PreCalculus '14-15
Dr. Quattrin
Rational Test Form A
CALCULATOR ALLOWED
Round to 3 decimal places.
Show all work.

Name: _Solution Key_

Score _____

1. Find the zeros and VAs of $y=\dfrac{4x^2-16x}{x^3-4x^2+x-4}$. Show the supporting algebraic work.

$$\frac{4x(x-4)}{(x^2+1)(x-4)}$$

Zeros: $(0,0)$

VAs: NONE

POE: $\left(4, \dfrac{16}{17}\right)$

2. Find the extreme points of $y=\dfrac{4x^2-16x}{x^3-4x^2+x-4}$. Show the derivative and algebra to support the critical values.

$$y \approx \frac{4x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{(x^2+1)(4)-(4x)(2x)}{(x^2+1)^2}$$

$$\frac{-4x^2+4}{(x^2+1)^2} = 0$$

$$x^2 - 1 = 0$$

$$x = \pm 1$$

$(1, 2)$

$(-1, -2)$

3. Find the equations of the lines tangent to and normal to $y = \dfrac{2x^2 - x - 3}{3 + 2x - x^2}$ at $x = 0$?

$y(0) = \dfrac{-3}{3} = -1$

$= \dfrac{(2x-3)(x+1)}{-(x-3)(x+1)}$

Tangent: $y+1 = \dfrac{1}{9}(x-0)$

$\approx \dfrac{2x-3}{-x+3}$

Normal: $y+1 = -9(x-0)$

$\dfrac{dy}{dx} = \dfrac{(-x+3)(2) - (2x-3)(-1)}{(-x+3)^2}$

$m = \dfrac{(3)(2) - (-3)(-1)}{(3)^2} = \dfrac{1}{9}$

4. Find the zeros, VAs, POEs and HA of $y = \dfrac{16 - x^2}{x^2 - 25}$. Show the supporting algebraic work.

$= \dfrac{(4-x)(4+x)}{(x-5)(x+5)}$

Zeros: $(\pm 4, 0)$

VAs: $x = \pm 5$

HA: $y = -1$

POE: NONE

5. Find the extreme points of $y = \dfrac{16 - x^2}{x^2 - 25}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = \frac{(x^2 - 25)(-2x) - (16 - x^2)(2x)}{(x^2 - 25)^2}$$

$$= \frac{-2x^3 + 50x - 32x + 2x^3}{(x^2 - 25)^2}$$

$$\frac{16x}{(x^2 - 25)^2} = 0$$

$$x = 0$$

$$y = \frac{-16}{25}$$
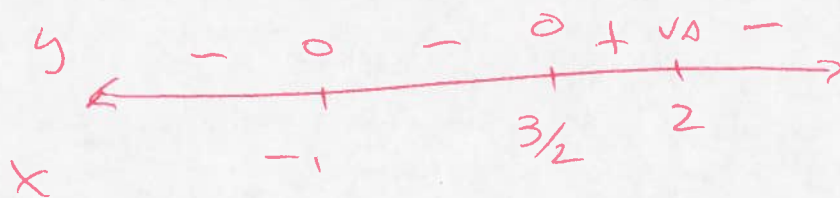
6. Write an equation of a rational function that has x–intercepts at (-2, 0), VA at $x = 4$, a POE at $x = -3$, and a HA at $y = \dfrac{3}{7}$.

$$y = \frac{3(x+2)(x+3)}{7(x-4)(x+3)}$$

7. Show the sign pattern and solve $\dfrac{2x^2 - x - 3}{3 + 2x - x^2} \leq 0$.

$$\frac{(2x-3)(x+1)}{-(x-2)(x+1)} \leq 0$$

$$x \in (-\infty, -1) \cup [1, \tfrac{3}{2}] \cup (2, \infty)$$

8. Find the traits and **sketch** $y = \dfrac{16 - x^2}{x^2 - 25}$.

Domain: $x \neq \pm 5$

Y – Intercept: $\left(0, -16/25\right)$

Zeros: $(\pm 4, 0)$

VAs: $x = \pm 5$

Range: $y \in (-\infty, -1) \cup \left[-\frac{16}{25}, \infty\right)$

End Behavior: $y = -1$

Extreme Points: $\left(0, \frac{-16}{25}\right)$

POE: NONE
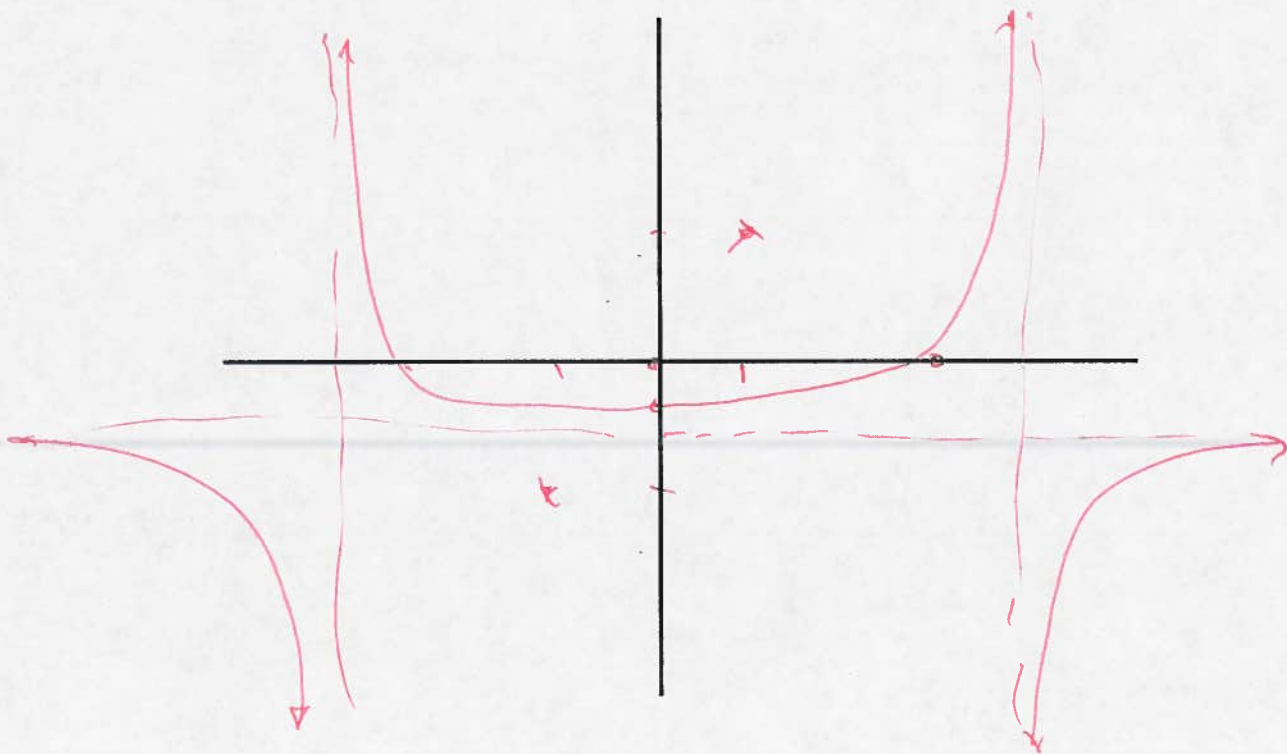
9. Find the traits and **sketch** of $y = \dfrac{4x^2 - 16x}{x^3 - 4x^2 + x - 4}$.

Domain: $x \neq 4$

Zeros: $(0, 0)$

VAs: NONE

POEs: $\left(4, \dfrac{16}{17}\right)$

Y – Intercept: $(0, 0)$

Range: $y \in [-2, 2]$

End Behavior: $y = 0$

Extreme Points: $(1, 2)\,(-1, -2)$