PreCalculus '14-'15 Name: Solution Key

Limits and Derivatives Test

CALCULATOR ALLOWED Score ___________

Round to 3 decimal places. Show all work.

1. Evaluate the following limits:

a. $\lim_{x \to 6} \frac{x^2 - 36}{x^2 - 4x - 12} =$ $\lim_{x \to 6} \frac{\cancel{(x-6)}(x+6)}{\cancel{(x-6)}(x+2)} = \frac{12}{8} = \frac{3}{2}$

b. $\lim_{x \to -2} \frac{2x^2 + x - 6}{x^4 + 12x^2 - 64}$ $= \lim_{x \to -2} \frac{(2x-3)\cancel{(x+2)}}{(x^2-4)(x^2+16)} = \frac{-7}{(-4)(20)} = \frac{7}{80}$

$(x-2)\cancel{(x+2)}$

c. $\lim_{x \to 4} \frac{2x^3 - 5x^2 - 17x + 20}{x^3 - 4x^2 - x + 4} =$ $\lim_{x \to 4} \frac{(x-4)(2x^2+3x-5)}{(x-4)(x^2-1)} = \frac{32+12-5}{15}$

$= \frac{39}{15} = \frac{13}{5}$

$$\begin{array}{r|rrrr} 4 & 2 & -5 & -17 & 20 \\ & & 8 & 12 & -20 \\ \hline & 2 & 3 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -1 & 4 \\ & & 4 & 0 & -4 \\ \hline & 1 & 0 & -1 & \end{array}$$

2. Use the equation of the line tangent to $f(x) = 4x^3 + 2x^4 - 9$ at $x = 1$ to approximate $f(.9)$

$y_1 = f(1) = 4 + 2 - 9 = -3$ $m = \left.\frac{dy}{dx}\right|_{x=1} = 12x^2 + 8x^3\Big]_{x=1}$

$= 20$

$y + 3 = 20(x - 1)$

$f(.9) \approx y(.9) = -3 + 20(.9 - 1) = -5.0$

3. The motion of a particle is described by $x(t) = 4t^3 - 63t^2 + 86t - 14$.

a) When the particle is stopped?

b) Which direction it is moving at $t = 4$?

c) Where is it when $t = 4$?

d) Where is it when $a(t) = 0$?

a) $v(t) = 12t^2 - 126t + 86 = 0$

$6t^2 - 63t + 43 = 0$

$t = \dfrac{63 \pm \sqrt{63^2 - 4(6)(43)}}{2(6)} = \begin{cases} 9.766 \\ .733 \end{cases}$

b) $v(4) = 12(4)^2 - 126(4) + 86 = -226$ $\therefore$ $\boxed{\text{DOWN}}$

c) $x(4) = 4(4)^3 - 63(4)^2 + 86(4) - 14 = -422$

d) $a(t) = 24t - 126 = 0 \rightarrow t = 21/4$

$\cancel{a(4) = 24(4) - 126 = \boxed{-30}}$ $x\left(\frac{21}{4}\right) = -720.125$

4. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent parallel to the line $2x - 4y = 3$?

$m = +1/2$

$\frac{dy}{dx} = x = 1/2$

$x = 1/2 \rightarrow y = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$

$\boxed{\left(\frac{1}{2}, \frac{1}{8}\right)}$

5. Set up, but do not solve, the limit definition of the derivative of
$y = 12x^7 - 11x^4 + x^2 - 31x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\left[12(x+h)^7 - 11(x+h)^4 + (x+h)^2 - 31(x+h)\right] - \left[12x^7 - 11x^4 + x^2 - 31x\right]}{h}$$

6. Find the following derivatives:

a. $\dfrac{dy}{dx}$ if $y = 6x^3 - x^2 + 8x + 42$

$$\frac{dy}{dx} = 18x^2 - 2x + 8$$

b. $D_x\left[16x^{15} - 18x^2 + 41x + e + 2 + \dfrac{3}{\sqrt{x^3}} + \dfrac{1}{x}\right] = 240x^{14} - 36x + 41 - \dfrac{9}{2}x^{-5/2} - \dfrac{1}{x^2}$

c. $D_x\left[\sqrt[3]{x^8} - \dfrac{6}{x^3} - \sqrt[7]{x} + \pi^3 + x\right] = D_x\left[x^{8/3} - 6x^{-3} - x^{+1/7} + \pi^3 + x\right]$

$$= \frac{8}{3}x^{5/3} + 18x^{-4} - \frac{1}{7}x^{-6/7} + 0 + 1$$