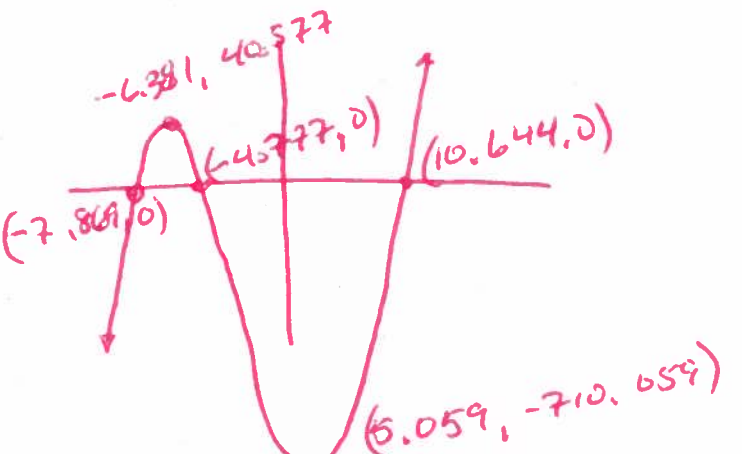
PreCalculus ACC-- Dr. Quattrin Name SOLUTION KEY
Fall Final 2009--Part I
Calculator required
Give all decimal answers to three places.

1. Given $f(x)=x^3+2x^2-97x-400$, sketch the complete graph, state the window used, and label the zeros and extremes.

$x \in [-10, 20]$ $y \in [-800, 50]$



2. In the game of golf, a golfer hits the ball from the start point (tee box) to the finishing point (the green). Sounds easy however, quite often the golfer is unable to hit the ball on a straight line directly to the green, as is the case here. The golfer hits the first shot 240 yds at 20 degrees and then hits the second shot 150 yds at 310 degrees, landing on the green. If the golfer had hit the ball directly from the tee to the green, what would the length and direction of the shot have to be?

$240\cos 20^\circ \vec{i} + 240 \sin 20^\circ \vec{j}$
$150\cos 310^\circ \vec{i} + 150 \sin 310^\circ \vec{j}$
$321.944\,\vec{i} + (-32.522)\vec{j}$

$|\vec{v}| = 323.613$

$\theta = -\cos^{-1}\left(\frac{321.944}{323.613}\right) = -5.821^\circ$

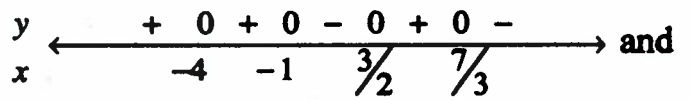
4. Prove $\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\cos x+\sin x$.

$\sqrt{2}\left[\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}\right]$

$=\sqrt{2}\left[\cos x\left(\frac{1}{\sqrt{2}}\right)+\sin x\left(\frac{1}{\sqrt{2}}\right)\right]$

$=\cos x+\sin x$

5. Find an inequality that has the following sign chart:



and

$x\in(-\infty,-4)\cup(-4,-1)\cup\left(\frac{3}{2},\frac{7}{3}\right)$

$-(x+4)^2(x+1)(2x-3)(3x-7)>0$

PreCalc ACC-- Dr. Quattrin
Fall Final 2009--Part II
No calculator allowed

Name SOLUTION KEY

1. Give exact values of:

a. $\cot\left(\frac{5\pi}{3}\right)$ $= -\frac{\sqrt{3}}{3}$

b. $\sec(240°)$ $= -2$

c. $x = \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $x \in (0, 540°)$

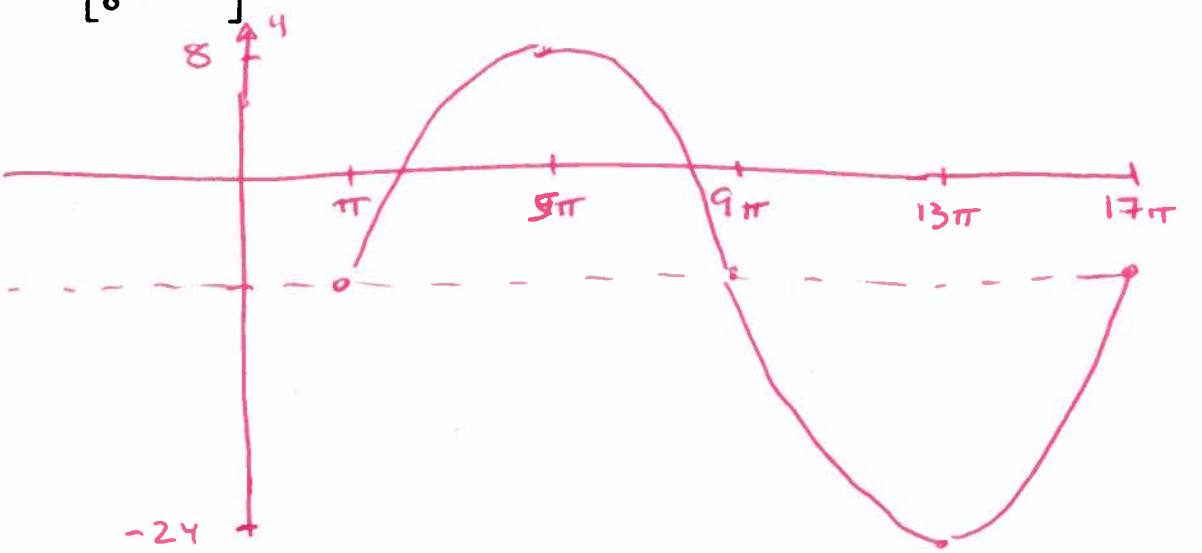
$= 60°, 240°, 420°, 600°$

d. $\theta = \sec^{-1}(-2)$ $x \in [-2\pi, 2\pi]$

$= \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}$

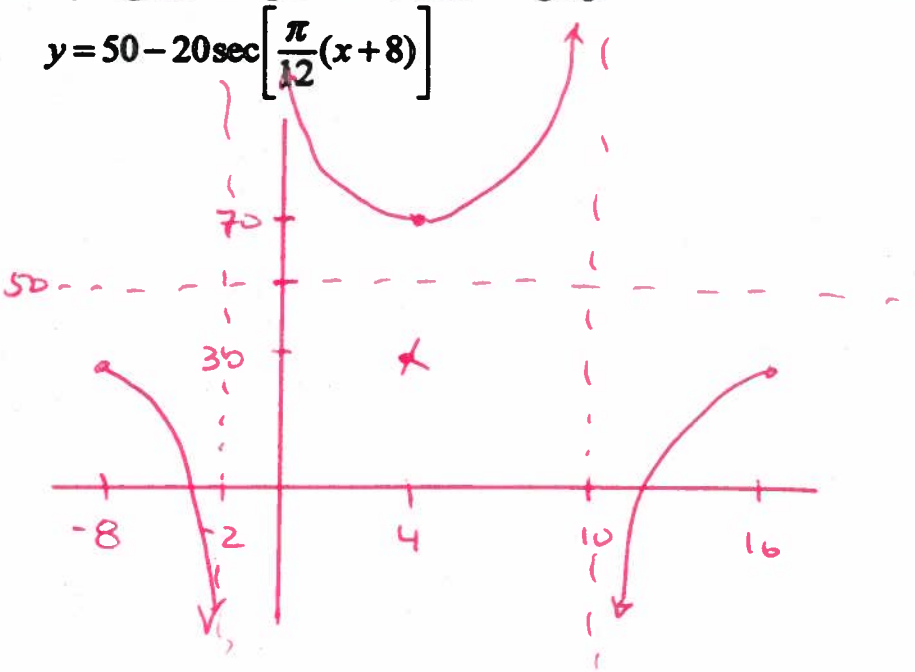
2. Graph the following equation over 1 cycle. Show all your work and show all necessary/significant points. Mark the graph as much as necessary.

$$y = -8 + 16\sin\left[\frac{1}{8}(x - \pi)\right]$$



3. Graph the following equation over 1 cycle. Show all your work and show all necessary/significant points. Mark the graph as much as necessary.

$$y = 50 - 20\sec\left[\frac{\pi}{12}(x+8)\right]$$



4. Given the following, find the exact values below:

$\cos A = -\frac{3}{5}$, $\sin A = \frac{4}{5}$ $\qquad 90° < A < 180°$

$\cos B = -\frac{24}{25}$, $\sin B = -\frac{7}{25}$ $\qquad 180 < B < 360$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\frac{4}{3} + \frac{7}{24}}{1 - \left(\frac{-4}{3}\right)\left(\frac{7}{24}\right)} = \frac{\frac{-96+21}{72}}{1 + \frac{28}{72}} = \frac{-75}{100} = \frac{-3}{4}$

$\sec 2B = \frac{1}{\cos^2 B - \sin^2 B} = \frac{1}{\left(\frac{-24}{25}\right)^2 - \left(\frac{-7}{25}\right)^2} = \frac{1}{527/625} = \frac{625}{527}$

$\sin\left(\frac{1}{2}B\right) = \pm\sqrt{\frac{1}{2}(1-\cos B)} = +\sqrt{\frac{1}{2}\left(1-\left(\frac{-24}{25}\right)\right)} = \sqrt{\frac{49}{50}} = \frac{7}{5\sqrt{2}}$

5. Find all the exact zeros algebraically of $f(x)=3x^4+5x^3-75x^2-125x$.

$f(x) = x\left[3x^3+5x^2-75x-125\right]$

$= x\left[x^2(3x+5) - 25(3x+5)\right]$

$= x(x^2-25)(3x+5) = 0$

$(0,0), (\pm 5, 0), (-5/3, 0)$

6. Prove the identity: $\dfrac{\cot\psi+\tan\psi}{\csc\psi} = \sec\psi$

$$\frac{\frac{\cos\psi}{\sin\psi}+\frac{\sin\psi}{\cos\psi}}{\frac{1}{\sin\psi}} = \frac{\cos^2\psi+\sin^2\psi}{\sin\psi\cos\psi}\cdot\frac{\sin\psi}{1}$$

$$= \frac{1}{\cos\psi}$$

$$= \sec\psi$$

7. Find the exact values of the six trigonometric functions of the angle W through the point (12, –8).

Sin W = $\frac{-2}{\sqrt{13}}$ Cot W = $-3/2$

Cos W = $\frac{3}{\sqrt{13}}$ Csc W = $-\sqrt{13}/2$

Tan W = $-2/3$ Sec W = $\sqrt{13}/3$

$r = \sqrt{12^2+8^2} = \sqrt{144+64} = \sqrt{208} = 4\sqrt{13}$

8. Solve for $\omega \in [-45°, 225°)$: $\sin\omega(\csc\omega - 2\sin\omega) = \frac{\sqrt{2}}{2}$

$1 - 2\sin^2\theta = \sqrt{2}/2$

$\cos 2\theta = \frac{\sqrt{2}}{2}$

$2\theta = \pm 45 \pm 360n$

$\theta = \pm 22.5 \pm 180n$

$\theta = \{-22.5, 22.5, 155.5, 205.5\}$

9. Find the general solution in radians: $\frac{1}{\sec^2\theta - 1} = 3$

$\frac{1}{\tan^2\theta} = 3$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\frac{1}{\sqrt{3}}$ or $\theta = \tan^{-1}\frac{-1}{\sqrt{3}}$

$\theta = \frac{\pi}{6} \pm \pi n$ or $\theta = -\frac{\pi}{6} \pm \pi n$