Name ___Solution Key___

Score_____

1. Find an equation of the line tangent to $f(x) = x^3 - 3x^2 + 15$ at the point

$(-2, -5)$.

$f' = 3x^2 - 6x$ $f'(-2) = 24 = m$

a) $24x - y = -43$

b) $x + 24y = -122$

c) $y - 24x = -53$

d) $24y + x = -122$

$y + 5 = 24(x + 2)$

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2. Find the polynomial of degree 3 whose zeros are $(-3,0)$, $\left(\frac{2}{3}, 0\right)$ and $(2,0)$

and goes through $(1, -3)$.

a) $g(x) = (x+3)(3x-2)(x-2)$

b) $g(x) = -\frac{3}{4}(x+3)(3x-2)(x-2)$

c) $g(x) = \frac{4}{3}(x+3)(3x-2)(x-2)$

d) $g(x) = -\frac{4}{3}(x+3)(3x-2)(x-2)$

e) $g(x) = \frac{3}{4}(x+3)(3x-2)(x-2)$

$y = a(x+3)(3x-2)(x-2)$

$-3 = a(4)(+1)(-1) \Rightarrow a = \frac{+3}{4}$

3. Solve the inequality $-3x^3 + 2x^2 + 147x - 98 > 0$.

a) $x \in (-\infty, -7)$

b) $x \in (-\infty, -7) \cup \left(\dfrac{2}{3}, 7\right)$

c) $x \in \left(-7, \dfrac{2}{3}\right) \cup (7, \infty)$

d) $x \in \left(-7, -\dfrac{2}{3}\right) \cup (7, \infty)$

e) $x \in (-\infty, -7) \cup \left(-\dfrac{2}{3}, 7\right)$

$=\cancel{3}$

$-x^2(3x-2) + 49(3x-2)$

$y\,\overset{+}{\frown}\,0 \quad - \quad \overset{+}{\frown}\,0 \quad -$
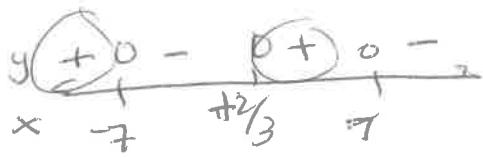
$x \qquad -7 \qquad +\sfrac{2}{3} \qquad \cdot 7$

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4. Given $g(x) = 3 - 2\cos\left[\dfrac{\pi}{6}(x+1)\right]$, which of the following statements is not true?

a) The amplitude of $g(x)$ is 2.

b) The period of $g(x)$ is 12.

c) The phase shift is 1.

d) The vertical shift is 3.

$A = 2$

$P = 12$

$h = -1$

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5. Which of the following is equivalent to $\sin(A + 30°) + \cos(A + 60°)$ for what values of A?

a) $\sin A$ (b)) $\cos A$ c) $\sqrt{3}\sin A + \cos A$

d) $\sqrt{3}\sin A$ e) $\sqrt{3}\cos A$

$\sin A \cos 30 + \cos A \sin 30 + \cos A \cos 60 - \sin A \sin 60$

$(\sin A)(\tfrac{\sqrt3}{2}) + \cos A (\tfrac12) + \cos A (\tfrac12) - \sin A (\tfrac{\sqrt3}{2})$

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6. If $f(x)$ is a linear function $f(1) = 2$ and $f(-2) = 4$, then $f(x) =$

a) ~~$f(x) = 2x + 8$~~   b) ~~$f(x) = \dfrac{2}{3}x + \dfrac{4}{3}$~~   c) ~~$f(x) = -\dfrac{3}{2}x + \dfrac{7}{2}$~~

d) $f(x) = -\dfrac{2}{3}x + \dfrac{4}{3}$ (e)) $f(x) = -\dfrac{2}{3}x + \dfrac{8}{3}$

$$m = \frac{4-2}{-2-1} = -\frac{2}{3}$$

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7. If $\csc\theta = \dfrac{9}{2}$ and $\cos\theta < 0$, then $\cos\theta =$

a. $-\dfrac{\sqrt{77}}{2}$ b) ~~$\dfrac{2}{\sqrt{77}}$~~   c) ~~$\dfrac{9}{\sqrt{77}}$~~

d. $-\dfrac{2}{\sqrt{77}}$ (e.) $-\dfrac{9}{\sqrt{77}}$

$r = 9 \quad y = 2 \rightarrow x^2 + y^2 = r^2 \rightarrow x^2 = 81 - 4 = 77$

$x = -\sqrt{77}$

8.   A particle moves along the x-axis so that at time $t \geq 0$ its position is given by

$x(t) = 2t^3 - 15t^2 + 24t - 60$. At what time t does the particle have no
acceleration?

$V = 6t^2 - 30t + 24 \longrightarrow a = 12t - 30 = 0$

a) $t = 1$ only   b) $t = 4$ only   c) $t = \dfrac{5}{4}$ only

2

d) $t = 1$ and $t = \dfrac{7}{2}$ e) $t = 1$ and $t = 4$

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9.   At the point where $x = 1$, the slope of the line normal to
$f(x) = 2x^3 - 5x^2 - 4x + 10$ is $f' = 6x^2 - 10x - 4$ $m_{TAN} = f'(1) = -8$

a)   -8   b)   8   c) $\dfrac{1}{8}$ d) $-\dfrac{1}{8}$ e)   9

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10. $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{2\sin x}{\sin^2 x - \cos^2 x}$ $= \lim\limits_{x \to \frac{\pi}{6}} \dfrac{2\sin x}{-\cos 2x} = \dfrac{2\sin \frac{\pi}{6}}{-\cos \frac{\pi}{3}} = \dfrac{1}{-\frac{1}{2}}$

a)   2   b)   0   c)   -2   d) $\dfrac{2\sqrt{3}}{3}$ e) $-\dfrac{2\sqrt{3}}{3}$

11. Given this sign pattern $f(x)$, which of the
x
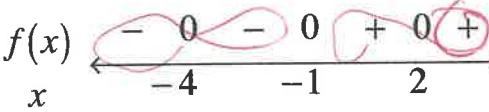
following **might** be the equation of $f(x)$? $(x+4)^2(x+1)(x-2)^2$

a) $f(x)=(x+4)(x+1)(x-2)$

b) $f(x)=-(x+4)^2(x+1)^3(x-2)^2$

c) $f(x)=(x+1)^3(x-2)^2(x+4)^2$

d) $f(x)=(x+1)^2(x-2)^3(x+4)^2$

e) $f(x)=-(x+4)^2(x+1)^3(x-2)^3$

12. Which of the following is NOT an equation for this graph
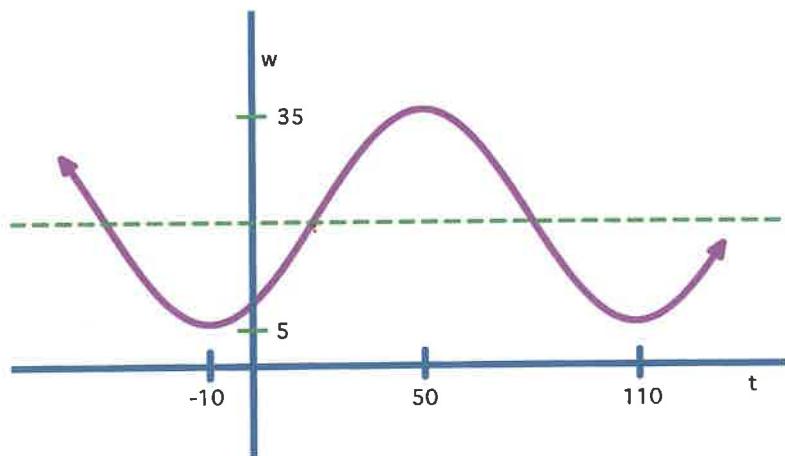
a) $y=20+15\sin\left[\dfrac{\pi}{60}(x-20)\right]$ b) $y=20+15\cos\left[\dfrac{\pi}{60}(x-50)\right]$

c) $y=20-15\cos\left[\dfrac{\pi}{60}(x+10)\right]$ d) $y=20-15\cos\left[\dfrac{\pi}{60}(x-10)\right]$

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Directions: Round to 3 decimal places. Show all work.

1a. $\displaystyle\lim_{x \to -2} \frac{x^3 + 8}{3x^3 + 6x^2 + x + 2}$ $= \displaystyle\lim_{x \to -2} \frac{(x+2)(x^2-2x+4)}{(x+2)(3x^2+1)}$

$$= \frac{12}{13}$$

1b. $\displaystyle\lim_{x \to 5} \frac{x^2 - 5x + 4}{x - 3}$ $= \frac{4}{2} = 2$

1c. $\displaystyle\lim_{x \to -1} \frac{x^4 + x^2 - 2}{x^3 + 7x^2 - 6}$ $= \displaystyle\lim_{x \to -1} \frac{(x+1)(x-1)}{(x+1)(x^2+6x-6)}$

$$\frac{(x-1)(x+1)}{(x^2-1)(x^2+2)}$$

$$= \frac{-6}{-16} = \frac{6}{16}$$

$$\begin{array}{r|rrr} -1 & 1 & 7 & 0 & -6 \\ & & -1 & -6 & 6 \\ \hline & 1 & 6 & -6 & 0 \end{array}$$

2. Prove $\dfrac{\tan^2 w - \sec w - 5}{\tan^2 w + 3\sec w + 3} = \dfrac{\sec w - 3}{\sec w + 1}$

$$\frac{\sec^2 w - 1 - \sec w - 5}{\sec^2 w - 1 + 3\sec w + 3}$$

$$\frac{\sec^2 w - \sec w - 6}{\sec^2 w + 3\sec w + 2}$$

$$\frac{(\sec w + 2)(\sec w - 3)}{(\sec w + 2)(\sec w + 1)}$$

$$\frac{u^2 - u - 6}{u^2 + 3u + 2} \; ; \qquad = \frac{(u - 3)(u + 2)}{(u + 2)(u + 1)}$$

3. Use the Power Rule to find:

a) $\dfrac{dy}{dx}$ if $y = -4x^3 - 5x^2 + 17x + 37$

$$\frac{dy}{dx} = -12x^2 + 10x + 17$$

b) $f'(x)$ if $f(x) = 12x^4 - 6x^3 + 4x + 7 + \left(\dfrac{1}{3x}\right) = \frac{1}{3}x^{-1}$

$$f'(x) = 48x^3 - 18x^2 + 4 \mp \frac{1}{3}x^{-2}$$

c) $D_x\left[\sqrt[7]{x^4} - \dfrac{3}{\sqrt{x^3}} - \sqrt[4]{x} + 8\pi^3 - 6x^3\right]$

$$= D_x\left[x^{4/7} - 3x^{-3/2} - x^{1/4} + 8\pi^3 - 6x^3\right]$$

$$= \frac{4}{7}x^{-3/2} + \frac{9}{2}x^{-5/2} - \frac{1}{4}x^{-3/4} - 18x^2$$

3. The motion of a particle is described by $x(t) = -t^3 + 3t^2 + 24t - 5$.

a) On what time intervals is the particle moving right?

$$v = -3t^2 + 6t + 24$$
$$= -3(t^2 - 2t - 8) = -3(t-4)(t+2)$$

$$v \quad - \quad 0 \quad + \quad 0 \quad -$$

$$t \quad -2 \qquad 4$$

$$t \in (-2, 4)$$

b) Find the position and acceleration when $v(t) = 0$.

$$v = 0 \Rightarrow t = 4, -2$$

$$x(4) = 75 \qquad a(4) = -18$$

$$x(-2) = -49 \qquad a(-2) = 18$$

c) Find the position and velocity when $a(t) = 0$.

$$a(t) = 0 \Rightarrow t = 1$$

$$x(1) = 21$$
$$v(1) = 27$$

4. Find the zeros algebraically of $y = 6x^4 + 17x^3 - 29x^2 - 2x + 8$. Show the synthetic division.

$-4 \rfloor$

	6	17	-29	-2	8
		-24	28	4	-8
	6	-7	-1	2	0

$1 \rfloor$

	6	-7	-1	2
		6	-1	2
	6	-1	-2	

$(x-1)(x+4)(6x^2 - x - 2)$

$(x-1)(x+4)(3x-2)(2x+1)$

$(1,0)$
$(-4,0)$
$(2/3,0)$
$(-1/2,0)$

5. Create the sign pattern for the inequality $6x^4 + 17x^3 - 29x^2 - 2x + 8 \geq 0$

y $+$ 0 $-$ 0 $+$ 0 $-$ $0+$

x -4 $-\frac{1}{2}$ $\frac{2}{3}$ 1

$x \in (-\infty, -4] \cup [-\frac{1}{2}, \frac{2}{3}] \cup [1, \infty)$

6. Mr. Jackanich's teaching varies sinusoidally with time. One month after the start of school, in September, he is teaching at his best, reaching all 100 of his students. Three months later he reaches his lowest, only 50 of his students.

a) Sketch a graph of the number of students reached as a function of months after school starts.
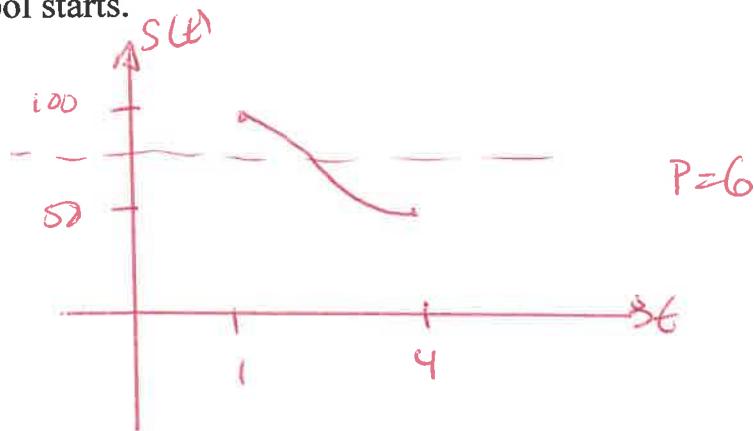
b) Write a sinusoidal equation that describes the number of students reached in terms of months after school starts.

$$S(t) = 75 + 25\cos\frac{\pi}{3}(t-1)$$

c) How many students were reached at the start of school?

$$S(0) = 87.5 \approx 87$$

d) When are the first two times he will reach 85 students?

$$85 = 75 + 25\cos\frac{\pi}{3}(t-1)$$

$$.4 = \cos\frac{\pi}{3}(t-1)$$

$$\pm 1.159 \pm 2\pi n = \frac{\pi}{3}(t-1)$$

$$\pm 1.107 \pm 6n = t-1$$

$$t = \begin{cases} 2.107 \pm 6n \\ -.107 \pm 6n \end{cases}$$

$$t = 2.107, \ 6.893$$

7. Use the equation of the line tangent to $f(x)=x^3-3x^2+4x-4$ at $x=-2$ to approximate $f(-1.9)$.

$f(-2) = \cancel{-8} \cancel{\ldots} -32$

$x=-2$

$f'(x) = 3x^2 \cancel{+2} - 6x + 4$

$m = f'(-2) = \cancel{20} \, 28$

$y+32 = 28(x+2)$

$f(-1.9) \approx y = -32 + 28 (+.1)$

$= -32 + 2.8$

$= \cancel{-36.8}$

$= \cancel{\ldots} -29.2$