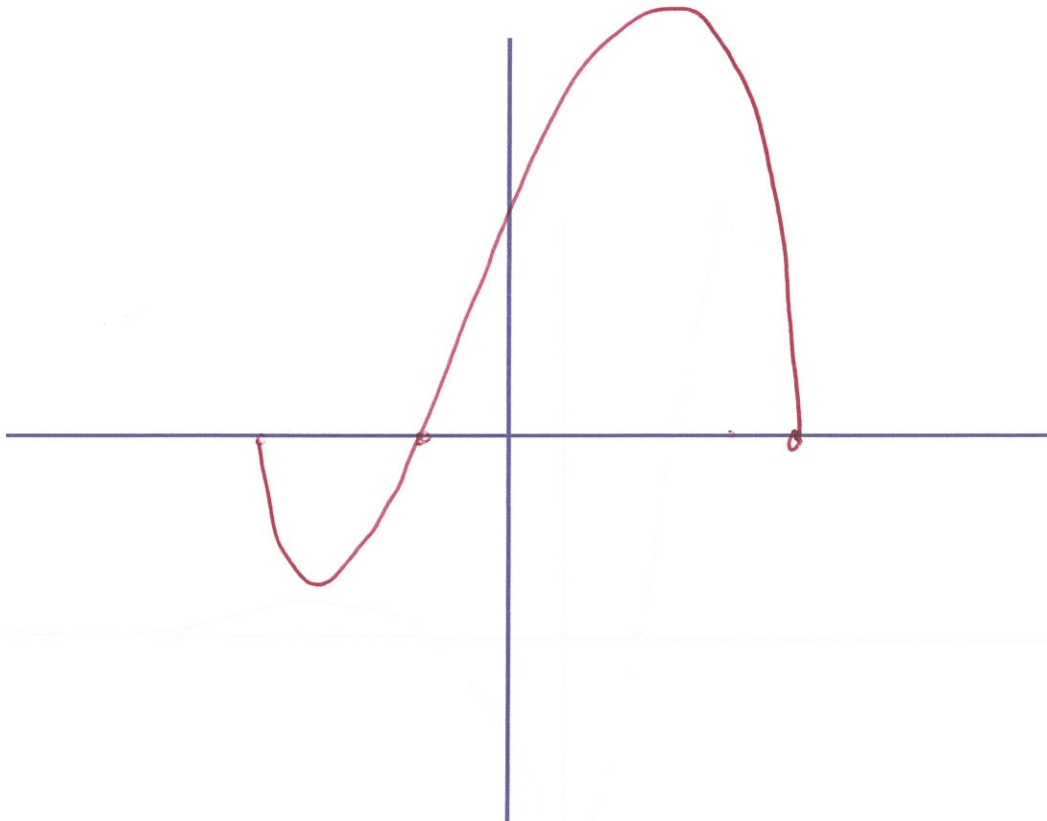
x - intercepts: $(-1, 0)$ $(\pm 3, 0)$

y - intercept: $(0, 3)$

Extreme Points: *SEE #3*

End Behavior (Left): *NONE*

End Behavior (Right): *NONE*



9. Find the traits and **sketch** of $y = (x^2 - 3x - 10)e^{-\frac{1}{2}x}$.

Domain: *ALL REALS*

Range: $y \geq -10.595$

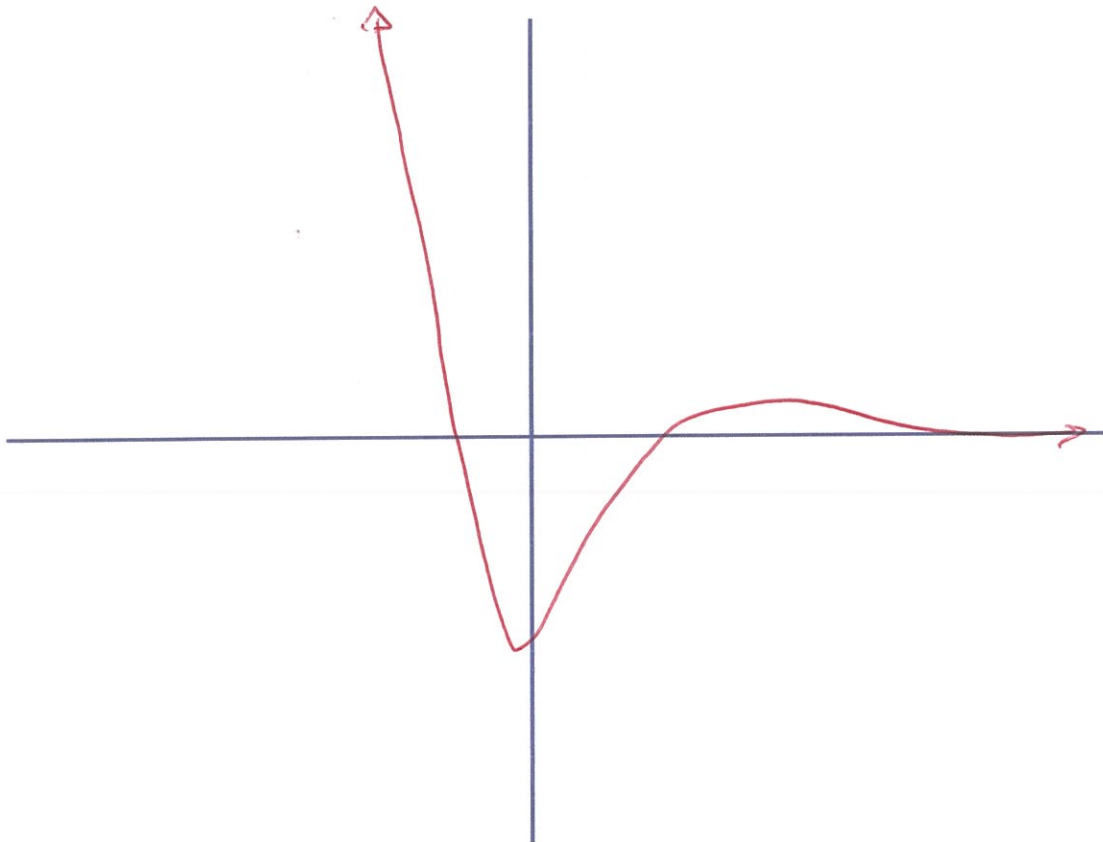
x - intercepts:

y - intercept: $(0, -10)$

Extreme Points: *SEE # 5*

End Behavior (Left): *UP*

End Behavior (Right): $y = 0$



10. Find the traits and **sketch** of $f(x) = \ln(x^4 - 13x^2 + 36)$ on $x \in [-4, 5]$.

Domain: *SEE #6*

Range: $y \in (-\infty, 5.817]$

VAs: $x = \pm 2, \pm 3$

y-intercept: $(0, 3.584)$

Extreme Points: *SEE #7*

End Behavior (Left): *NONE*

End Behavior (Right): *NONE*

