

PreCalculus '13-14
Take-Home Midterm
Dr. Quattrin

Name: SOLUTION KEY

Score _____

CALCULATOR ALLOWED

Round to 3 decimal places. Show all work.

1. Find the equations of the lines tangent and normal to

$$y = \frac{1}{4}x^4 + \frac{1}{3}x^3 + 2x^2 + 2x - 7 \text{ at } x = -1.$$

$$y(-1) = \frac{1}{4} - \frac{1}{3} + 2 - 2 - 7 = -\frac{85}{12}$$

$$\frac{dy}{dx} = x^3 + x^2 + 4x + 2 \quad m = -1 + 1 - 4 + 2 = -2$$

TANGENT $y + \frac{85}{12} = -2(x+1)$

NORMAL $y + \frac{85}{12} = \frac{1}{2}(x+1)$

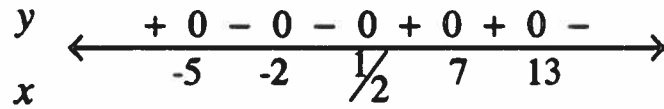
2. Find all zeros, algebraically, of $f(x) = -x^4 + x^3 + 41x^2 - 105x$.

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -41 & 105 \\ & & 3 & +6 & -105 \\ \hline & 1 & +2 & -35 & 0 \end{array}$$

$$\begin{aligned} &= -x(x^3 - x^2 - 41x + 105) \\ &= -x(x-3)(x^2 + 2x - 35) \\ &= -x(x-3)(x+7)(x-5) \end{aligned}$$

$$\begin{aligned} &(0, 0) \\ &(3, 0) \\ &(-7, 0) \\ &(5, 0) \end{aligned}$$

3. Find an inequality that has this sign pattern and solution:



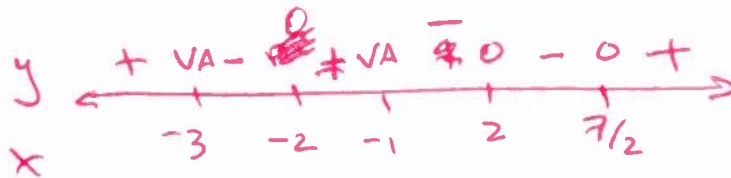
and $x \in (-\infty, -5] \cup \{-2\} \cup [0, 13]$

$$y \geq -(x+5)(x+2)^2(2x-1)(x-7)^2(x-13)$$

4. Write an equation of a rational function that has x -intercepts at $(-3, 0)$, VA at $x = 5$, a POE at $x = -2$, and a HA at $y = \frac{6}{5}$.

$$y = \frac{6(x+3)(x+2)}{5(x-5)(x+2)}$$

5. Solve this inequality: $0 \leq \frac{(x-2)(x+2)(2x-7)}{(x+3)(x-2)(x+1)}$.



$$x \in (-\infty, -3) \cup [-2, -1) \cup [2, 7/2] \cup [7/2, \infty)$$

6. Find the extreme values of $y = \frac{1}{2}x^4 - x^3 - 9x^2 + 27x$, algebraically.

$$\frac{dy}{dx} = 2x^3 - 3x^2 - 18x + 27 = 0$$

$$x^2(2x-3) - 9(2x-3) = 0$$

$$(x^2-9)(2x-3) = 0$$

$$x = \pm 3, 3/2$$

$$y(3) = 13.5$$

$$y(-3) = -94.5$$

$$y(1.5) = 19.406$$

7. Find VAs, HA, POEs and zeros of $y = \frac{6x^3 - 11x^2 - 3x + 2}{x^4 - x^2 - 12} = \frac{(x-2)(6x^2+x-1)}{(x^2-4)(x^2+3)}$

$$\begin{array}{r} 2 \overline{) 6 \quad -11 \quad -3 \quad 2} \\ \underline{12 \quad -2 \quad -2} \\ 6 \quad +1 \quad -1 \quad 0 \end{array}$$

$$= \frac{(x-2)(3x+1)(2x-1)}{(x-2)(x+2)(x^2+3)}$$

$$\text{Zeros: } \left(\frac{1}{3}, 0\right) \left(-\frac{1}{2}, 0\right)$$

$$\text{VA: } x = -2$$

$$\text{POE } \left(2, \frac{25}{28}\right)$$

$$\text{HA: } y = 0$$

8. Find all the asymptotes and zeros of $y = \frac{x^2 - 16}{x^2 - 7}$

$$\text{Zeros: } (\pm 4, 0)$$

$$\text{VAs: } x = \pm \sqrt{7}$$

$$\text{HA: } y = 1$$

9. Find the extremes of $y = \frac{x^2 - 16}{x^2 - 7}$ algebraically.

$$\frac{dy}{dx} = \frac{(x^2 - 7)(2x) - (x^2 - 16)(2x)}{(x^2 - 7)^2}$$

$$= \frac{2x^3 - 14x - 2x^3 + 32x}{(x^2 - 7)^2}$$

$$= \frac{18x}{(x^2 - 7)^2} = 0 \quad x = 0 \quad \left(0, \frac{16}{7}\right)$$
$$y = \frac{16}{7}$$

10. List all the traits and sketch $y = \frac{6x^3 - 11x^2 - 3x + 2}{x^4 - x^2 - 12}$

Domain: $x \neq -2, 2$

Range: All Real

X-Intercepts: $(-\frac{1}{3}, 0)$ $(\frac{1}{2}, 0)$

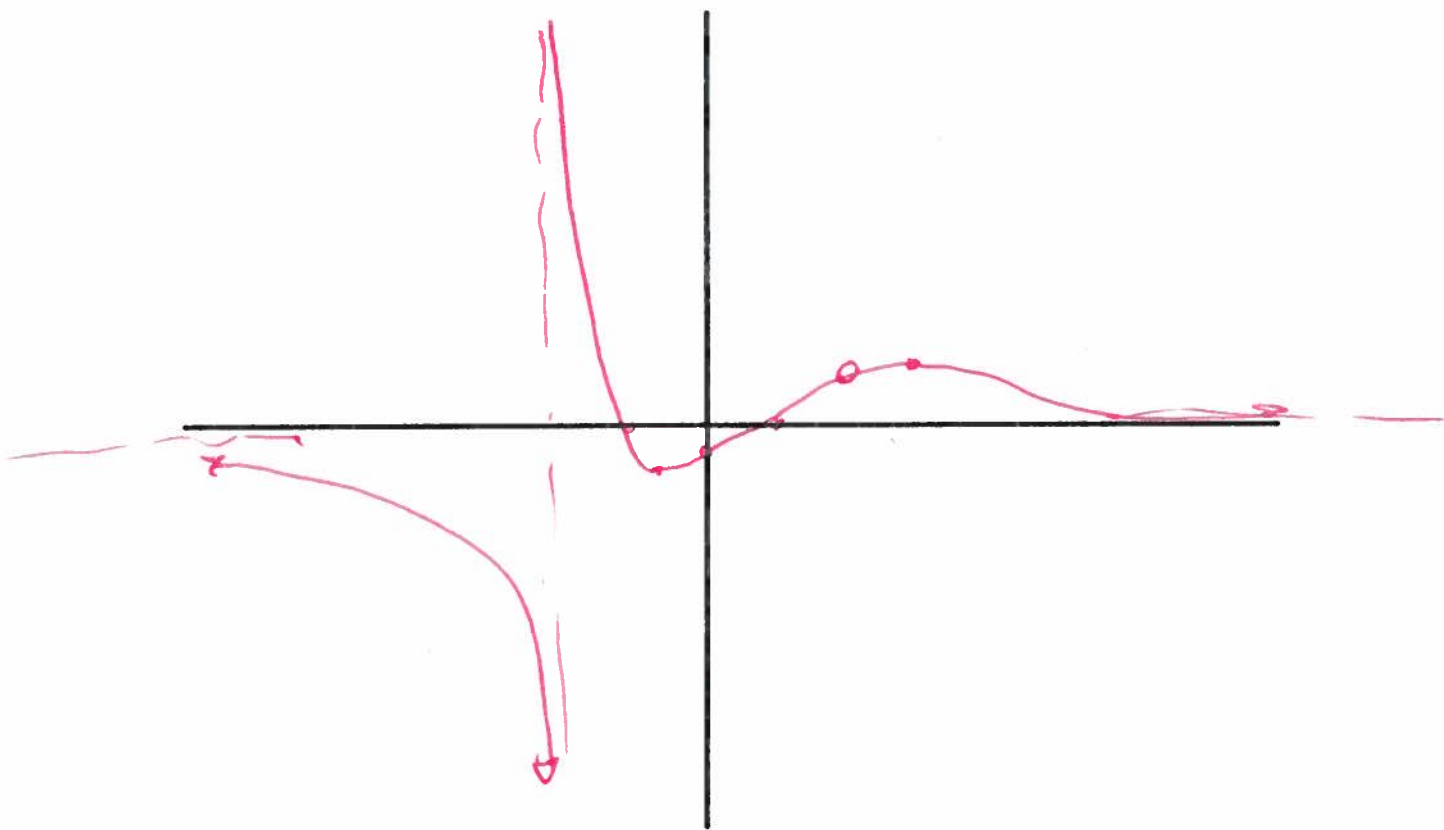
Y-Intercept: $(0, -\frac{1}{6})$

VAs: $x = -2$

POEs: $(2, \frac{25}{28})$

End Behavior: $y = 0$

Extremes: $(-0.122, -1.82)$
 $(2.683, 0.940)$



11. List all the traits and sketch $y = \frac{x^2 - 16}{x^2 - 7}$

Domain: $x \neq \pm\sqrt{7}$

Range: $y \in (-\infty, 1) \cup [\frac{16}{7}, \infty)$

X-Intercepts: ~~$(0, \frac{16}{7})$~~ $(\pm 4, 0)$

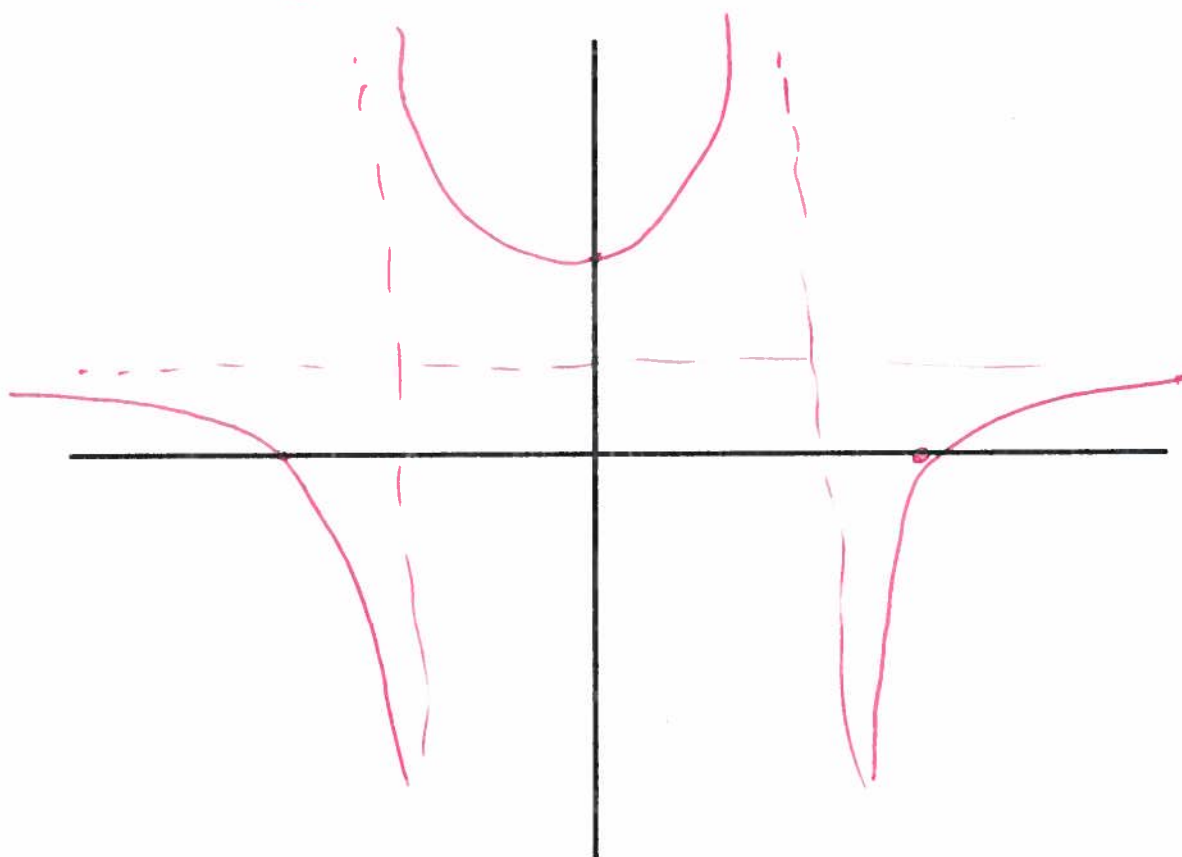
Y-Intercepts: $(0, \frac{16}{7})$

VAs: $x = \pm\sqrt{7}$

POEs: NONE

End Behavior: $y = 1$

Extremes: $(0, \frac{16}{7})$



12. List all the traits and sketch $f(x) = -x^4 + x^3 + 41x^2 - 105x$ on

Domain: ALL REALS

Range: $y \in (-\infty, 807.582]$

X-Intercepts: $(0,0)$ $(3,0)$ $(-7,0)$
 $(5,0)$

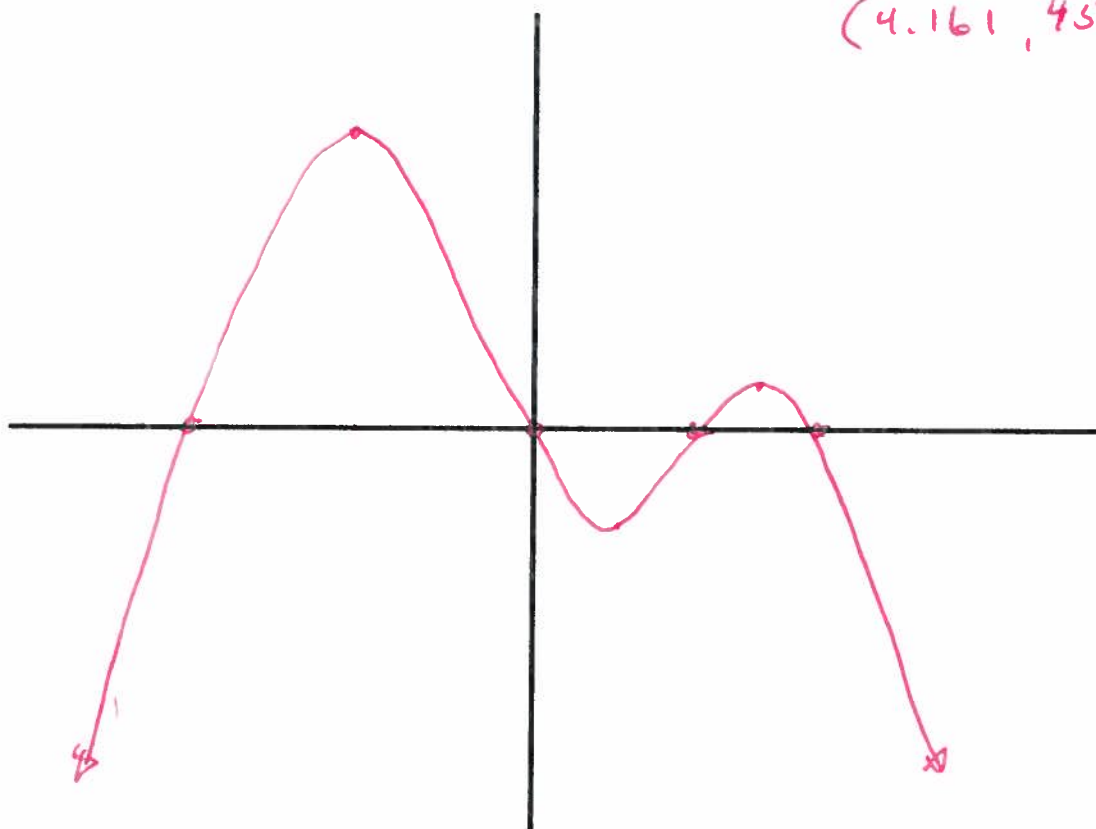
Y-Intercepts: $(0,0)$

VAs: NONE

POEs: NONE

End Behavior: $\downarrow \downarrow$

Extremes: $(-4.741, 807.582)$
 $(1.331, -67.901)$
 $(4.161, 45.237)$



$$\frac{dy}{dx} = -4x^3 + 3x^2 + 82x - 105 = 0$$

$$0 : x = \text{~~2.310~~, 4.045, -4.}$$