PreCalculus '14
Trig Identities Group Test
Page 1

Name Solution Key

Score 70

For Problem 1-6, use:

(-4, -3) is on the terminal side of A and
$180° \le A \le 270°$

$Sec\ B = 17/8$ and $0° \le B \le 90°$; and

$Cot\ C = 12/5$ and $-180° \le C \le -90°$

to find the exact values of:

$\sin A = \frac{-3}{5}$ $\cos A = \frac{-4}{5}$ $\tan A = \frac{3}{4}$

$\sin B = \frac{15}{17}$ $\cos B = \frac{8}{17}$ $\tan B = \frac{15}{8}$

$\sin C = \frac{-5}{13}$ $\cos C = \frac{-12}{13}$ $\tan C = \frac{5}{12}$

1. Sin (A-B) $= \sin A \cos B - \cos A \sin B$

$= \left(\frac{-3}{5}\right)\left(\frac{8}{17}\right) - \left(\frac{-4}{5}\right)\left(\frac{15}{17}\right)$

$= \frac{36}{85}$

2. Cos (2A) $= \cos^2 A - \sin^2 A$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$

$= \frac{7}{25}$

3. $\tan\frac{1}{2}C = \frac{\sin C}{1+\cos C} = \frac{-5/13}{1 + -12/13}$

$= -5$

4. Csc (A+B)

$= \frac{1}{\sin A \cos B + \cos A \sin B}$

$= \frac{1}{\frac{-3}{5}\left(\frac{8}{17}\right) + \left(\frac{-4}{5}\right)\left(\frac{15}{17}\right)}$

$= \frac{-85}{84}$

5. Tan (2B) $= \frac{2\tan B}{1 - \tan^2 B}$

$= \frac{2(15/8)}{1 - \left(\frac{15}{8}\right)^2} = -\frac{240}{161}$

6. $\cos\frac{1}{2}A = \pm\sqrt{\frac{1}{2}(1 + \cos A)}$

$= -\sqrt{\frac{1}{2}\left(1 + \left(\frac{-4}{5}\right)\right)} = -\frac{1}{\sqrt{10}}$

7. Prove: $\dfrac{2\sin^2 w - 5\cos w + 1}{6\sin^2 w - 5\cos w - 2} = \dfrac{\cos w + 3}{3\cos w + 4}$

$$\frac{2(1-\cos^2 w) - 5\cos w + 1}{6(1-\cos^2 w) - 5\cos w - 2}$$

$$= \frac{-2\cos^2 w - 5\cos w + 3}{-6\cos^2 w - 5\cos w + 4}$$

$$= \frac{-(\cancel{2\cos w - 1})(\cos w + 3)}{-(\cancel{2\cos w - 1})(\cos w + 4)}$$

9. Solve exactly for A: $\cos^4 A - \sin^4 A = 1$

$$(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$$

$$(\cos 2A)(1) = 1$$

$$2A = 0 \pm 2\pi n$$

$$A = 0 \pm \pi n$$

8. Prove:
$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

$$(\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$$

$$(\sin A\cos B)^2 - (\cos A\sin B)^2$$

$$\sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$$

$$\sin^2 A - \cancel{\sin^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 A\sin^2 B}$$

$$\sin^2 A - \sin^2 B$$

10. Solve exactly for $x \in [0, \pi)$:

$$\frac{\tan\frac{1}{2}x + \cot\frac{1}{2}x}{\cot\frac{1}{2}x - \tan\frac{1}{2}x} = 2$$

$$\frac{\frac{1+\cos x}{\sin x} + \frac{1-\cos x}{\sin x}}{\frac{1-\cos x}{\sin x} - \frac{1+\cos x}{\sin x}} = 2$$

$$\frac{2}{-2\cos x} = 2$$

$$\cos x = -1/2$$

$$x = \pm\frac{2\pi}{3} \pm 2\pi n$$

$$x = \left\{\frac{2\pi}{3}\right\}$$

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For Problem 1-6, use:

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$Sec\ B = 17/8$ and $0° \le B \le 90°$; and

$Cot\ C = 12/5$ and $-180° \le C \le -90°$

to find the exact values of:

1. cos (A-B)

$\cos A \cos B + \sin A \sin B$

$\left(\frac{-4}{5}\right)\left(\frac{8}{17}\right) + \left(\frac{-3}{5}\right)\left(\frac{15}{17}\right)$

$\frac{-77}{85}$

2. Sin 2B $= 2\sin B\cos B$

$= 2\left(\frac{15}{17}\right)\left(\frac{8}{17}\right) = \frac{240}{289}$

3. Sin $\left(\frac{1}{2}C\right)$ $= \pm\sqrt{\frac{1}{2}(1-\cos C)}$

$= -\sqrt{\frac{1}{2}\left(1-\left(\frac{-12}{13}\right)\right)} = \frac{-5}{\sqrt{26}}$

4. Tan (A+B) $= \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{3}{4} + \frac{15}{8}}{1 - \left(\frac{3}{4}\right)\left(\frac{15}{8}\right)} = \frac{-84}{13}$

5. Cot 2A $= \frac{1-\tan^2 A}{2\tan A}$

$= \frac{1-\left(\frac{3}{4}\right)^2}{2(3/4)} = \frac{7}{24}$

6. Sec $\left(\frac{1}{2}B\right)$ $= \frac{1}{\pm\sqrt{\frac{1}{2}(1+\cos B)}}$

$= \frac{1}{+\sqrt{\frac{1}{2}(1+8/17)}} = \frac{\sqrt{34}}{5}$

7. Prove:

$$\frac{\cos(A+2B)\cos(B)+\sin(A+2B)\sin(B)}{\sin A\cos B+\cos A\sin B} = \cancel{\frac{\tan A+\tan B}{1-\tan A\tan B}} \quad \frac{1-\tan A\tan B}{\tan A+\tan B}$$

$$\frac{\cos(A+2B-B)}{\sin\cos B+\cos A\sin B} = \cot(A+B)$$

$$\frac{\cos(A+B)}{\sin(A+B)} =$$

$$\cot \cancel{\tan} (A+B)$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}$$

9. Prove: $\tan x\sin 2x-2\cos^2 x=-2\cos 2x$

$$\frac{\sin x}{\cancel{\cos x}}(2\sin x\cancel{\cos x})-2\cos^2 x$$

$$2\sin^2 x-2\cos^2 x$$

$$2(\sin^2 x-\cos^2 x)$$

$$2(-\cos 2x)$$

$$-2\cos 2x$$

8. Solve for x:

$\sec\left(x-\frac{\pi}{4}\right)=2+2\sec\left(x-\frac{\pi}{4}\right)$

$$\sec\left(x-\frac{\pi}{4}\right)=-2$$

$$\cos\left(x-\frac{\pi}{4}\right)=\frac{-1}{2}$$

$$x-\frac{\pi}{4}=\begin{cases}2\pi/3 \pm 2\pi n\\ -2\pi/3 \pm 2\pi n\end{cases}$$

$$x=\begin{cases}-\frac{5\pi}{12} \pm 2\pi n\\ +\frac{11\pi}{12} \pm 2\pi n\end{cases}$$

10. Solve for $A\in(-2\pi,2\pi)$:

$\left(\sin\frac{1}{2}A-\cos\frac{1}{2}A\right)^2=\frac{1}{2}$

$$\sin^2\tfrac{1}{2}A-2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A+\cos^2\tfrac{1}{2}A$$

$$1-\sin A=1/2$$

$$\sin A=1/2$$

$$A=\begin{cases}\pi/6 \pm 2\pi n\\ 5\pi/6 \pm 2\pi n\end{cases}$$

$$X=\left\{\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}\right\}$$

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For Problem 1-6, use:

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$Sec\ B = 17/8$ and $0° \le B \le 90°$; and

$Cot\ C = 12/5$ and $-180° \le C \le -90°$

to find the exact values of:

1. Tan (A-B)

$$= \frac{\tan A - \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{3}{4} - \frac{15}{8}}{1 + \frac{3}{4}\left(\frac{15}{8}\right)} = \frac{-36}{77}$$

2. Cot 2B $= \frac{1 - \tan^2 B}{2 \tan B}$

$$= \frac{1 - \left(\frac{15}{8}\right)^2}{2\left(15/8\right)} = \frac{-161}{240}$$

3. $\mathrm{Csc}\left(\frac{1}{2}C\right) = \frac{1}{\pm\sqrt{\frac{1}{2}(1 - \cos C)}}$

$$= -\frac{1}{\sqrt{\frac{1}{2}\left(1 - \left(\frac{-12}{13}\right)\right)}} = \frac{-\sqrt{26}}{5}$$

4. Cos (A+C)

$$= \cos A \cos C - \sin A \sin C$$

$$= \left(\frac{-4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right)$$

$$= \frac{33}{65}$$

5. Csc 2C $= \frac{1}{2 \sin C \cos C}$

$$= \frac{1}{2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)}$$

$$= \frac{169}{120}$$

6. $\mathrm{Sin}\left(\frac{1}{2}B\right) = \pm\sqrt{\frac{1}{2}(1 - \cos B)}$

$$= \sqrt{\frac{1}{2}\left(1 - 8/17\right)} = 3/\sqrt{34}$$

7. Prove: $\dfrac{\cos^4\phi - \sin^4\phi}{\sin\phi\cos\phi} = \dfrac{1-\tan^2\phi}{\tan\phi}$

$$\frac{(\cos^2\phi - \sin^2\phi)(\cos^2\phi + \sin^2\phi)}{\sin\phi\cos\phi}$$

$$\frac{\cos^2\phi - \sin^2\phi}{\sin\phi\cos\phi}\left[\frac{1/\cos^2\phi}{1/\cos^2\phi}\right]$$

$$\frac{1-\frac{\sin^2\phi}{\cos^2\phi}}{\frac{\sin\phi}{\cos\phi}}$$

$$= \frac{1-\tan^2\phi}{\tan\phi}$$

8. Prove: $2\cot f = \cot\frac{f}{2} - \tan\frac{f}{2}$

$$\frac{2\cos f}{\sin f} = \cancel{\frac{1-\cos f}{\sin f}} \; \cancel{\frac{1+\cos f}{\sin f}}$$

$$= \cancel{\frac{-2\cos f}{\sin f}}$$

$$= \frac{1+\cos f}{\sin x} - \frac{1-\cos f}{\sin f}$$

$$= \frac{2\cos f}{\sin f}$$

9. Solve exactly for $x \in [0, \pi)$:
$\tan 3x - \tan x + \tan 3x \tan x = -1$

$$\tan 3x - \tan x = -1 - \tan 3x \tan x$$

$$\frac{\tan 3x - \tan x}{1 + \tan 3x \tan x} = -1$$

$$\tan 2x = -1$$

$$2x = \frac{3\pi}{4} \pm \pi n$$

$$x = \frac{3\pi}{8} \pm \frac{\pi}{2} n$$

$$x = \left\{\frac{3\pi}{8}, \frac{7\pi}{8}\right\}$$

10. Solve exactly for x: $\dfrac{1-\tan x \tan\frac{\pi}{12}}{\tan x + \tan\frac{\pi}{12}} = \sqrt{3}$

$$\cot\left(x + \frac{\pi}{12}\right) = \sqrt{3}$$

$$\tan\left(x + \frac{\pi}{12}\right) = \frac{1}{\sqrt{3}}$$

$$x + \frac{\pi}{12} = \frac{\pi}{6} \pm \pi n$$

$$x = \frac{+\pi}{12} \pm \pi n$$

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For Problem 1-6, use:

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$Cot\ C = 12/5$ and $-180° \le C \le -90°$

to find the exact values of:

1. Csc (A-B)

$$\frac{1}{\sin A\cos B - \cos A\sin B}$$

$$= \frac{1}{\left(\frac{-3}{5}\right)\left(\frac{8}{17}\right) - \left(\frac{-4}{5}\right)\left(\frac{15}{17}\right)}$$

$$= \frac{85}{36}$$

2. Sin 2A $= 2\sin A\cos A$

$$= 2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right)$$

$$= \frac{24}{25}$$

3. $\cos\left(\frac{1}{2}C\right) = \pm\sqrt{\frac{1}{2}(1+\cos C)}$

$$= +\sqrt{\frac{1}{2}\left(1+\frac{-12}{13}\right)} = \frac{1}{\sqrt{26}}$$

4. Sin (B+C)

$\sin B\cos C + \cos B\sin C$

$$= \left(\frac{15}{17}\right)\left(\frac{-12}{13}\right) + \frac{8}{17}\left(\frac{-5}{13}\right)$$

$$= \frac{-220}{221}$$

5. Cos 2C $= \cos^2 C - \sin^2 C$

$$= \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2$$

$$= \frac{119}{169}$$

6. $\cot\left(\frac{1}{2}A\right) = \frac{1+\cos A}{\sin A}$

$$= \frac{1 + -4/5}{-3/5} = -\frac{1}{3}$$

7. Prove: $\sin^2\theta\tan\frac{\theta}{2}=\sin\theta-\sin\theta\cos\theta$

$\sin^2\theta\left(\frac{1-\cos\theta}{\cancel{\sin\theta}}\right)$

$\sin\theta-\sin\theta\cos\theta$

9. Prove: $\frac{\sec\theta}{\cot\theta+\tan\theta}=\sin\theta$

$$\frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}\frac{(\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$

$$\frac{\sin\theta}{\cos^2\theta+\sin^2\theta}$$

$\sin\theta$

8. Solve for $x\in[-2\pi, 2\pi]$:

$3-3\sin x-2\cos^2 x=0$

$3-3\sin x-2(1-\sin^2 x)$

$2\sin^2 x-3\sin x+1=0$

$(\sin x-1)(2\sin x-1)=0$

$\sin x=1$ $\quad$ $\sin x=1/2$

$x=\frac{\pi}{2}\pm 2\pi n$ $\quad$ $x=\begin{cases}\pi/6\pm 2\pi n\\ 5\pi/6\pm 2\pi n\end{cases}$

$x=\left\{\frac{\pi}{2}, \frac{-3\pi}{2}, \frac{\pi}{6}, \frac{-11\pi}{6}, \frac{5\pi}{6}, \frac{-7\pi}{6}\right\}$

10. Solve for x:

$\left(\cos^2\frac{1}{2}x-\sin^2\frac{1}{2}x\right)^2-\left(2\sin\frac{1}{2}x\cos\frac{1}{2}x\right)^2=\frac{1}{2}$

$(\cos x)^2-(\sin x)^2$

$\cos 2x=\frac{1}{2}$

$2x=\pm\frac{\pi}{3}\pm 2\pi n$

$x=\pm\frac{\pi}{6}\pm\pi n$