PreCalculus '13-'14 Name: Solution Key

Limits and Derivatives Test v6

CALCULATOR ALLOWED Score ___________

Round to 3 decimal places. Show all work.

1. $\lim_{x\to 1} \frac{x^2+2x-3}{x^2-1} =$ $\lim_{x\to 1} \frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x+1)} = \frac{4}{2} = 2$

(a) $\lim_{x\to 3} \frac{x^2-9}{2x^2-7x+3} =$ $\lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(2x-1)} = \frac{6}{5}$

(b) $\lim_{x\to -1} \frac{3x^2+2x-1}{x^3+x^2+4x+4} =$ $\lim_{x\to -1} \frac{\cancel{(x+1)}(3x-1)}{\cancel{(x+1)}(x^2+4)} = \frac{-4}{5}$

2. Use the equation of the line tangent to $y=6x^3-3x^2+5x-4$ at $x=1$ to approximate $f(.9)$

$y(1) = 4$

$\frac{dy}{dx} = 18x^2-6x+5$

$m = \left.\frac{dy}{dx}\right|_{x=1} = 17$

$y-4 = 17(x-1)$

3. The motion of a particle is described by $y(t) = 2t^3 + 5t^2 - 4t + 3$.

a) When the particle is stopped?

b) Which direction it is moving at $t = 4$?

c) Where is it when $t = 4$?

d) Find $a(4)$.

$v(t) = 6t^2 + 10t - 4$

$a(t) = 12t + 10$

a) $v = 0 \rightarrow 6t^2 + 10t - 4 = 0$

$3t^2 + 5t - 2 = 0$

$(3t - 1)(t + 2) = 0 \quad t = 1/3, -2$

b) $v(4) = + \therefore$ UP

c) $x(t) = 195$

d) $a(4) = 58$

4. At what point on the graph of $y = \frac{1}{3}x^3$ is the tangent parallel to the line $2x - 8y = 3$

$m = +\frac{1}{4}$

$\frac{dy}{dx} = x^2$

$\frac{1}{4} = x^2$

$\therefore x = \pm \frac{1}{2}$

$\left(\frac{1}{2}, \frac{1}{24}\right)$

$\left(\frac{-1}{2}, \frac{-1}{24}\right)$

5. Set up, but do not solve, the limit definition of the derivative for $\frac{d}{dx}\left[5x^4 - x^3 + 7x^2 + 3^4\right]$

$$\lim_{h \to 0} \frac{\left[5(x+h)^4 - (x+h)^3 + 7(x+h)^2 + 3^4\right] - \left[5x^4 - x^3 + 7x^2 +\right.}{h}$$

6. Find the following derivatives:

a. $\frac{dy}{dx}$ if $y = 6x^7 - 19x^4 + 3x^2 - 12x - 13$

$$\frac{dy}{dx} = 42x^6 - 76x^3 + 6x - 12$$

b. $D_x\left[\sqrt[4]{x^7} - \frac{6}{x^5} - \sqrt[3]{x} + \pi^2 - x\right]$ $= D_x\left[x^{7/4} - 6x^{-5} - x^{1/3} + \cancel{\pi^2} - x\right]$

$$= \frac{7}{4}x^{3/4} + 30x^{-6} - \frac{1}{3}x^{-2/3} - 1$$

c. $\frac{d}{dx}\left[x^7 - 4\sqrt[8]{x^7} + 7^3 - \frac{1}{\sqrt[7]{x^4}} + \frac{1}{5x}\right]$ $= \frac{d}{dx}\left[x^7 - 4x^{7/8} + 7^3 - x^{-4/7} + \frac{1}{5}x^{-1}\right.$

$$= 7x^6 - \frac{7}{2}x^{-1/8} + \frac{4}{7}x^{-11/7} - \frac{1}{5}x^{-2}$$