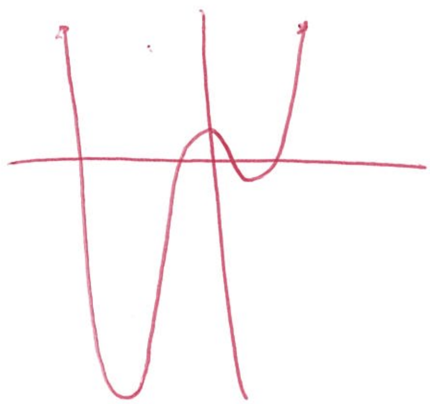
PreCalculus Acc 2019-20 Name Solution Key
Fall Final
Dr. Quattrin score ___________
PART I: Calculator allowed

1. Given $y = 21x^4 + 47x^3 - 59x^2 - 27x + 18$, use your calculator to sketch the complete graph and state the window used.



$x \in [-4.7 \times 4.7]$

$y \in [-250, 100]$

2. Use your calculator to find the Zeros and Extreme Points of
$y = 21x^4 + 47x^3 - 59x^2 - 27x + 18$

Zeros: $(-2/3, 0)$ $(-3, 0)$, $(.429, 0)$ $(1, 0)$

MIN $(-2.241, -217.109)$
$(.753, -8.966)$

max $(-.190, 20.705)$

3. Find:

a. $\lim_{x\to 5} \dfrac{x^2-8x+15}{x^3-5x^2+6x-30}$

$$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x-3)}{x^2\cancel{(x-5)}+6\cancel{(x-5)}}$$

$$= \frac{2}{31}$$

$$\begin{array}{r|rrr} 5 & 1 & -8 & 15 \\ & & 5 & -15 \\ \hline & 1 & -3 & 0 \end{array}$$

5

b. $\lim_{x\to -\frac{2}{3}} \dfrac{12x^2+5x-2}{6x^2+13x+6}$

$$= \lim_{x \to -2/3} \frac{\cancel{(x+2/3)}(12x-3)}{\cancel{(x+2/3)}(6x+9)}$$

$$= \frac{12(-2/3)-3}{6(-2/3)+9} = \frac{-11}{5}$$

$$\begin{array}{r|rrr} -2/3 & 12 & 5 & -2 \\ & & -8 & 2 \\ \hline & 12 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrr} -2/3 & 6 & 13 & 6 \\ & & -4 & -6 \\ \hline & 6 & 9 & 0 \end{array}$$

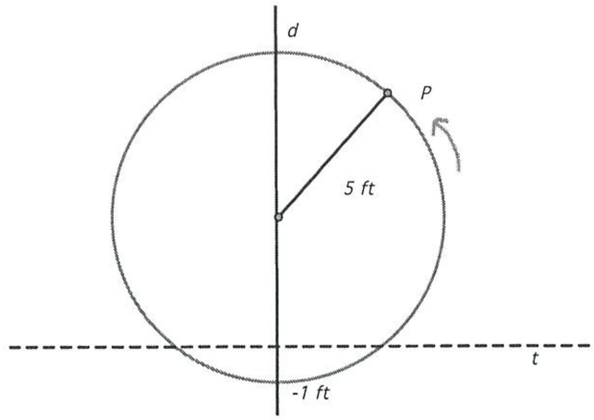
c. $\lim_{x\to 2} \dfrac{x^4-5x^3+10x^2+3x+10}{x^3+7x-22}$

$$= \frac{32}{0}$$

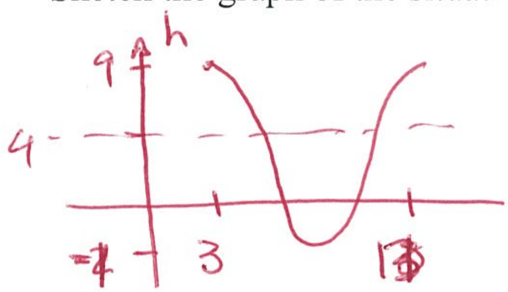
$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 10 & 3 & 10 \\ & & 2 & -6 & 8 & 22 \\ \hline & 1 & -3 & 4 & 11 & 32 \end{array}$$

4. Gold was discovered in California at Sutter's Mill in 1848 when James Marshal; was digging a millrace in the America River near Coloma to build a water-driven sawmill. A millrace is a trench for water to flow through so it can turn a wheel that drives the saws.

Suppose the waterwheel has a five-foot radius and sits one foot into the millrace as pictured here. Assume that it rotates at 5 revolutions per minute and that the height h from point P to the water's surface varies sinusoidally with time. Three seconds after you start timing the motion, point P is at its highest.



a. Sketch the graph of the situation.



b. Create an equation that models the height h in terms of time t.

$$h = 4 + 5\cos\frac{\pi}{6}(t-3)$$

c. What is the first time t when P ***emerges*** from the water?

$$0 = 4 + 5\cos\frac{\pi}{6}(t-3)$$

$$-4/5 = \cos\frac{\pi}{6}(t-3)$$

$$\left.\begin{array}{l} 2.498 \pm 2\pi n \\ -2.498 \pm 2\pi n \end{array}\right\} = \frac{\pi}{6}(t-3)$$

$$\left.\begin{array}{l} 4.771 \pm 18n \\ -4.771 \pm 18n \end{array}\right\} = t-3$$

$$= t = \left\{\begin{array}{l} 7.771 \pm 12n \\ -1.771 \pm 12n \end{array}\right. \rightarrow t = 10.229$$

5. Find the zeros of $y = 2x^3 - 3x^2 - 12x + 18$. Show the algebraic work (factoring) to support the zeros.

$y = x^2(2x-3) - 6(2x-3)$

$y = (x^2-6)(2x-3)$

$(3/2, 0)$ $(\cancel{3/2, 0})$ $(\pm\sqrt{6}, 0)$

6. Find the extreme points of $y = 2x^3 - 3x^2 - 12x + 18$. Show the derivative and algebra to support the critical values.

$\frac{dy}{dx} = 6x^2 - 6x - 12 = 0$

$= x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$(2, -2)$ $(1, 25)$

7. Prove the identity: $\dfrac{2\cot^2 w - 7\csc w - 2}{5\cot^2 w + 23\csc w + 17} = \dfrac{2\csc w + 1}{5\csc w - 3}$

$$\frac{2(\csc^2 w - 1) - 7\csc w - 2}{5(\csc^2 w - 1) - 23\csc w + 17}$$

$$\frac{2\csc^2 w - 7\csc w - 4}{5\csc^2 w - 23\csc w + 12}$$

$$\frac{(2\csc w + 1)\cancel{(\csc w - 4)}}{(5\csc w - 3)\cancel{(\csc w - 4)}}$$

8. $\sec B = -\frac{5}{4}$ in Quadrant III. Find the other five exact trig values and the approximate value of B: $y = -3$

$\sin B =$ $-3/5$ $\cos B =$ $-4/5$ $\tan B =$ $3/4$

$\cot B =$ $4/3$ $\csc B =$ $-5/3$ $\sec B = -\frac{5}{4}$

$B =$ $-2.498 \pm 2\pi n$ or

$-143.130 \pm 360n$

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PART II: **NO** calculator allowed

9. Use the equation of the line tangent to $f(x) = 3x^3 + x^2 - 3x - 1$ at $x = 1$ to approximate $f(1.1)$.

$f'(x) = 9x^2 + 2x - 3$

$f(1) = 0$

$f'(1) = 8$

$y = 8(x-1)$

$f(1.1) \approx 8(1.1-1) = .8$

10. Find the sign pattern and solution to $2x^3 - 3x^2 - 12x + 18 < 0$

[Note: This is the same polynomial in #5.)

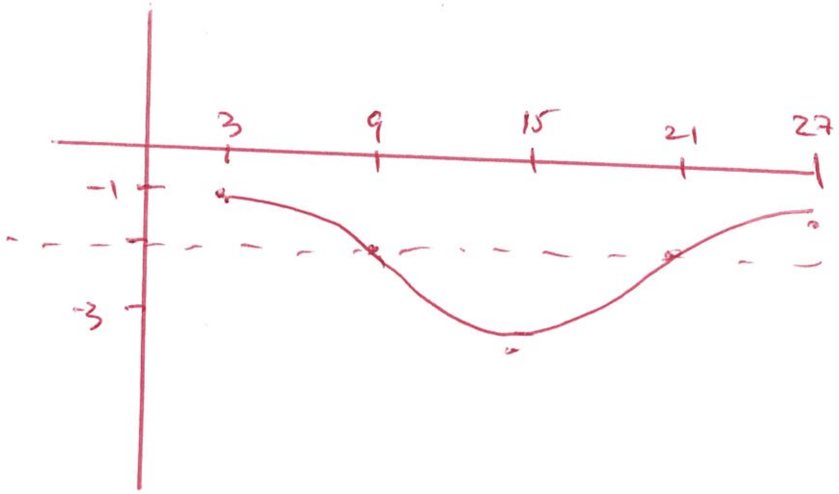
$(2x-3)(x^2-6) < 0$

$y \quad - \quad 0 \quad + \quad 0 \quad - \quad 0 \quad +$

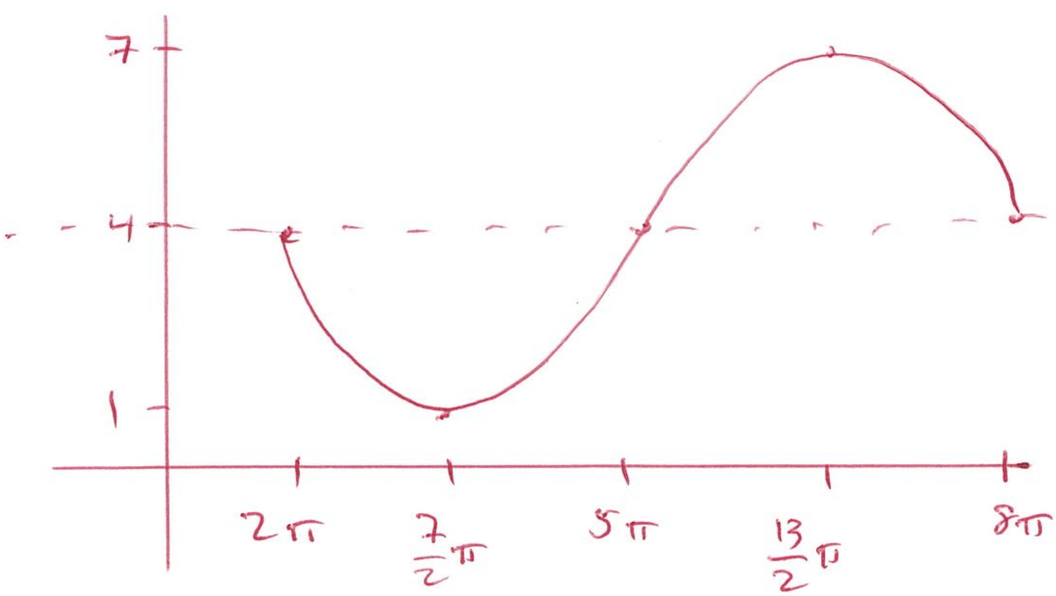
$x \quad -\sqrt{6} \quad \frac{3}{2} \quad \sqrt{6}$

$x \in (-\infty, -\sqrt{6}) \cup (3/2, \sqrt{6})$

11. Graph on cycle of $y = -2 + \cos\left[\frac{\pi}{12}(x-3)\right]$.



12. Graph one cycle of $y = 4 - 3\sin\left[\frac{1}{3}(x-2\pi)\right]$.



13. Use the Power Rule to find:

a. $\frac{dy}{dx}$ if $y = -2x^5 - 13x^3 + 5x^2 - 9x + 7$

$$\frac{dy}{dx} = -10x^4 - 39x^2 + 10x - 9$$

b. $f'(x)$ if $f(x) = x^4 - 3\sqrt[6]{x^7} + e^2 - \frac{1}{\sqrt[3]{x^8}} + \frac{2}{3x}$

$$f(x) = x^4 - 3x^{7/6} + e^2 - x^{-8/3} + \frac{2}{3}x^{-1}$$

$$f'(x) = 4x^3 - \frac{7}{2}x^{1/6} + \frac{8}{3}x^{-11/3} - \frac{2}{3}x^{-2}$$

c. $\frac{d}{dx}\left[\frac{2}{x^3} - 9\sqrt[3]{x^7} + \sqrt{x^5} - \pi x\right]$

$$= \frac{d}{dx}\left[2x^{-3} - 9x^{7/3} + x^{5/2} - \pi x\right]$$

$$= -6x^{-4} - 21x^{4/3} + \frac{5}{2}x^{3/2} - \pi$$

14. Find the traits and **sketch** $y = 2x^3 - 3x^2 - 12x + 18$.

Domain: $x \in \{\text{ALL REALS}\}$

Range: $y \in \{\text{ALL REALS}\}$

Zeros: $(2/3, 0)\ (\pm\sqrt{6}, 0)$

y – intercept: $(0, 18)$

Extreme Points: $(2, -2)\ (1, 25)$

End Behavior (Left): DOWN

End Behavior (Right): UP