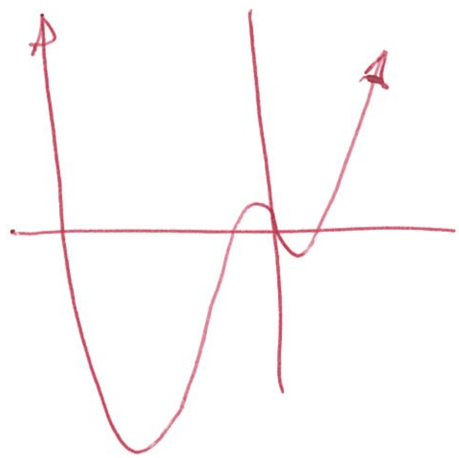
PreCalculus Acc 2019-20
Practice Fall Final
Dr. Quattrin
PART I: Calculator allowed

Name Solution Key

score ________

1. Given $f(x)=5x^4+122x^3-655x^2+486x-72$, use your calculator to sketch the complete graph and state the window used.



$x \in [-40, 5]$

$y \in [-500{,}000, 10{,}000]$

2. Use your calculator to find the Zeros and Extreme Points of $f(x)=5x^4+122x^3-655x^2+486x-72$

Zeros: $(-29.029, 0)$ $(3.766, 0)$
$(.162, 0)$ $(.701, 0)$

Ext Pts: $(-21.412, -457\,432.1)$
$(.422, 35.774)$
$(2.690, -857.757$

3. Find:

a. $\lim_{x\to\frac{1}{3}} \dfrac{6x^2+x-1}{3x^2-16x+5}$ $= \lim_{x\to 1/3} \dfrac{\cancel{(3x-1)}(2x+1)}{\cancel{(3x-1)}(x-5)}$

$= \dfrac{2/3+1}{2/3-5} = \dfrac{5/3}{-14/3} = -\dfrac{5}{14}$

b. $\lim_{x\to-4} \dfrac{x^4-14x^2+3x-20}{x^3+6x^2-2x-40}$

$= \lim_{x\to-4} \dfrac{\cancel{(x+4)}(x^3-4x^2+2x-5)}{\cancel{(x+4)}(x^2+2x-10)}$

$= \dfrac{(-4)^3-4(-4)^2+2(-4)-5}{(-4)^2+2(-4)-10} = \dfrac{141}{2}$

$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -14 & 3 & -20 \\ & & -4 & 16 & -8 & 20 \\ \hline & 1 & -4 & 2 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 6 & -2 & -40 \\ & & -4 & -8 & 40 \\ \hline & 1 & 2 & -10 & 0 \end{array}$$

c. $\lim_{x\to-1} \dfrac{x^4+2x^3+x^2}{x^3-7x^2-6x}$ $= x\ \dfrac{0}{-2}$

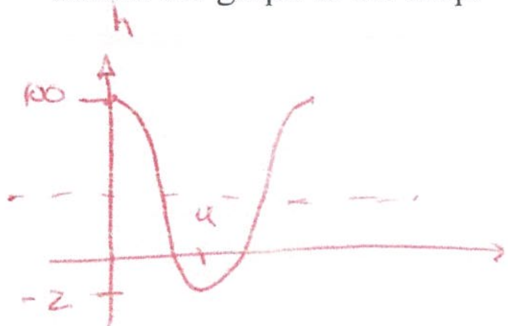
$= \lim_{x\to-1} \dfrac{(x+1)(x^3+x^2)}{}$

$$\begin{array}{r|rrr} -1 & 1 & 2 & 1 \\ & & -1 & -1 \\ \hline & 1 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrr} -1 & 1 & -7 & -6 \\ & & -1 & 8 \\ \hline & 1 & -8 & 2 \end{array}$$

5. In episode 218 of the Mythbusters, Jamie Hyneman bungee-jumped from a 100-foot-high lift to bob for apples in a pool. Like the Bouncing Spring Problem, Jamie's height from the water's surface varies sinusoidally with time. It took four seconds for Jamie to fall from the tower to the water and two feet under the surface.

a. Sketch the graph of his drop.



b. Create an equation that models Jamie's height h in terms of time t.

$h = 49 + 5\phi \cos \frac{\pi}{4} t$

c. How far above the water is Jamie at 1.4 seconds?

$h(1.4) = 72.154$ FT

d. At what time did Jamie come back out of the water?

$0 = 49 + 51 \cos \frac{\pi}{4} t$

$-.961 = \cos \frac{\pi}{4} t$

$\left.\begin{matrix} 2.861 \pm 2\pi n \\ -2.861 \pm 2\pi n \end{matrix}\right\} = \frac{\pi}{4} t$

$t = \begin{matrix} 3.642 \pm 8n \\ -3.642 \pm 8n \end{matrix}$

$t_2 = 4.358$ SEC

5. Find the zeros of $y = -2x^3 + 9x^2 + 24x - 108$ on $x \in [-5, 3]$. Show the algebraic work (factoring) to support the zeros.

$y = -x^2(2x-9) + 12(2x-9)$

$= (12 - x^2)(2x-9)$

$(\pm 2\sqrt{3}, 0)$ $(9/2, 0)$

6. Find the extreme points of $y = -2x^3 + 9x^2 + 24x - 108$ on $x \in [-5, 3]$. Show the derivative and algebra to support the critical values.

$\frac{dy}{dx} = -6x^2 + 18x + 24 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

$\cancel{(4, 4)}$ $(-5, 463)$

$(-1, -121)$ $(3, +9)$

7. Prove the identity: $\dfrac{\tan^2 w - \sec w - 5}{\tan^2 w + 3\sec w + 3} = \dfrac{\sec w - 3}{\sec w + 1}$

$$\frac{\sec^2 w - 1 - \sec w - 5}{\sec^2 w - 1 + 3\sec w + 3}$$

$$\frac{\sec^2 w - \sec w - 6}{\sec^2 w + 3\sec w + 2}$$

$$\frac{(\sec w - 3)\cancel{(\sec w + 2)}}{(\sec w + 1)\cancel{(\sec w + 2)}}$$

8. $\csc B = \dfrac{29}{20}$ in Quadrant II. Find the other five exact trig values and the approximate value of B:

$\sin B =$ $20/29$ $\cos B =$ $-21/29$ $\tan B =$ $-20/21$

$\cot B =$ $-21/20$ $\csc B = \dfrac{29}{20}$ $\sec B =$ $-29/21$

$B =$ $2.781 \pm 2\pi n$ or $136.397 \pm 360n$

$0.$

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PART II: **NO** calculator allowed

9. Use the equation of the line tangent to $f(x)=5x^3+3x^2-4x$ at $x=-1$ to approximate $f(-0.9)$.

$f'(x) = 15x^2 + 6x - 4)$

$f(-1) = -5 + 3 + 4 = 2$

$m = f'(-1) = 5$

$y - 2 = 5(x+1)$

$f(-.9) \approx y - 2 = 5(-.9+1)$

$y = 2.5$

10. Find the sign pattern and solution to $-2x^3+9x^2+24x-108<0$
[Note: This is the same polynomial in #5.)

$(12 - x^2)(2x+9) < 0$

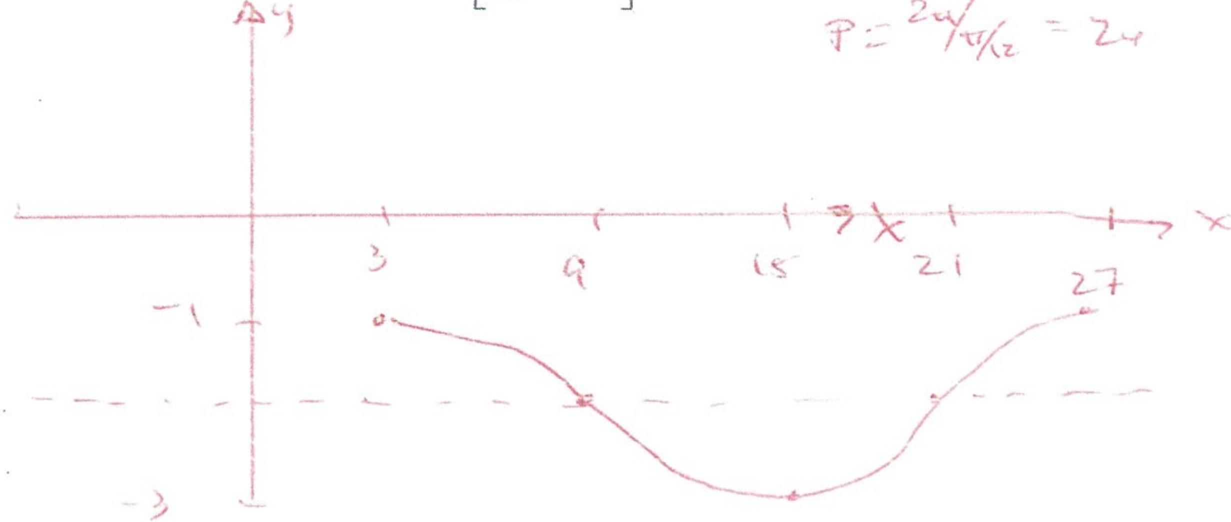
y: $+$ 0 $-$ 0 $+$ 0 $-$

$-2\sqrt{3}$ $2\sqrt{3}$ 4.5

$x \in (-2\sqrt{3}, 2\sqrt{3}) \cup (4.5, \infty)$

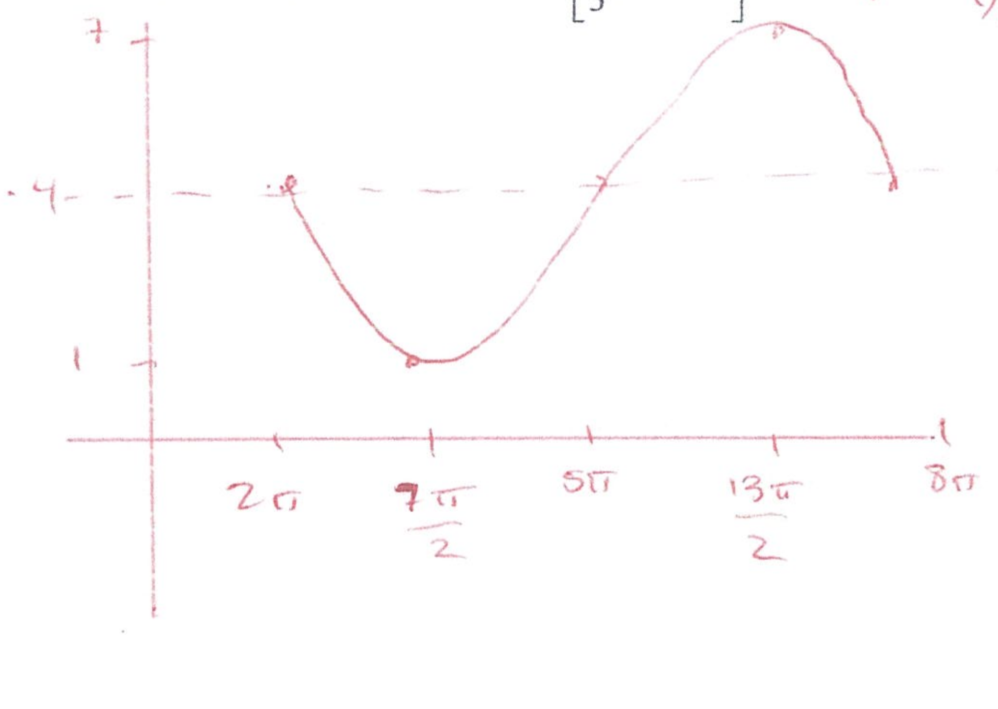
11. Graph on cycle of $y=-2+\cos\left[\frac{\pi}{12}(x-3)\right]$.

$P = 2\pi / \frac{\pi}{12} = 24$



12. Graph one cycle of $y=4-3\sin\left[\frac{1}{3}(x-2\pi)\right]$.

$P = \frac{2\pi}{1/3} = 6\pi$



13. Use the Power Rule to find:

a. $\frac{dy}{dx}$ if $y = 12x^7 - 31x^4 + 25x^2 - 7x + 5$

$\frac{dy}{dx} = 84x^6 - 124x^3 + 50x - 7$

b. $f'(x)$ if $f(x) = x^5 - 7\sqrt[5]{x^7} + 7^2 - \frac{1}{\sqrt[3]{x^{10}}} + \frac{5}{6x}$ $= x^5 - 7x^{7/5} + 7^2 - x^{-10/3} + \frac{5}{6}x^{-1}$

$f'(x) = 5x^4 - \frac{49}{5}x^{2/5} + 0 + \frac{10}{3}x^{-13/3} - \frac{5}{6}x^{-2}$

c. $\frac{d}{dx}\left[\frac{12}{x^4} - 8\sqrt[4]{x^3} + \sqrt{x^7} - \pi x^3\right]$

$= \frac{d}{dx}\left[12x^{-4} - 8x^{3/4} + x^{7/2} - \pi x^3\right]$

$= -48x^{-5} + 6x^{-1/4} + \frac{7}{2}x^{5/2} - 3\pi x^2$

14. Find the traits and **sketch** $y = 2x^3 - 9x^2 - 24x + 108$ on $x \in [-5, 3]$.

Domain: $x \in [-5, 3]$

Range: $y \in [-121, 463]$

Zeros: $(\pm 2\sqrt{3}, 0)$ $(4.5, 0)$

y – intercept: $(0, 108)$

Extreme Points:

SEE #6

End Behavior (Left): NONE

End Behavior (Right): NONE