Precalc ACC '23 (Quattrin) Name: Solution Key

Practice Spring Final – Part I; 30 Minutes

Calculator Allowed score ______

1. Which of the following statements must be **false**?

(A) $\frac{d}{dx}\sqrt{e^x+3} = \frac{1}{2\sqrt{e^x+3}}$ F

(B) $\frac{d}{dx}(\ln\sin x) = \cot x$ T

(C) $\frac{d}{dx}\left(4x^4 - e + \sqrt[7]{x^2} - \frac{2}{x^3}\right) = 16x^3 + \frac{2}{7\sqrt[7]{x^5}} + \frac{6}{x^4}$ T

(D) $\frac{d}{dx}\left[\ln\sqrt{4x+1}\right] = \frac{2}{4x+1}$ T

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2. If $y = x^2e^{2x}$, then $\frac{dy}{dx} =$ $x^2e^{2x}(2) + e^{2x}(2x) = 2xe^{2x}(x+1)$

a) $2xe^{2x}$ b) $4xe^{2x}$ c) $xe^{2x}(x+1)$

d) $2xe^{2x}(x+1)$ e) $xe^{2x}(x+2)$

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3. The functions f(x) and g(x) are continuous and differentiable, and have values given in the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

Given that $k(x) = \dfrac{f(x)}{g(x)}$, find $k'(4)$. $= \dfrac{g(4)\,f'(4) - f(4)\,g'(4)}{[g(4)]^2} = \dfrac{4\cdot(8) - 10\,(3)}{4^2}$

(A) $\frac{5}{2}$ (B) $\frac{8}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{8}$ (E) $\frac{3}{5}$

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4. Find the equation of the line tangent to the curve $f(x) = 4x^4 - 5x^2 + x$ at the point where $x = -1$.

$f' = 16x^3 - 10x + 1$

$m = f'(-1) = -5$

(A) $y + 2 = -5(x+1)$

(B) $y + 2 = -6(x+1)$

(C) $y - 5 = -2(x+1)$

(D) $y - 5 = -6(x+1)$

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5. Which of these functions has a point of exclusion at $(1,3)$ and a vertical asymptote at $x = \frac{1}{2}$?

(A) $g(x) = \dfrac{x-1}{4x^2-1}$

(B) $h(x) = \dfrac{x-1}{2x^2-3x+1}$

(C) $f(x) = \dfrac{2x+1}{4x^2-1}$

(D) $k(x) = \dfrac{3x-3}{2x^2-3x+1}$ $= \dfrac{3(x-1)}{(2x-1)(x-1)}$

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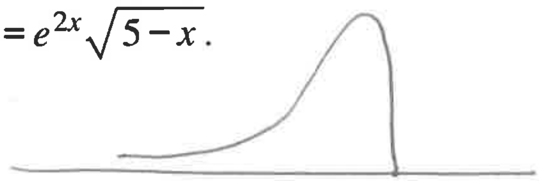
6. Find the end behavior, if any, for $g(x) = e^{2x}\sqrt{5-x}$.



a) Left end none; $y = 0$ on the right

b) Left end down; $y = 0$ on the right

c) Left end $y = 0$; right end up

d) Left end $y = 0$; right end none

e) None on the left and right

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f' — 0 + 0 — 0 —

-5 -1 3

7. Suppose $f(x)$ **has the derivative** $f'(x) = -(x-3)^2(x+5)(x+1)$. Then

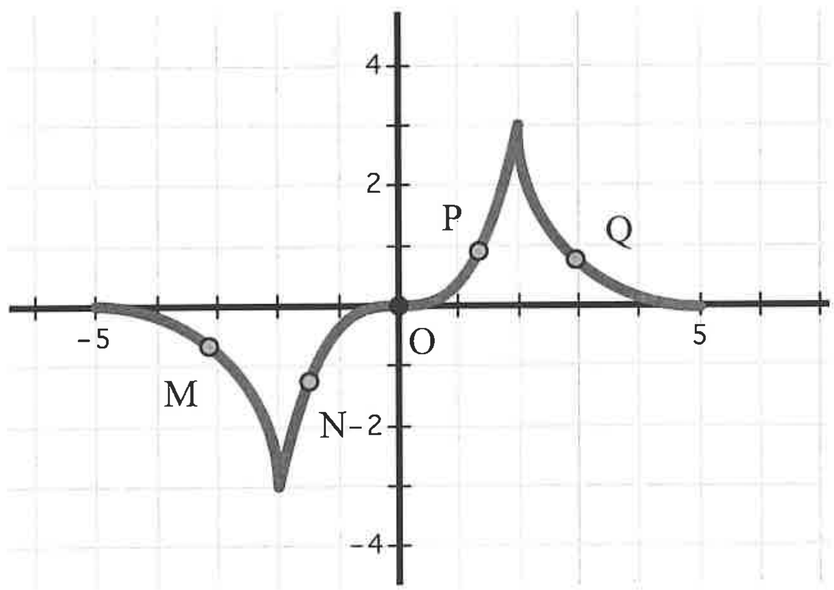
a) $f(x)$ has a relative minimum at $x = -5$ **and** $\cancel{x = 3}$

b) $f(x)$ has a relative maximum at $x = -5$ and a relative minimum at $x = -1$

c) $f(x)$ has a relative maximum at $x = -1$ and a relative minimum at $x = -5$ and $\cancel{x = 3}$

d) $f(x)$ has a relative maximum at $x = -1$ and a relative minimum at $x = -5$

e) $f(x)$ has a relative maximum at $x = -1$ and a relative minimum at $\cancel{x = 3}$



8. At what point on the above curve is $\dfrac{dy}{dx} > 0$ **and** $\dfrac{d^2y}{dx^2} < 0$

a) M b) N c) P d) Q

INCREASING & CONCAVE DOWN

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9. A particle's velocity is given by $v(t)=\cos^2\left(\frac{\pi}{3}t\right)$. The particle's acceleration at $t=1$ is:

$a(1) = 2\left(\cos\frac{\pi}{3}\right)\left(-\sin\frac{\pi}{3}\right)\left(\frac{\pi}{3}\right)$

$\left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)$

(A) $\frac{-\pi\sqrt{3}}{6}$ (B) $\frac{-\sqrt{3}}{4}$ (C) $\frac{3}{4}$ (D) $\frac{\pi}{4}$ (E) DNE

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10. The domain of $y=\ln\left(x^3-9x\right)$ is

e^y $-$ 0 $+$ 0 $-$ 0 $+$

0 -3 0 3

a) $x\in(-\infty,\ -3]\cup[0,\ 3]$

b) $x\in(-\infty,\ -3)\cup(0,\ 3)$

c) $x\in[-3,\ 0]\cup[3,\ \infty)$

d) $x\in(-3,\ 0)\cup(3,\ \infty)$

e) $x\in(-\infty,\ \infty)$

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11. Which of the following is true of $g(x)=\frac{1+e^x}{e^x-1}$?

~~POE~~

POE @ x=0

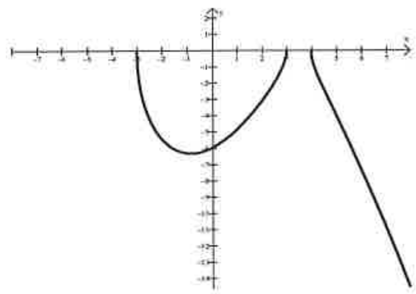
a) $g(x)$ has a y-intercept at $x=0$ F

b) $g(x)$ attains a relative maximum at $x=1$

c) $g(x)$ is increasing for $x\neq 0$

d) $g(x)$ has a zero at $x=0$ F

e) $g(x)$ has a vertical asymptote at $x=0$ F



12. Given the graph above, which of the following might be the sign pattern of $f(x)$?

a)
$f(x)$ + 0 − 0 + 0 −
x −3 3 4

b)
$f(x)$ − 0 + 0 − 0 +
x −3 3 4

c)
$f(x)$ + 0 0 + 0
x −3 3 4

d)
$f(x)$ − 0 0 − 0
x −3 3 4

e)
$f(x)$ 0 − 0 0 −
x −3 3 4

Precalc ACC '23 (Quattrin) Name: Solution Key

Practice Spring Final – Part 60 Minutes

Dr. Quattrin

Calculator Allowed score _____

a. $\frac{d}{dx}\left(\tan 4x^2\right) = \sec^2(4x^2)\cdot 8x$

$= 8x\sec^2 4x^2$

b. $\frac{d}{dx}\left(\ln\left(x^2+7x\right)\right) = \frac{1}{x^2+7x}(2x+7) = \frac{2x+7}{x^2+7x}$

c. $\frac{d}{dx}\left(e^{-\frac{1}{2}x}\csc x\right) = e^{-1/2x}(-\csc x\cot x) + \csc x\left(e^{-1/2x}(-1/2)\right)$

$= -e^{-1/2x}\csc x\left[\cot x - \frac{1}{2}\right]$

d. $\frac{d}{dx}\left(\frac{\sin 5x}{25+x^2}\right) = \frac{(25+x^2)\cos 5x(5) - \sin 5x(2x)}{(25+x^2)^2}$

2. Find the end behavior of each of the following functions. Show the limits that lead to your conclusions.

a) $y=\left(4x^2-16x\right)e^{-0.25x}$

Left end: UP

Right End: $y=0$

b) $y=\ln\left(-x^3-6x^2+5x+30\right)$

Left end: UP

Right End: NONE

c) $y=-\sqrt{\dfrac{16x}{x^2+4}}$.

Left end: NONE

Right End: $y=0$

3. Find the domain and Zeros of $y = -\sqrt{\dfrac{16x}{x^2+4}}$. Show the supporting derivative work.

Domain: $x \in [0, \infty)$

Zeros: $(0, 0)$

4. Find the extreme points of $y = -\sqrt{\dfrac{16x}{x^2+4}}$. Show the algebraic work to support the critical values.

Extreme Points: $(0, 0)$ $(2, -2)$

$$\frac{dy}{dx} = \frac{-1}{2}\left(\frac{16x}{x^2+4}\right)^{-1/2}\left[\frac{(x^2+4)(16) - 16x(2x)}{(x^2+4)^2}\right]$$

$$= \frac{8x^2-32}{(x^2+4)^{3/2}(16x)^{1/2}}$$

i) $\frac{dy}{dx} = 0 \rightarrow x = \pm 2$

$(2, -2)$

$(0, 0)$

ii) $\frac{dy}{dx}$ DNE $\rightarrow x = 0$

iii) NONE

5. Find the domain and Zeros of $y = (4x^2 - 16x)e^{-0.25x}$. Show the supporting derivative work.

Domain: ALL REALS

Zeros: $(0,0), (4,0)$

6. Find the extreme points of $y = (4x^2 - 16x)e^{-0.25x}$. Show the algebraic work to support the critical values.

Extreme Points: $(.528, -10.312)$ $(10.422, 19.776)$

$$\frac{dy}{dx} = (4x^2 - 16x)e^{-.25x}(-.25) + e^{-.25x}(8x - 16)$$

$$= e^{-.25x}[-x^2 + 4x + 8x - 16]$$

$$= e^{-.25}[-x^2 + \cancel{8}12x - 16]$$

i) $\frac{dy}{dx} = 0 \Rightarrow \frac{-12 \pm \sqrt{144 - 4(-1)(16)}}{2(-1)} = \begin{cases} 1.528 \\ 10.472 \end{cases}$

ii) NONE

iii) NONE

7. Find the domain and Zeros of $y=\ln\left(-x^3-6x^2+5x+30\right)$ on $x\in[-7,5]$.

Domain: $x \in [-7, -6) \cup (-\sqrt{5}, \sqrt{5})$

$-x^2(x+6)+5(x+6)$

$+ \; 0 \; - \; 0 \; + \; 0 \; -$

$-6 \quad -\sqrt{5} \quad \sqrt{5}$

VAS: $x=-6, \pm\sqrt{5}$

Zeros: $(-6.632, 0)\ (\pm 2.177, 0)$

8. Find the extreme points of $y=\ln\left(-x^3-6x^2+5x+30\right)$ on $x\in[-7,5]$.
Show the algebraic work to support the critical values.

Extreme Points: $(.380, 3.433)\ (-7, 3.784)$

$$\frac{dy}{dx}=\frac{-3x^2-12x+5}{-x^3-6x^2+5x+30}$$

i) $-3x^2-12x+5=0 \Rightarrow x=\begin{cases}\cancel{-4.380}\\ .380\end{cases}$

ii) $x=\pm\sqrt{5}, \cancel{-6}$

iii) $x=-7, \cancel{5}$

Do **TWO** of the following three problems:

9. Find the traits and **sketch** of $y=(4x^2-16x)e^{-0.25x}$.

Domain: ALL REALS

Range: $y \in [-10.312, \infty)$

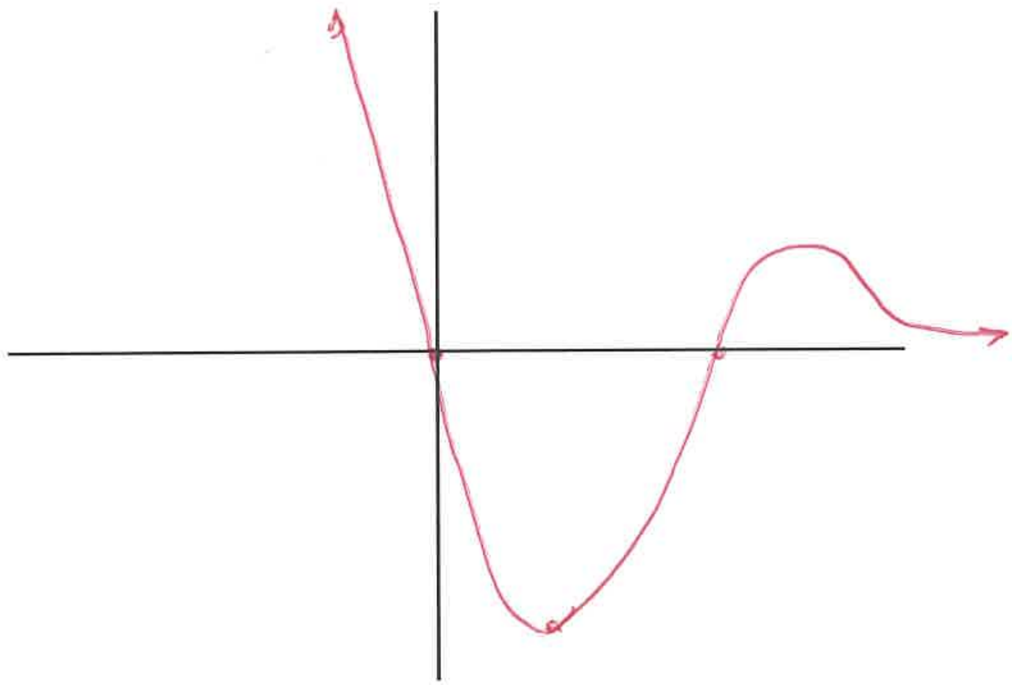
x – intercepts: (0,0) (4,0)

y – intercept:

Extreme Points: (.528, -10.312) (10.422, 14.776

End Behavior (Left): UP

End Behavior (Right): $y=0$



10. Find the traits and sketch $y = \ln\left(-x^3 - 6x^2 + 5x + 30\right)$ on $x \in [-7, 5]$.

Domain: $x \in [-7, -6) \cup (-\sqrt{5}, \sqrt{5})$

Range: $y \in (-\infty, 3.784]$

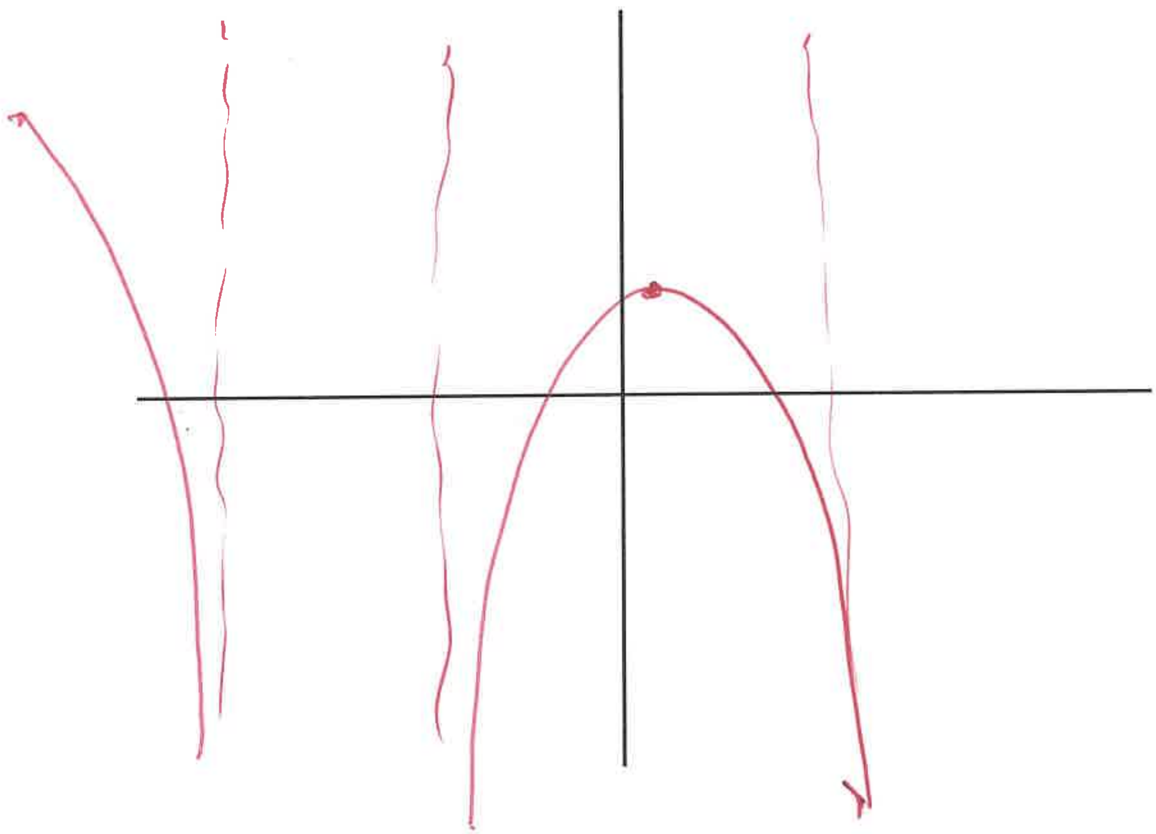
x – intercepts: $(-6.~~632~~, 0)$
$(\pm 2.177, 0)$

y – intercept: $(0, \ln 30)$

Extreme Points: $(.380, 3.433)$
$(-7, 3.784$

End Behavior (Left): NONE

End Behavior (Right): NONE



11 ~~10.~~ Find the traits and sketch $y = -\sqrt{\dfrac{16x}{x^2+4}}$.

Domain: $x \in [0, \infty)$

Range: $y \in [-2, 0]$

x – intercepts: $(0, 0)$

y – intercept: $(0, 0)$

Extreme Points: $(0, 0)$ $(+2, -2)$

End Behavior (Left): NONE

End Behavior (Right): $y = 0$