PreCalculus ACC '19-20
Trig Identities Group Test
Page 1

Name: Solution Key

Score__________

For Problem 1-6, use:

(15, -8) is on the terminal side of C and $\sin C = \frac{-8}{17}$ $\cos C = \frac{15}{17}$ $\tan C = \frac{-8}{15}$

$\sin Q = \dfrac{24}{25}$ in QII $\cos Q = \frac{-7}{25}$ $\tan Q = -\frac{24}{7}$

to find the exact values of:

1. sin (C-Q)

$$= \left(\frac{-8}{17}\right)\left(\frac{\cancel{24}\ -7}{25}\right) - \left(\frac{\cancel{7}\ 24}{25}\right)\left(\frac{15}{17}\right)$$

$$= \frac{56\ \cancel{192} - \cancel{105}\ 360}{425}$$

$$= \frac{\cancel{87} -304}{425}$$

2. cos (Q-C)

$$= \left(\frac{-7}{25}\right)\left(\frac{15}{17}\right) + \left(\frac{24}{25}\right)\left(\frac{-8}{17}\right)$$

$$= \frac{-105 - 192}{425}$$

$$= \frac{-297}{425}$$

3. $\tan(C-Q)$ $= \dfrac{\frac{-8}{15} - \left(\frac{-24}{7}\right)}{1 + \left(\frac{-8}{15}\right)\left(\frac{-24}{7}\right)}$

$$= \frac{-56 + 360}{105} \cdot \frac{105}{105 + 192}$$

$$\frac{304}{297}$$

4. sec (2Q) $= \dfrac{1}{\cos^2 Q - \sin^2 Q}$

$$= \frac{1}{\left(\frac{-7}{25}\right)^2 - \left(\frac{24}{25}\right)^2}$$

$$= -\frac{625}{527}$$

5. cot (2Q) $= \dfrac{1 - \left(\frac{-24}{7}\right)^2}{2\left(\frac{-24}{7}\right)}$

$$= \frac{-527}{49} \cdot \frac{7}{-48} = \frac{527}{336}$$

6. $\csc(2Q)$ $= \dfrac{1}{2\left(\frac{24}{25}\right)\left(\frac{-7}{25}\right)}$

$$= \frac{-625}{336}$$

7. Prove: $\dfrac{\tan^2 w - \sec w - 5}{\tan^2 w + 3\sec w + 3} = \dfrac{\sec w - 3}{\sec w + 1}$

$$\frac{\sec^2 w - 1 - \sec w - 5}{\sec^2 w - 1 + 3\sec w + 3}$$

$$\frac{\sec^2 w - \sec w - 6}{\sec^2 w + 3\sec w + 2}$$

$$\frac{\cancel{(\sec w + 2)}(\sec w - 3)}{\cancel{(\sec w + 2)}(\sec w + 1)}$$

9. Solve exactly for $x \in [0°, 360°)$:
$\sin\theta = \sin 2\theta$

$$0 = \sin 2\theta - \sin\theta$$

$$= 2\sin\theta\cos\theta - \sin\theta$$

$$0 = \sin\theta(2\cos\theta - 1)$$

$\sin\theta = 0$ $\qquad$ $\cos\theta = 1/2$

$\theta = \begin{cases} 0 \pm 360n \\ 180 \pm 360n \end{cases}$ $\qquad$ $\theta = \begin{cases} 60 \pm 360n \\ -60 \pm 360n \end{cases}$

$\theta = 0°, 180°, \cancel{360}, 60°, 300°$

8. Solve exactly for x:
$2\left(\cot\frac{1}{2}x\right)(1 - \cos x)\dfrac{\csc x}{\sec x} = \sqrt{3}$

$$2(\cot w)(1 - \cos 2w)\frac{\cos w}{\sin w}$$

$$2\,\frac{\cos^2 w}{\cancel{\sin^2 w}}\cancel{(1 - (1 - 2\sin^2 w))}$$

$$\cos^2 w = \frac{\sqrt{3}}{2}$$

PreCalculus ACC '19-20
Trig Identities Group Test
Page 2

Name: Solution Key

Score__________

For Problem 1-6, use:

(15, -8) is on the terminal side of C and
$\sin Q = \frac{24}{25}$ in QII

to find the exact values of:

1. sin (C+Q)

$$\left(\frac{-8}{17}\right)\left(\frac{\cancel{24}\ -7}{25}\right)+\left(\frac{24}{25}\right)\left(\frac{15}{17}\right)$$

$$= \frac{\cancel{192}\ 56 + \cancel{105}\ 360}{425}$$

$$= \frac{\cancel{-297}\ 416}{425}$$

2. cos (C-Q)

$$= \left(\frac{24}{25}\right)\left(\frac{-8}{17}\right)+\left(\frac{-7}{25}\right)\left(\frac{15}{17}\right)$$

$$= \frac{-192-105}{425}$$

$$= \frac{-297}{425}$$

3. $\tan(C+Q)$

$$= \frac{\frac{-8}{15}+\left(\frac{-24}{7}\right)}{1-\left(\frac{-8}{15}\right)\left(\frac{-24}{7}\right)}$$

$$= \frac{-56-360}{105}\cdot\frac{105}{105-192}$$

$$= \frac{-416}{87}$$

4. sec (2C)

$$= \frac{1}{\left(\frac{15}{17}\right)^2-\left(\frac{-8}{17}\right)^2}$$

$$= \frac{289}{225-64}$$

$$= \frac{289}{161}$$

5. cot (2C)

$$= \frac{1-\left(\frac{-8}{15}\right)^2}{2\left(\frac{-8}{15}\right)}$$

$$= \frac{(225-64)}{225}\cdot\frac{15}{-16}$$

$$= -\frac{161}{240}$$

6. $\csc 2C$

$$= \frac{1}{2\left(\frac{15}{17}\right)\left(\frac{-8}{17}\right)}$$

$$= -\frac{289}{240}$$

7. Prove: $\dfrac{2\sin^2 w-5\cos w+1}{6\sin^2 w-5\cos w-2}=\dfrac{\cos w+3}{3\cos w+4}$

$$\frac{2(1-\cos^2 w)-5\cos w+1}{6(1-\cos^2 w)-5\cos w-2}$$

$$\frac{-2\cos^2 w-5\cos w+3}{-6\cos^2 w-5\cos w+4}$$

$$=\frac{2\cos^2 w+5\cos w-3}{6\cos^2 w+5\cos w-4}$$

$$=\frac{(\cos w+3)\cancel{(2\cos w-1)}}{(3\cos w+4)\cancel{(2\cos w-1)}}$$

8. Solve for A: $\cos^4 A-\sin^4 A=1$

$(\cos^2 A-\sin^2 A)(\cos^2 A+\sin^2 A)$

$(\cos 2A)\,(1)=1$

$\cos 2A=1$

$2A=\frac{\pi}{2}\pm 2\pi n$

$A=\pm\frac{\pi}{4}\pm \pi n$

9. Solve exactly for $x\in[0,\pi)$:
$\dfrac{\tan x+\cot x}{\cot x-\tan x}=2$

$$\frac{\tan x+\frac{1}{\tan x}}{\frac{1}{\tan x}-\tan x}$$

$$\frac{\tan^2 x+1}{1-\tan^2 x}=2$$

$\tan^2 x+1=2-2\tan^2 x$

$3\tan^2 x=1$

$\tan^2 x=\frac{1}{\sqrt{3}}$

$\tan x=\pm\frac{1}{\sqrt{3}}$

$x=\pm\frac{\pi}{6}\pm 2\pi n$

$\boxed{x=\frac{\pi}{6},\frac{5\pi}{6}}$

PreCalculus ACC ‘19-20
Trig Identities Group Test
Page 3

Name: Solution Key

Score____________

For Problem 1-6, use:

(15, -8) is on the terminal side of C and
$\sin Q = \dfrac{24}{25}$ in QII
to find the exact values of:

1. sin (2Q) $= 2\left(\frac{24}{25}\right)\left(\frac{-7}{25}\right)$

$= \frac{-336}{625}$

2. cos (2Q) $= \left(\frac{-7}{25}\right)^2 - \left(\frac{24}{25}\right)^2$

$= -\frac{527}{625}$

3. $\tan(2Q) = \dfrac{2\left(-\frac{24}{7}\right)}{1-\left(-\frac{24}{7}\right)^2}$

$= \frac{-48}{7} \cdot \left(\frac{49}{-527}\right)$

$= \frac{336}{527}$

4. sec (C+Q) $= \dfrac{1}{\left(\frac{15}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-8}{17}\right)\left(\frac{24}{25}\right)}$

$= \dfrac{425}{-105+192}$

$= \dfrac{425}{87}$

5. cot (C-Q) $= \dfrac{297}{304}$

See p1 #3

6. $\csc(Q+C) = \cancel{\dfrac{-425}{304}}$ $\dfrac{425}{416}$

See
Page 2 #1

7. Prove:

$$\frac{\cos(A+2B)\cos(B)+\sin(A+2B)\sin(B)}{\sin A\cos B+\cos A\sin B}=\frac{1-\tan A\tan B}{\tan A+\tan B}$$

$$\cancel{\frac{(\cos A\cos 2B-\sin A\sin 2B)\cos B+(\sin A\cos 2B+\cos A\sin 2B)\sin B}{\sin A\cos B+\cos A\sin B}}$$

$$\cos A\cos 2B\cos B-\sin A\sin 2B\cos B+\sin A\cos 2B\sin B+\cos A\sin 2B\sin B$$

$$\frac{\cos(A+2B-B)}{\sin A+B}=\frac{\cos(A+B)}{\sin(A+B)}$$

$$=\cot(A+B)$$

$$=\frac{1}{\tan(A+B)}$$

8. Solve for $\theta\in[0°,360°]$:

$$\frac{1-\tan\theta\tan 15°}{\tan\theta+\tan 15°}=\sqrt{3}$$

$$\cot(\theta+15)=\sqrt{3}$$

$$\tan(\theta+15)°=\frac{1}{\sqrt{3}}$$

$$\theta+15°=\begin{cases}30\pm 360n\\210\pm 360n\end{cases}$$

$$\theta=\begin{cases}15\pm 360n\\195\pm 360n\end{cases}$$

$$\theta=15°,\ 195°$$

9. Solve for $A\in(-2\pi,2\pi)$:

$$\left(\sin\frac{1}{2}A-\cos\frac{1}{2}A\right)^2=\frac{1}{2}$$

$$\sin^2\tfrac{1}{2}A-2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A+\cos\tfrac{1}{2}A=\frac{1}{2}$$

$$1-\sin 2(\tfrac{1}{2}A)=\tfrac{1}{2}$$

$$\frac{1}{2}=\sin A$$

$$A=\begin{cases}\pi/6\pm 2\pi n\\5\pi/6\pm 2\pi n\end{cases}$$

$$A=\frac{\pi}{6},\ -\frac{11\pi}{6},\ \frac{5\pi}{6},\ -\frac{7\pi}{6}$$

PreCalculus ACC '19-20
Trig Identities Group Test
Page 4

Name Solution Key

Score__________

For Problem 1-6, use:

(15, -8) is on the terminal side of C and
$\sin Q = \frac{24}{25}$ in QII
to find the exact values of:

1. sin (2C) $= 2\left(\frac{-8}{17}\right)\left(\frac{15}{17}\right)$

$= \frac{-240}{289}$

2. cos (2C) $= \frac{1}{\sec 2C}$

$= \frac{161}{289}$

(SEE p 2 #4)

3. $\tan(2C) = \frac{1}{\cot 2C}$

$= \frac{-240}{161}$

SEE PAGE 2 #5

4. sec (Q-C) $= \frac{1}{\cos(Q-C)}$

$= \frac{-425}{297}$ (SEE PAGE 1 #2

5. cot (Q-C)

$\tan(Q-C) = \dfrac{\left(\frac{-24}{7}\right) - \left(\frac{-8}{15}\right)}{1 + \left(\frac{-8}{17}\right)\left(\frac{-24}{17}\right)}$

$= -\frac{297}{304}$

$\cot(Q-C) = \frac{-304}{297}$

6. $\csc(Q-C) = \frac{1}{\sin(Q-C)}$

$= \frac{1}{304/425}$ (SEE PAGE 1 #1)

$= \frac{+425}{304}$

7. Prove: $\frac{\cos^4\phi-\sin^4\phi}{\sin\phi\cos\phi}=\frac{(1-\tan^2\phi)\,2}{2\tan\phi}$

$\frac{(\cos^2\phi-\sin^2\phi)(\cos^2\phi+\sin^2\phi)}{\sin\phi\cos\phi}$

$\frac{(\cos 2\phi)(1)}{\frac{1}{2}\sin 2\phi} = (\cot 2\phi)(2)$

$= 2\cot 2\phi$

9. Solve for $x\in[-2\pi,\,2\pi]$:

$3-3\sin x-2\cos^2 x=0$

$3-3\sin x-2(1-\sin^2 x)=$

$2\sin^2 x-3\sin x+1=0$

$(2\sin x-1)(\sin x-1)=0$

$\sin x=\frac{1}{2}$ $\quad$ $\sin x=1$

$x=\begin{cases}\pi/6\pm 2\pi n\\ 5\pi/6\pm 2\pi n\end{cases}$ $\quad$ $x=+\frac{\pi}{2}+2\pi n$

$x=\frac{\pi}{6},\frac{5\pi}{6},\frac{-11\pi}{6},\frac{-7\pi}{6},\frac{\pi}{2},\frac{-3\pi}{2}$

8. Solve for x:

$(\cos^2 x-\sin^2 x)^2-(2\sin x\cos x)^2=\frac{1}{2}$

$(\cos 2x)^2-(\sin 2x)^2$

$\cos 4x=\frac{1}{2}$

$4x=\pm\frac{1\pi}{3}\pm 2\pi n$

$x=\pm\frac{\pi}{12}\pm\frac{\pi}{2}n$