PreCalculus ACC '22-23 Name: Solution Key

Chapter 10 Test

CALCULATOR ALLOWED (20 min) Score__________

Round to 3 decimal places. Show all work.

1. If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$ $(x-1)(3(x^2+2)^2(2x)) + (x^2+2)^3$

$= (x^2+2)^2[6x(x-1) + x^2+2]$

a) $6x(x^2+2)^2$

b) $6x(x-1)(x^2+2)^2$

c) $(x^2+2)^2(x^2+3x-1)$

d) $(x^2+2)^2(7x^2-6x+2)$

e) $-3(x-1)(x^2+2)^2$

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2. If $h(t) = e^{2t}(t+1)$, then $h'(0) =$ $e^{2t}(1) + (t+1)e^{2t}(2) = 1+2$

a) 0 b) 1 c) 2 d) 3 e) 4

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3. A particle is moving along the x-axis in such a way that its velocity at time $t > 0$ is given by $v(t) = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(a) 1 (b) $e^{1/2}$ (c) e (d) $e^{3/2}$

(e) There is no maximum value of v.

v' + 0 −, 0, e

$$\frac{dv}{dt} = \frac{t\left(\frac{1}{t}\right) - \ln t}{t^2} = \frac{1 - \ln t}{t^2}$$

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4. Let f be a differentiable function with $f(4) = 3$ and $f'(4) = -2$, and let g be a function defined by $g(x) = x\,f(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = \cancel{3}\ 4$?

a) $y - 3 = -2(x - 4)$

b) $y - 3 = \frac{1}{2}(x - 4)$

c) $y - 3 = -5(x - 4)$

d) $y - 3 = \frac{1}{5}(x - 4)$

e) $y - 3 = -\frac{1}{5}(x - 4)$

$g' = xf' + f$

$m = 4(-2) + 3 = -5$

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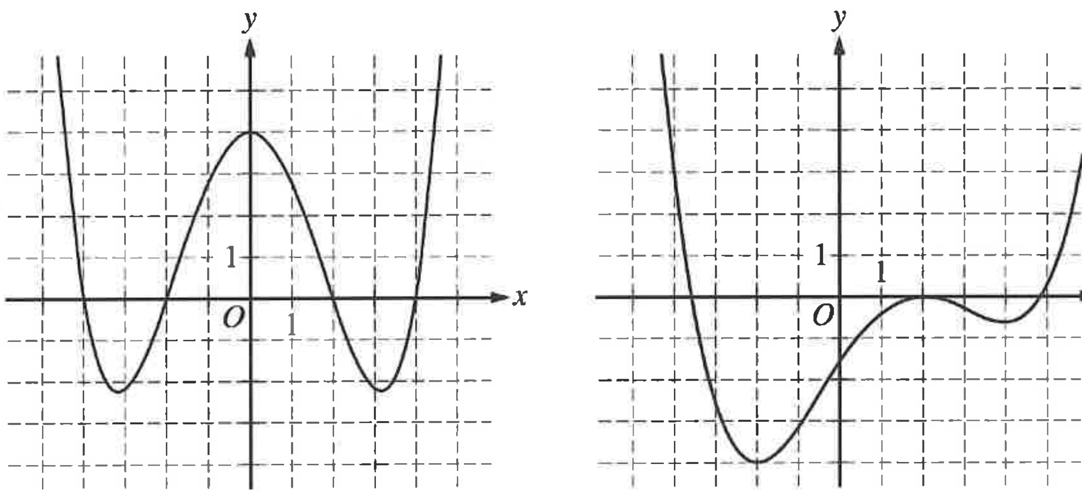
5. $\lim_{x \to \infty} \dfrac{x^2}{e^x} =$ 0

a) 0 b) ∞ c) $-\infty$ d) 1 e) e^x



Graph of f Graph of g

6. The graphs of the differentiable functions $f(x)$ **and** $g(x)$ are shown above. If $P(x) = f(x)g(x)$, which of the following will be true about P'?

a) $P'(2) < 0$

b) $P'(2) > 0$

c) $P'(0) > 0$

d) $P'(0) < 0$

e) $P'(0) = 0$

$P' = f \cdot g' + g \cdot f'$

$P'(0) = 4 \cdot (+) + (-) \cdot 0 > 0$

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7. Given the functions $f(x)$ **and** $g(x)$ that are both continuous and differentiable, and that have values given on the table below, find $h'(4)$, given that $h(x) = g(x) \cdot f(x)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

a) -12 b) 24 c) 0 d) -48 e) 62

$h'(4) = g(4) \cdot f'(4) + f(4) \cdot g'(4)$

$= 4(8) + (10)(3) = 62$

8. Find the end behavior, if any, for $g(x) = e^{-2x}\sqrt{x+1}$.

LEFT: NONE

a) Left end none; $y = 0$ on the right

b) Left end down; $y = 0$ on the right

c) Left end $y = 0$; right end up

d) Left end $y = 0$; right end none

e) $y = 0$ on the left and right

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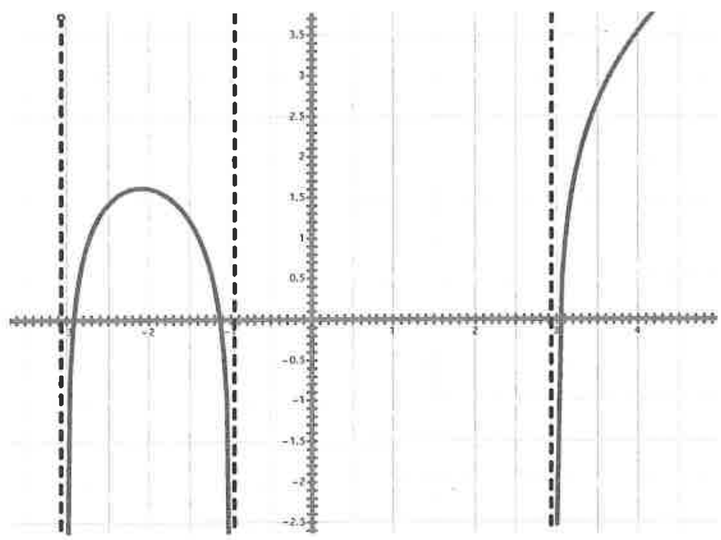
9. Which of the following is the equation of this graph?



ln $(x+3)(x+1)(x-3)$

a) $y = \ln\left(\left(x^2+9\right)(x-1)\right)$

b) $y = \ln\left(\left(x^2-9\right)(x+1)\right)$

c) $y = \ln\left(\left(9-x^2\right)(x+1)\right)$

d) $y = \ln\left(\left(x^2-9\right)(x-1)\right)$

PreCalculus ACC '22-23 Name: SOLUTION KEY

Chapter 10 Test – Form A

CALCULATOR ALLOWED Score __________

Round to 3 decimal places. Show all work.

1. Find domain and x – intercepts of $y=\left(x^2-8\right)e^{-\frac{1}{2}x}$.

DOMAIN: $x \in$ ALL REALS

ZEROS: $(\pm\sqrt{8}, 0)$

2. Find the extreme points of $y=\left(x^2-8\right)e^{-\frac{1}{2}x}$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = (x^2-8)\,e^{-1/2x}(-1/2) + e^{-1/2x}(2x)$$

$$= e^{-1/2x}\left[-\frac{1}{2}x^2+2x+4\right]$$

i) $\frac{dy}{dx}=0 \rightarrow x=\begin{cases}-1.464\\ 5.464\end{cases}$ $(-1.464, -12.177)$ $(5.464, 1.423)$

ii) $\frac{dy}{dx}$ DNE $\rightarrow$ NONE

iii) ENDPOINTS: NONE

3. Find domain and $x -$ intercepts of $y = (x+1)\sqrt{9-x^2}$.

Zeros: $(-1, 0)(\pm 3, 0)$

Domain $x \in [-3, 3]$

4. Find the extreme points of $y = (x+1)\sqrt{9-x^2}$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = (x+1)\left(\frac{-x}{(9-x^2)^{1/2}}\right) + (9-x^2)^{1/2} = \frac{-x^2 - x + 9 - x^2}{(9-x^2)^{1/2}}$$

i) $\frac{dy}{dx} = 0 \rightarrow \begin{cases} -2.386 \\ 1.886 \end{cases}$

$(-2.386, -2.520)$
$(1.886, 6.733)$
$(\pm 3, 0)$

ii) $\frac{dy}{dx}$ DNE $x = \pm 3$

iii) NONE

5. Find domain, VAs, and x – intercepts of $f(x) = \ln(x^3 - 9x)$ on $x \in [-4,\ 5]$.

DOMAIN: $x \in (-3, 0) \cup (3, 5)$

$x^3 - 9x$: $-$ 0 $+$ 0 $-$ 0 $+$

-3 $\quad$ 0 $\quad$ 3

VA: $x = 0, \pm 3$

ZEROS: $(-2.943, 0)$

$(-.111, 0)$

$(3.054, 0)$

6. Find the extreme points of $f(x) = \ln(x^3 - 9x)$ on $x \in [-4,\ 5]$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = \frac{3x^2 - 9}{x^3 - 9x} = \frac{3(x^2 - 3)}{x^3 - 9x}$$

i) $\frac{dy}{dx} = 0 \Rightarrow x = \pm\sqrt{3}$

ii) $\frac{dy}{dx}$ DNE: $x = 0, \pm 3$

iii) $x = 5$

$(-1.732,\ 2.341)$

$(5,\ 4.382)$

DO TWO OF THE FOLLOWING THREE SKETCHING PROBLEMS

7. Find the traits and **sketch** $y=\left(x^2-8\right)e^{-\frac{1}{2}x}$.

Domain: ALL REALS

Range: $x \in [-12.177, \infty)$

x – intercepts: $(\pm\sqrt{8}, 0)$

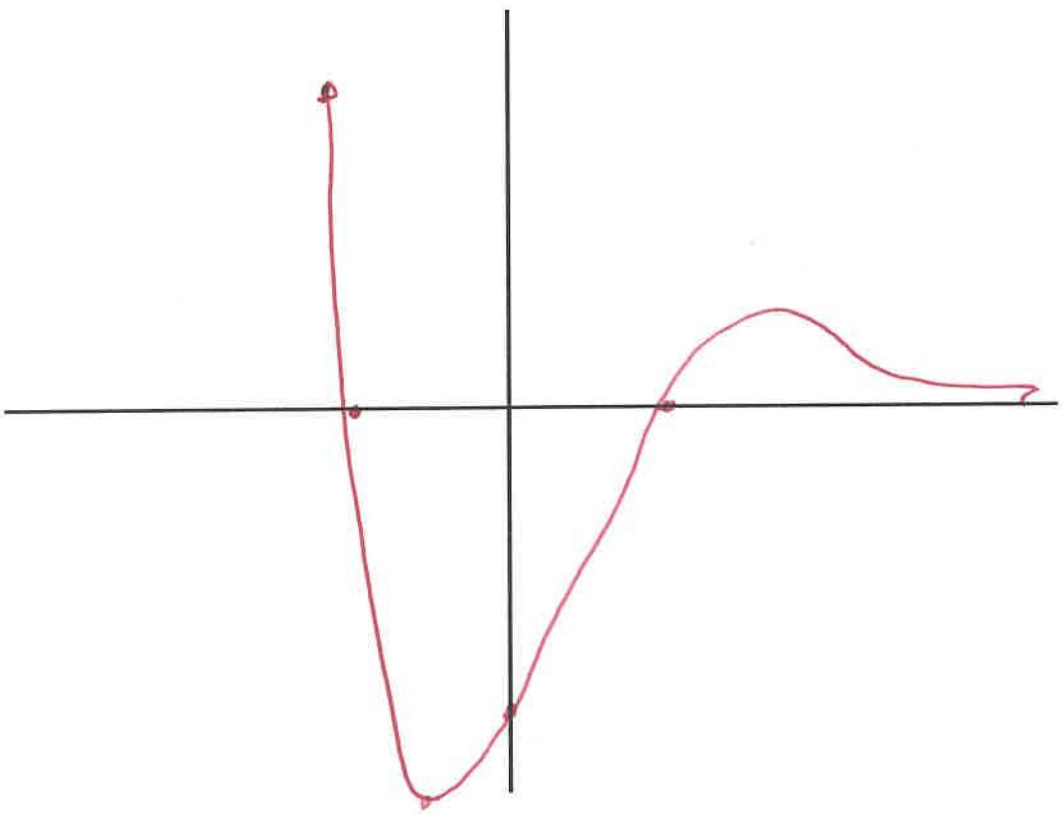
y – intercept: $(0, -8)$

Extreme Points: $(-1.464, -12.177)$

$(5.464, 1.423$

End Behavior (Left): UP

End Behavior (Right): $y = 0$



8. Find the traits and **sketch** of $f(x) = \ln(x^3 - 9x)$ on $x \in [-4, 5]$.

Domain: $x \in (-3, 0) \cup (3, 5)$

Range: $y \in (-\infty, 4.382]$

x – intercepts: ~~NONE~~ SEE #5

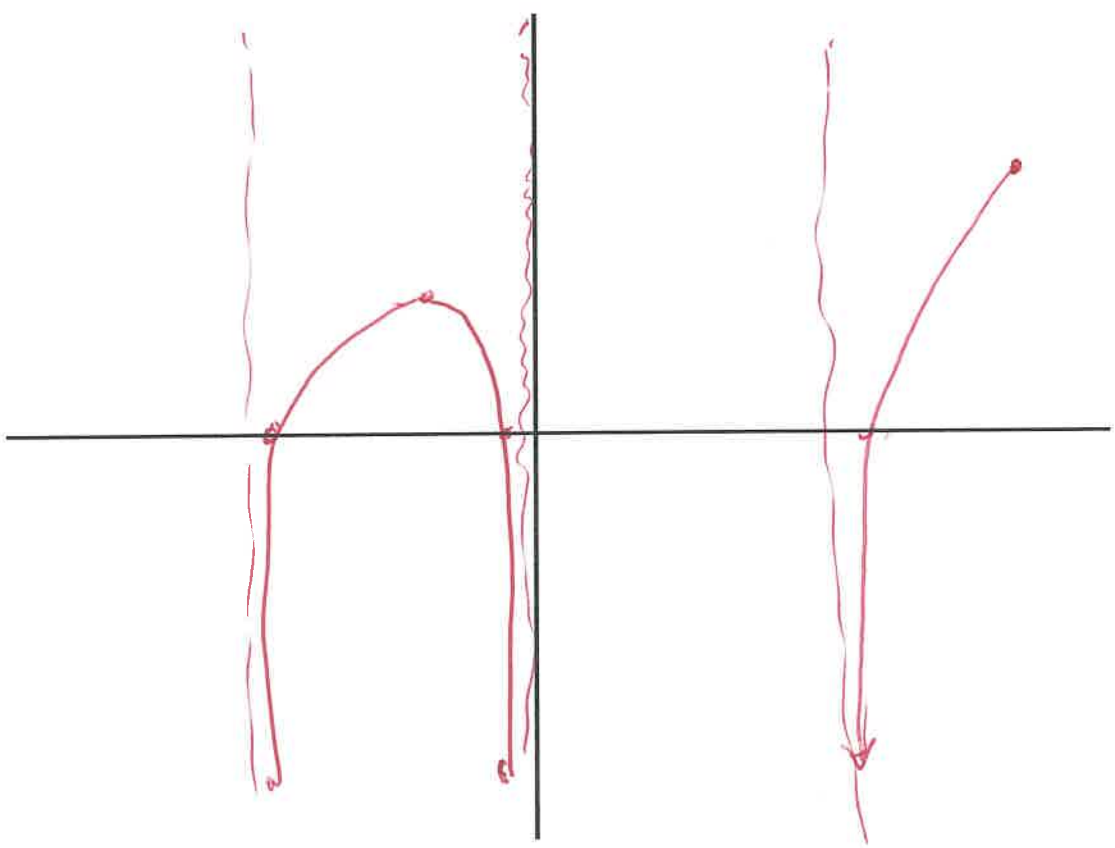
y – intercept: NONE

VAs: $x = \pm 3, 0$

Extreme Points: $(-1.732, 2.341)$ $(5, 4.382)$

End Behavior (Left): NONE

End Behavior (Right): NONE



9. Find the traits and **sketch** of $y = (x+1)\sqrt{9-x^2}$.

Domain: $x \in [-3, 3]$

Range: $y \in [\cancel{-2.386}\ -2.520,\ 6.733]$

VAs: NONE

y – intercept: $(0, 3)$

Extreme Points: SEE #4

End Behavior (Left): NONE

End Behavior (Right): NONE