

PreCalculus '14-15

Dr. Quattrin

Spring Final-- CALCULATOR ALLOWED

Round to 3 decimal places.

Show all work.

Name: SOLUTION KEY

Score _____

1. Find the zeros and Domain of $f(x) = x^4 - 19x^2 + 48$. Show the supporting algebraic work.

Domain: $x \in \text{ALL REALS}$

Zeros $x^4 - 19x^2 + 48 = 0$

$$(x^2 - 16)(x^2 - 3) = 0$$

$$x = \pm 4, \pm \sqrt{3}$$

$$(\pm 4, 0) (\pm \sqrt{3}, 0)$$

2. Find the extreme points of $f(x) = x^4 - 19x^2 + 48$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = 4x^3 - 38x = 0$$

$$2x(2x^2 - 19) = 0$$

$$x = 0, \pm \sqrt{19/2} = 3.082$$

$$(0, 48) (\pm 3.082, -42.25)$$

3. Find the traits and sketch $f(x) = x^4 - 19x^2 + 48$.

Domain: All Reals

Range: $y \in [-42.25, \infty)$

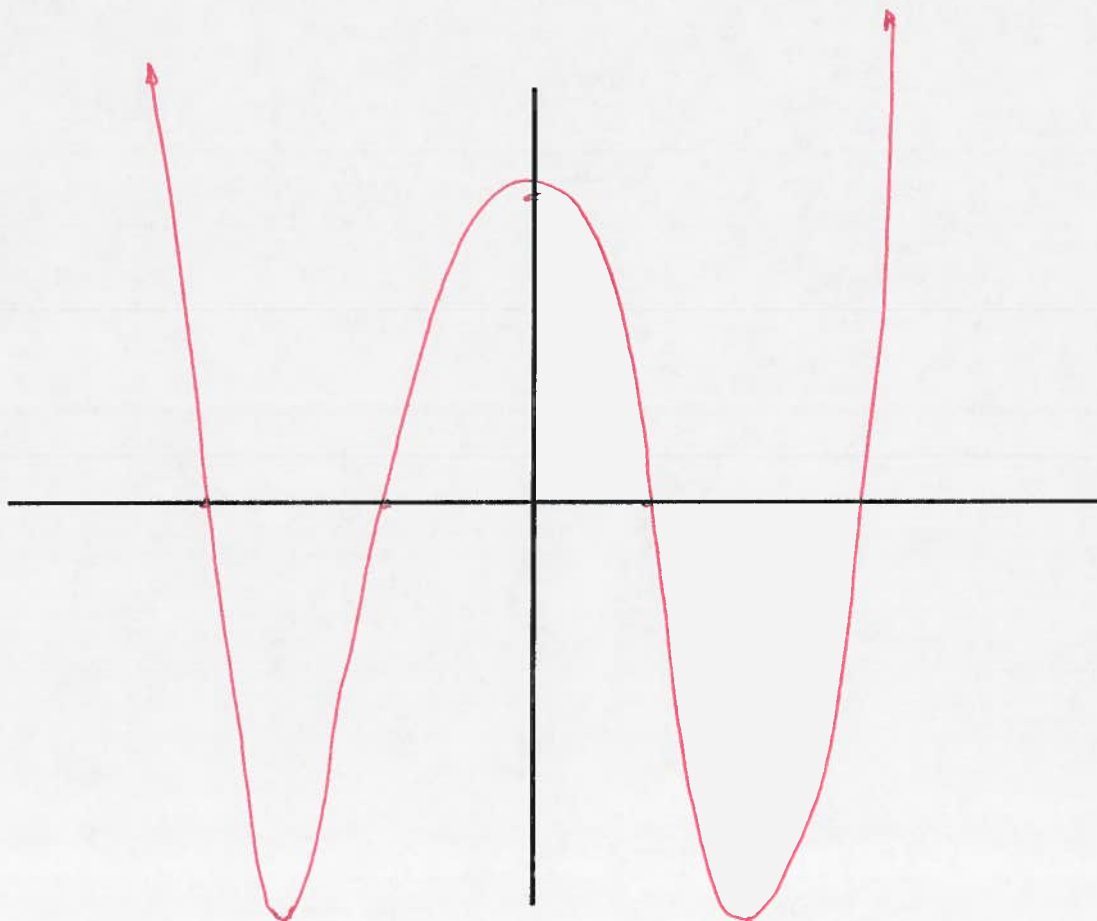
Y-Int: $(0, 48)$

EB Left: UP

Zeros: $(\pm 4, 0)$ $(\pm \sqrt{3}, 0)$

EB Right: UP

Extreme Points: $(0, 48)$
 $(\pm 3.082, -42.25)$



4. Find the zeros, VAs, and POEs of $y = \frac{x^2 - 3x}{x^2 - 9}$. Show the supporting algebraic work.

$$= \frac{x(x-3)}{(x-3)(x+3)}$$

$$\text{VA: } x = -3$$

$$\text{Zero } (0, 0)$$

$$\text{POE } (3, \frac{1}{2})$$

$$\text{POE: } x = +3$$

$$\lim_{x \rightarrow 3} \frac{x(x-3)}{(x-3)(x+3)} = \frac{3}{6} = \frac{1}{2}$$

5. Find the extreme points of $y = \frac{x^2 - 3x}{x^2 - 9}$. Show the derivative and algebra to support the critical values.

$$y \approx \frac{x}{x+3}$$

$$\frac{dy}{dx} = \frac{(x+3)(1) - x(1)}{(x+3)^2} = \frac{3}{(x+3)^2} = 0$$

NO SOLUTION

\therefore NO EXTREMES

6. Find the traits and sketch $y = \frac{x^2 - 3x}{x^2 - 9}$.

Domain: $x \neq \pm 3$

Range: $y \neq \frac{1}{2}, 1$

Y-Int: $(0, 0)$

EB Left: $y = 1$

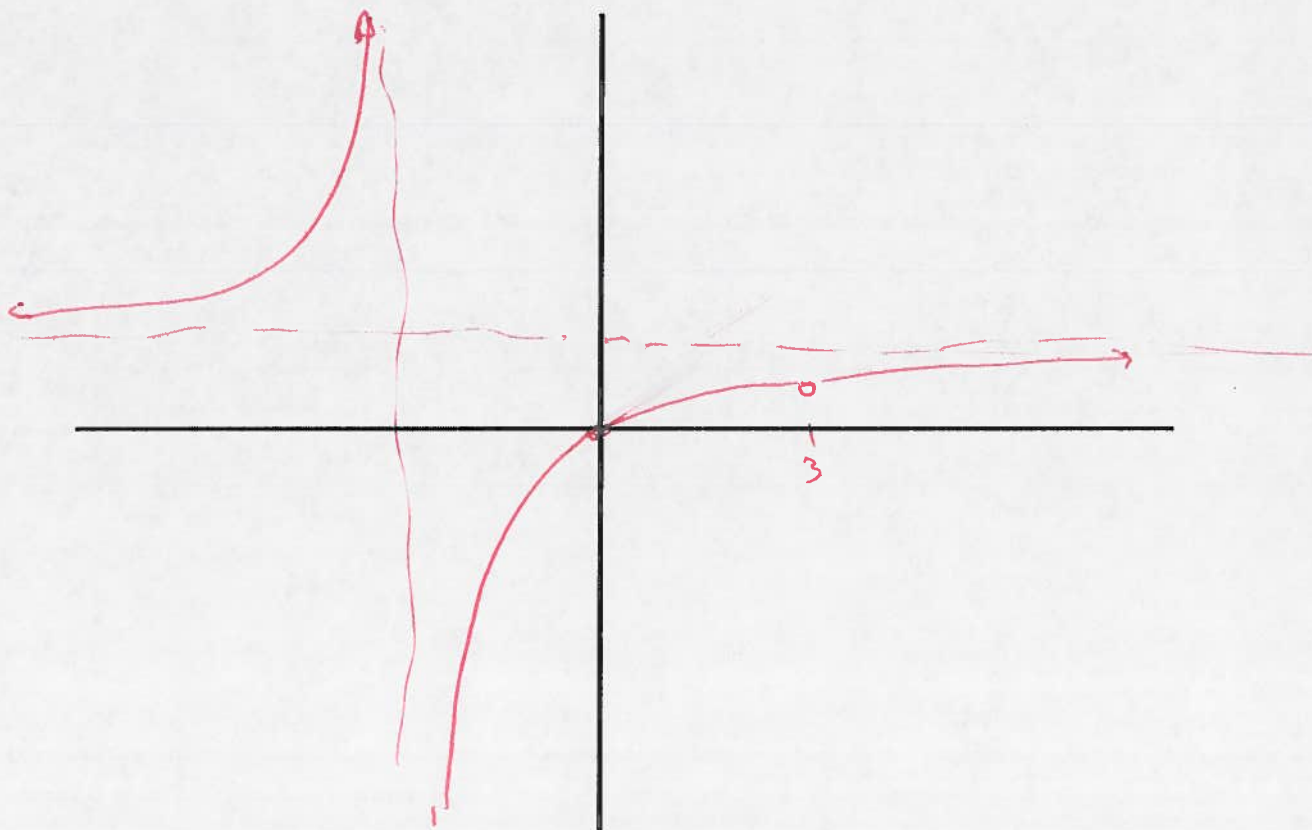
Zeros: $(0, 0)$

EB Right: $y = 1$

VAs: $x = -3$

POEs: $(3, \frac{1}{2})$

Extreme Points: NONE



7. Find the zeros and domain of $y = (4 - x^2)\sqrt{16 - x^2}$. Show the supporting algebraic work.

Zeros $(\pm 2, 0)$, $(\pm 4, 0)$

DOMAIN $16 - x^2 \geq 0$ $x \in [-4, 4]$

8. Find the extreme points of $y = (4 - x^2)\sqrt{16 - x^2}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = (4 - x^2) \cdot \frac{1}{2} (16 - x^2)^{-1/2} (-2x) + (16 - x^2)^{1/2} (-2x)$$

$$= \frac{-x(4 - x^2)}{(16 - x^2)^{1/2}} - 2x(16 - x^2)^{1/2}$$

$$= \frac{-4x + x^3 - 32x + 2x^3}{(16 - x^2)^{1/2}} = \frac{-36x + 2x^3}{(16 - x^2)^{1/2}}$$

i) $\frac{dy}{dx} = 0 \Rightarrow 3x(x^2 - 12) = 0$ $(0, 16)$ $(\pm\sqrt{12}, -16)$
 $x = 0, \pm\sqrt{12}$

ii) $\frac{dy}{dx} \text{ DNE} \Rightarrow 16 - x^2 = 0 \Rightarrow x = \pm 4$ $(\pm 4, 0)$

9. Find the traits and sketch of $y = (4 - x^2)\sqrt{16 - x^2}$.

Domain: $x \in [-4, 4]$

Y-Int: $(0, 16)$

Zeros: $(\pm 4, 0)$ $(\pm 2, 0)$

Range:

VAs: NONE

EB Left: NONE

Extreme Points:

EB Right: NONE

