PreCalculus '12-13
Dr. Quattrin
Ln/Exponential Test-- CALCULATOR ALLOWED
Round to 3 decimal places.
Show all work.

Name: $\underline{\text{Solution Key}}$

Score _____

Formulas: $S = P\left(1+\dfrac{r}{n}\right)^{nt}$

$$S = \dfrac{P\left(\left(1+\dfrac{r}{n}\right)^{nt} - 1\right)}{\dfrac{r}{n}}$$

$$A = \dfrac{P\left(1-\left(1+\dfrac{r}{n}\right)^{-nt}\right)}{\dfrac{r}{n}}$$

1. Suppose you can take out a 30 year loan for a $575,000 house, at a fixed APR of 4.25% compounded monthly. What are your monthly payments and how much will you actually pay the bank?

$$575000 = P\,\dfrac{\left(1-\left(1+\dfrac{.0425}{12}\right)^{-360}\right)}{.0425/12} \Rightarrow P = 2828.65$$

$$\text{TOTAL} = \$1,018,315.58$$

2. Suppose you invest $500 per month at 6% for 10 years, then invest the total at 10% compounded monthly for 20 years. How much will you have 30 years from now?

$$S = 500\,\dfrac{\left[\left(1+\dfrac{.06}{12}\right)^{120}-1\right]}{.06/12} = 81,939.67$$

$$S = 81,939.67\left(1+\dfrac{.1}{12}\right)^{240} = \$600,459.96$$

3a. $\dfrac{d}{dx}\left[5x^2 e^{-x}+1\right]$ $u=5x^2$ $v=e^{-x}$

$du=10x$ $Dv=-e^{-x}$

$= -5x^2 e^{-x} + e^{-x}(10x)$

$= 5xe^{-x}(2-x)$

3b. $D_x\left[(4x^2-5)^6(3x^2+7)^4\right]$

$u=(4x^2-5)^6$ $D_u=6(4x^2-5)^5(8x) = 48x(4x^2-5)^5$

$v=(3x^2+7)^4$ $D_v=4(3x^2+7)^3(6x) = 24x(3x^2+7)^3$

$(4x^2-5)^6\left(24x(3x^2+7)^3\right) + (3x^2+7)^4\left(48x(4x^2-5)^5\right)$

$= 24x(4x^2-5)^5(3x^2+7)^3\left[4x^2-5 + 2(3x^2+7)\right]$

$= 24x(4x^2-5)^5(3x^2+7)^3(10x^2+9)$

4. Find the zeros and Domain of $y = (3x - x^2)e^x$. Show the supporting algebraic work.

Zeros: $3x - x^2 = 0$ $x = 0, 3$

$(3, 0)$ $(0, 0)$

DOMAIN: $x \in$ ALL REALS

5. Find the critical values and extreme values of $y = (3x - x^2)e^x$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = (3x - x^2)e^x + e^x(3 - 2x)$$

$$= e^x(-x^2 + x + 3) = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 12}}{-2} = \begin{cases} -1.303 \\ 2.303 \end{cases}$$

$(-1.303, -1.524)$

$(2.303, 16.059)$

6. Find the traits and **sketch** $y = (3x - x^2)e^x$.

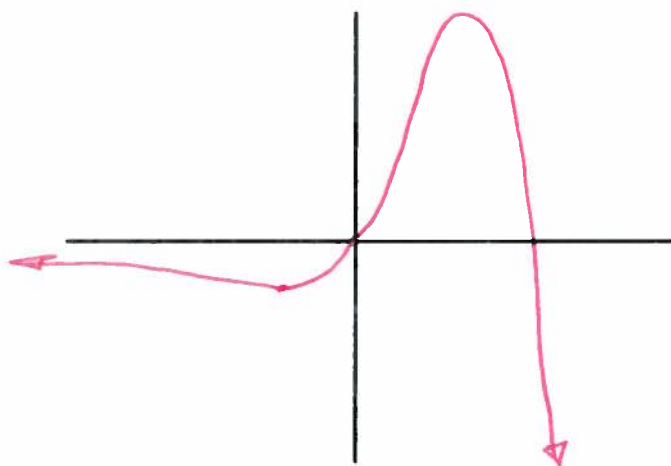Domain: ALL REALS

Range: $y \in (-\infty, 16.059]$

Y – Int: $(0, 0)$

End Behavior: LEFT $y = 0$
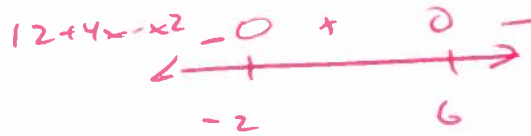RIGHT DOWN

Zeros: $(0, 0)$ $(3, 0)$

Extreme Points:
$(-1.303, -1.524)$
$(2.303, 16.059)$

7. Find the zeros and Domain of $y = x\sqrt{12+4x-x^2}$. Show the supporting algebraic work.

$(x+2)(x-6)$

$(6,0)$
$(0,0)$
$(-2,0)$

$12+4x-x^2$

$x \in [-2, 6]$

8. Find the critical values and extreme values of $y = x\sqrt{12+4x-x^2}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = x\left[\frac{1}{2}(12+4x-x^2)^{-\frac{1}{2}}\overset{2-x}{(4-2x)}\right] + (12+4x-x^2)^{\frac{1}{2}}$$

$$= \frac{2x-x^2 + 12+4x-x^2}{(12+4x-x^2)^{\frac{1}{2}}} = \frac{12+6x-2x^2}{(12+4x-x^2)^{\frac{1}{2}}}$$

i) $\frac{dy}{dx} = 0$ $x = \frac{-6 \pm \sqrt{6^2-4(12)(-2)}}{-4} = \begin{cases} -1.372 \\ 4.372 \end{cases}$

ii) $\frac{dy}{dx} = DNE \Rightarrow 12+4x-x^2 = 0$ $x = -2, 6$

$(-2,0) (6,0)$ $(-1.372, -2.952) (4.572, 14.006)$

9. Find the traits and **sketch** of $y = x\sqrt{12 + 4x - x^2}$.

Domain: $x \in [-2, 6]$ Y – Int: $(0, 0)$

Zeros: $(0, 0)$ $(-2, 0)$ $(6, 0)$ Range: ~~$(-2, 0)$~~ ~~$(6, 0)$~~ $y \in [-2.952,$
                                          $(-1.372, -2.952)$ $14.006]$
End Behavior: NONE $4.572,$ $14.606)$

Extreme Points: $(-2, 0)$ $(6, 0)$