Chapter 0:

Right Triangle Trigonometry

Chapter 0 Overview

Trigonometry is, literally, the study of triangle measures. We learned in geometry that the relationships between the angles and sides of a triangle hold special significance, especially in a right triangle. Those relationships were those of the Pythagorean Theorem and SOHCAHTOA.

In this chapter, we will review those relationships and explore how they compare to those in oblique (non-right) triangles. The Law of Cosines and the Law of Sines are the rules for oblique triangles that are comparable to the Pythagorean Theorem and SOHCAHTOA, respectively. In fact, those right triangle rules are specific cases of the laws of Sines and Cosines, where one of the angles is 90°.

0-1: Right Triangle Review:

There are two key pieces of knowledge required in right triangle Trig:

<u>Pythagorean Theorem</u>: $a^2 + b^2 = c^2$, where c is the hypotenuse

<u>SOHCAHTOA:</u>

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

Of course, these definitions and formulas, as we memorized them, leave out some key details.

1. Angles and their opposite sides in a triangle are labeled with the same letter--upper case for the angle and lower case for the side.

2. Implied in the Pythagorean Theorem is that a and b are lengths of the legs of a right triangle and c is the length of the hypotenuse.

3. SOHCAHTOA is a mnemonic for the relations of the sides of a right triangle with an angle. Sin, Cos, and Tan mean **NOTHING** without an angle for reference. They should be "sin A, cos A, tan A."

4. Most importantly, Sin A does not mean "Sine times A." Sine is the operation and there is no multiplication implied. Therefore, you do not isolate A by dividing by Sin. Sin is a verb, not a noun. (See EX 1.)

OBJECTIVE

Use the relations of sides and angles in a right triangle to find missing sides and angles.

EX 1. If ΔABC is a right triangle with hypotenuse c and a = 4 and c = 5, find the missing angle and side measures.

$$a^2 + b^2 = c^2$$
$$4^2 + b^2 = 5^2$$
$$b = 3$$

$$\sin A = \frac{4}{5}$$
$$A = \sin^{-1}\frac{4}{5}$$
$$m\angle A = 53.130°$$

$$\sin B = \frac{3}{5}$$
$$B = \sin^{-1}\frac{3}{5}$$
$$m\angle B = 36.869°$$

For the sake of consistency, we will always round our final answers at three decimal places.

As in point 4 above, we did not isolate A by dividing. We applied the inverse (or Arc) function. While -1 as an exponent does mean reciprocal in other instances, it does not mean that here. Actually, the -1 means the inverse operation, the operation that cancels the one being referred to. Reciprocals cancel when we multiply, but we are not multiplying here. The sin and cos are the operations. So,

$$\sin^{-1}\theta \neq \csc\theta \qquad \cos^{-1}\theta \neq \sec\theta$$
$$\tan^{-1}\theta \neq \cot\theta$$

In fact, you will note in EX 1 that the angle value A is not the object of the inverse operations. **<u>The angle A is equal to the inverse of a number</u>**, that number being the trig ratio. If the angle is in radians, the inverse is often referred to as an Arcsin or Arccos because they equal the arc length on the unit circle.

Of course, we want to use these facts in real situations.

Vocabulary:

1. *Angle of inclination*--the angle measured upward from a horizontal line.
2. *Angle of depression*-- the angle measured downward from a horizontal line.

EX 2. A person whose eye level is at 5 feet stands 30 feet away from a building. With a clinometer, she measures the angle to the top of the tree is 48°. How tall is the building? (Draw your own picture first.)

The 30' is the adjacent side. The height of the tree is the opposite side. Therefore, the function we use will be tangent.

$$\tan 48^\circ = \frac{y}{30}$$
$$y = 30\tan 48^\circ$$
$$= 33.318$$

This is the side of the triangle. The height of the building is this size plus the 5' from her eye level to the ground, or

38.318 ft

EX. 3 A tree falls on your teacher's house. The base of the tree had been 40 feet from the house and the house is 25 feet tall. If six feet of the tree is on the roof, how long was the tree and what is its angle of inclination? (Again, draw your own picture.)

The Pythagorean Theorem will tell us the length of the tree from the base to the edge of the house:

$$25^2 + 40^2 = c^2$$
$$c = 47.170$$

plus the six feet on the house is

53.170 feet

Since we now know all three sides of the triangle, any of the three trig functions will tell us the angle of inclination. It might be best to use tangent, in case our tree length is incorrect.

$$\tan A = \frac{25}{40}$$

$$A = \tan^{-1}\frac{5}{8}$$

$$m\angle A = 32.005^\circ$$

0-1 Homework

Find the missing sides and angles (correct to 3 decimal places) of the given right triangles.

1. In ΔABC, $m\angle C = 90°$, a = 60 and c = 61

2. In ΔBFT, $m\angle T = 90°$, b = 16 and f = 13

3. In ΔJEQ, $m\angle Q = 90°$, j = 2 and q = 5

4. In ΔMUG, $m\angle U = 90°$, m = u = 8

5. Jerry, whose eye height is 5.5 feet, is standing 65 feet from the S.I. Library. Using his clinometer, he finds that the angle of inclination is 19º. How tall is the library? Draw a diagram first.

6. Joanne knows that the height of the Transamerica Pyramid in San Francisco is 853 feet. Her eye height is 5 feet and she uses her clinometer to find an angle of inclination of 23º. How far away from the Pyramid is Joanne standing? Draw a diagram first.

7. Brad is standing two miles from San Francisco International Airport watching planes land. When United Flight 361 is at an altitude of 2000 feet, what is the angle of inclination? (A mile is 5,280 feet.) Draw a diagram first.

8. The pilot of Jet Blue Flight 12 has just seen her fuel warning light come on. Her instruments tell her that her altitude is 16 km and her horizontal distance from the airport is 42 km. If she has enough fuel to stay in the air for 50 km, will she make it to the airport? What is her angle of descent (depression)? Draw a diagram first.

9. A telephone pole falls and crashes into your car, shearing off the top 12 feet of the pole. If the base of the pole was originally 18 feet from your car, and your car is 5 feet tall, find the angle of inclination and the original height of the pole. Draw a diagram first.

0-2: The Law of Cosines

The Pythagorean Theorem and SOHCAHTOA are very powerful tools. But they are somewhat limited in that they only apply to right triangles. In this chapter, we will expand these tools to fit non-right (oblique) triangles.

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Law of Cosines

$$a^2 + b^2 - 2ab\cos C = c^2$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The Law of Cosines is the oblique version of the Pythagorean Theorem. In fact, the Pythagorean Theorem is a specific instance of the Law of Cosines. If $m\angle C = 90°$, Cos C = 0, the "extra term" in the Law (below) drops out, and we have the Pythagorean Theorem.

Remember that angles and their opposite sides in a triangle are designated by the same letter--upper case for angles and lower case for sides. This is important here because there is no longer a hypotenuse but the angle in the cosine must be opposite the side on the other side of the equation. The other two sides are interchangeable because addition and multiplication are associative and commutative.

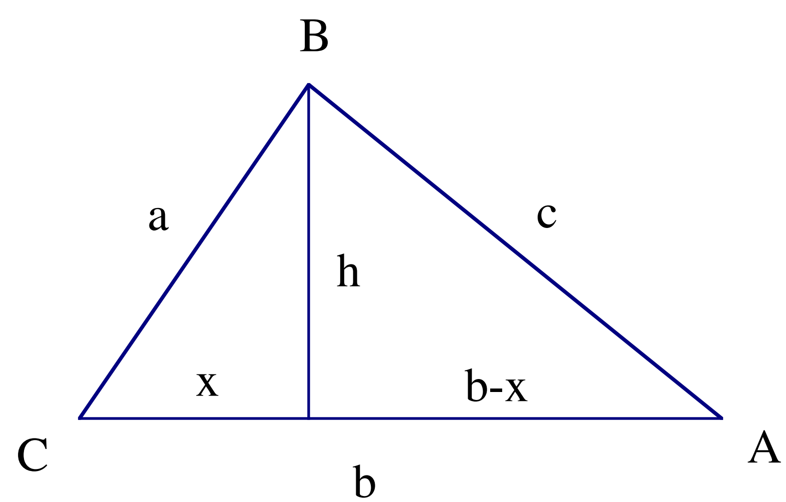
There are two main situations in which to use the Law of Cosines:

1. when given SAS information and asked to find the 3rd side;

2. when given SSS information and asked to find any angles.

Proof of the Law of Cosines

Consider ΔABC below, where sides a and b and $m\angle C$ are known.



By looking at the right triangle on the left and based on SOHCAHTOA, we can say that $h = a\sin C$ and $x = a\cos C$. Looking at the right triangle on the right and the Pythagorean Theorem, we can know $h^2 + (b-x)^2 = c^2$. Substituting for h and x we get:

$$(a\sin C)^2 + (b - a\cos C)^2 = c^2$$

Multiplying and simplifying gives us:

$$a^2\sin^2 C + b^2 - 2ab\cos C + a^2\cos^2 C = c^2$$
$$a^2(\sin^2 C + \cos^2 C) + b^2 - 2ab\cos C = c^2$$

In our work with the Unit Circle, we found that (cos C, sin C) would be a point on the circle where C is the angle. But points on the circle must satisfy the equation of that circle and the equation of the Unit Circle is $x^2 + y^2 = 1$. Therefore, $\cos^2 C + \sin^2 C = 1$ and

$$a^2(\sin^2 C + \cos^2 C) + b^2 - 2ab\cos C = c^2$$
$$a^2 + b^2 - 2ab\cos C = c^2$$

OBJECTIVE

Use the Law of Cosines to find missing sides and angles in an oblique triangle.

EX 1 If in ΔABC, a = 14 and b = 5 and $m\angle$C=130°, find c.

$$a^2+b^2-2ab\cos C=c^2$$
$$14^2+5^2-2(14)(5)\cos 130°=c^2$$
$$196+25-(-89.990)=c^2$$
$$c^2=310.990$$

$$\text{c}=17.635$$

EX 2 If in ΔXYZ, x = 6 and y = 7 and z = 9, find $m\angle$Y.

$$\cos\ C=\frac{a^2+b^2-c^2}{2ab}$$
$$\cos\ Y=\frac{6^2+9^2-7^2}{2(6)(9)}$$
$$\cos\ Y=.6296...$$

$$m\angle \text{Y}=50.977$$

Notice that we did not round until the answer. Interim steps kept the full floated decimal.

EX 3 In ΔCEK, c = 4 and e = 8 and k = 14. Find $m\angle$K.

$$\cos K = \frac{4^2 + 8^2 - 14^2}{2(4)(8)}$$
$$\cos K = -1.8125$$
$$m\angle K = error$$

There is no solution here because these lengths do not form a triangle. We could have figured that out at the start because the first two sides of this "triangle" do not add to more than the third side.

0-2 Homework

1. In ΔABC, $m\angle C = 35°$, $a = 6$ and $b = 7$, find c.

2. In ΔDEF, $m\angle D = 130°$, $e = 10$ and $f = 8$, find d.

3. In ΔMZK, $m\angle K = 13°$, $m = 5$ and $z = 61$, find k.

4. In ΔPDQ, $m\angle P = 114°$, $d = 17$ and $q = 45$, find p.

5. In ΔBFD, b = 5, d = 6 and $f = 9$, find $m\angle B$.

6. In ΔBFD, b = 5, d = 6 and $f = 9$, find $m\angle D$.

7. In ΔJAM, $j = 9$, $m = 11$ and $a = 17$, find $m\angle A$.

8. In ΔKRQ, $k = 8$, $r = 15$ and $q = 6$, find $m\angle R$.

0-3: Area and the Law of Sines

The Law of Sines serves as the oblique version of the SOH part of SOHCAHTOA. It relates the angles to the sides in a triangle. As with the Law of Cosines and the Pythagorean Theorem, SOH is the specific case of the Law of Sines when the angle = 90°.

Law of Sines

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

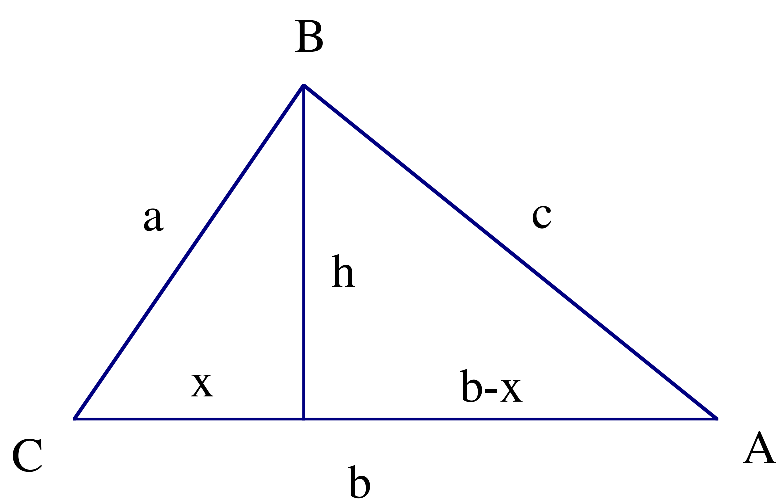
or

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

The main situation in which to use the Law of Sines is when given information about two angles and a side.

Proof of the Law of Sines

As we saw when setting up the proof of the Law of Cosines in the last section, $h = a\sin C$.



Since the area of a triangle is $Area = \frac{1}{2}bh$, then in this case

$$Area = \frac{1}{2}ab\sin C$$

But we could have draw the altitude from A or B, which would have yielded $Area = \frac{1}{2}bc\sin A$ and $Area = \frac{1}{2}ac\sin B$ respectively. Since all three must give the same area, then

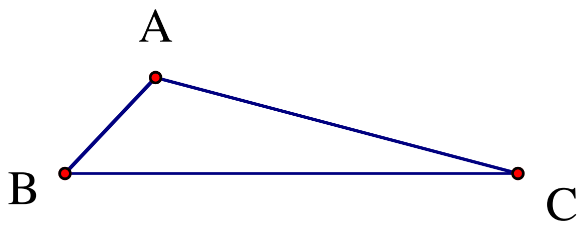
$$\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C.$$

Dividing by $\frac{1}{2}abc$, we get $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

OBJECTIVE

Use the Law of Sines to find missing sides in an oblique triangle.

EX 1 In ΔABC, a = 6, $m\angle$B=50° and $m\angle$C=13°. Find b and c.



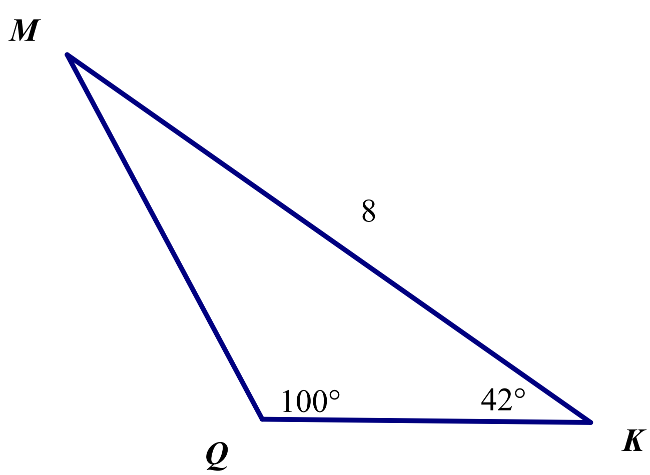
Given two angles, we immediately know the third $m\angle$A=117° because the three angles of a triangle add to 180°.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{6}{\sin 117°} = \frac{b}{\sin 50°} \quad \text{and} \quad \frac{6}{\sin 117°} = \frac{c}{\sin 13°}$$

$$b = 5.159 \qquad c = 1.515$$

EX 2 In ΔMQK, l = 8, $m\angle$M=42° and $m\angle Q$=100°. Find m and k.



$$\frac{8}{\sin 100°} = \frac{m}{\sin 42°}$$

$$\boxed{m = 5.436}$$

Again, because the three angles of a triangle add to 180°, $m\angle K = 38°$.

$$\frac{8}{\sin\ 100°} = \frac{k}{\sin\ 38°}$$

$$k = 5.001$$

EX 3 Find the areas of ΔABC and ΔMQK above.

ΔABC: $\quad Area = \frac{1}{2}ab\sin\ C = \frac{1}{2}(6)(5.159)\sin\ 13° = 3.482$

ΔMQK: $\quad Area = \frac{1}{2}(qm)\ \sin\ K = \frac{1}{2}(8)(5.436)\sin\ 38° = 13.387$

If three sides are given, there is another way to find the area. It is called Heron's Formula:

$$\text{Area of } \Delta ABC = \sqrt{(s)(s-a)(s-b)(s-c)},$$
$$\text{where } s = \frac{a+b+c}{2}.$$

EX 4 Find the area of ΔCEK, if $c = 4$ and $e = 8$ and $k = 11$.

$$s = \frac{4+8+11}{2} = 11.5, \text{ so } \Delta\text{CEK} = \sqrt{(11.5)(7.5)(3.5)(0.5)} = 12.286$$

0-3 Homework

Find the missing sides and the areas of these triangles.

1. In ΔABC, $m\angle C = 35°$, $m\angle A = 46°$, and b = 7

2. In ΔDEF, $m\angle D = 35°$, $m\angle E = 47°$, and f = 8.

3. In ΔMQK, $m\angle K = 63°$, m = 5, and $m\angle L = 61°$.

4. In ΔPDQ, $m\angle P = 114°$, $m\angle D = 17°$ D = 17° and q = 45.

5. In ΔBFC, $m\angle B = 135°$, $m\angle C = 29°$, and c = 6.

6. In ΔRCQ, $m\angle R = 33°$, $m\angle Q = 57°$ and q = 11.

There is another way to find the area of a triangle. Find the areas of the following triangles.

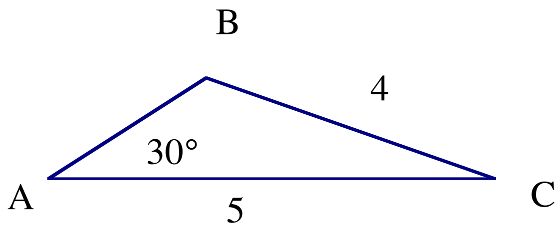
7. In ΔBFD, b = 5, d = 6 and f = 9.

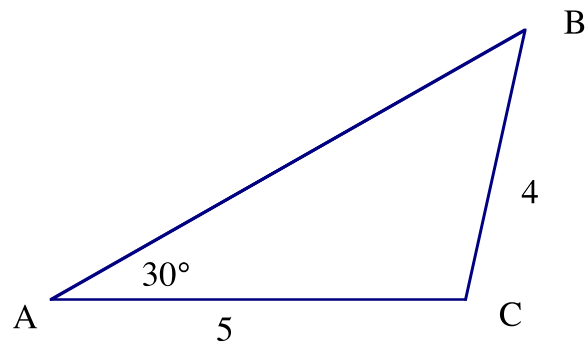
8. In ΔKRV, k = 8, r = 15 and v = 6.

9. In ΔJAM, j = 9, a = 17 and m = 11.

0-4: SSA--The Ambiguous Case

In Geometry, there is no SSA congruence theorem. The reason is that there might be two different ways to draw a figure with two known sides and a known non-included angle. Consider $\triangle ABC$ where a = 4, b = 5 and $m\angle A = 30°$. Both these pictures fit the criteria.





This is an "ambiguous" situation--it can be understood in two different ways. If we were trying to solve for side c, there would be two answers. In fact, there are even two ways to solve for c.

EX 1 In ΔABC, a = 4, b = 5 and $m\angle A = 30°$. Find c.

We could use the Law of Cosines. Since the known angle is opposite a known side, the set up looks like this:

$$a^2 + b^2 - 2ab\cos C = c^2$$
$$5^2 + c^2 - 2(5)(c)\cos 30° = 4^2$$
$$c^2 - 8.660c + 9 = 0$$

Solving with the Quadratic Formula, we get

$$c = \frac{8.660 \pm \sqrt{8.660^2 - 4(1)(9)}}{2}$$

$$\boxed{c = 7.330 \text{ or } 1.330}$$

The $\pm$ in the quadratic formula is what gives the two possible sides.

EX 1 again: In ΔABC, a = 4, b = 5 and $m\angle A = 30°$. Find c.

We could use the Law of Sines and find the other angles first. If we do this, the ambiguity arises from the fact that there are two angles that have any given sign value--an acute angle and an obtuse angle. (We will explore why in Chapter 2.) The second answer--which your calculator does not give you directly--is the supplement of the first angle.

$$\frac{\sin\ B}{5} = \frac{\sin\ 30°}{4}$$
$$\sin\ B = \frac{5\sin\ 30°}{4} = .625$$
$$B\ = \sin^{-1}\ .625 = 38.682°$$
$$\frac{4}{\sin\ 30°} = \frac{c}{\sin\ 8.682°}$$

$$c = 1.732$$

But notice that we only got one of the two answers that the Law of Cosines gave us. This is because the triangle is not a right triangle. **Once we consider angles larger than 90°, there are two angles with sine equal to 0.625.** As we will explore more carefully in Chapter 1, inverse trigonometric operations always yield two answers, *only one of which the calculator will give*. When dealing with $\sin^{-1}$, the other angle is the supplement of the first. That is:

$$\sin^{-1}\frac{y}{r} = \begin{Bmatrix} \text{calculator} \\ 180° - \text{calculator} \end{Bmatrix}$$

EX 1 yet again: In ΔABC, a = 4, b = 5 and $m\angle A = 30°$. Find c.

$$\frac{\sin B}{5} = \frac{\sin 30°}{4}$$

$$\sin B = \frac{5\sin 30°}{4} = .625$$

$$B = \sin^{-1} .625 = \begin{Bmatrix} 38.682° \\ 141.318° \end{Bmatrix}$$

The two answers for B yield two answers for C, based on the angle sum.

$$C = \begin{Bmatrix} 111.318° \\ 8.682° \end{Bmatrix}$$

$$\frac{4}{\sin 30°} = \frac{c}{\sin 8.682°} \qquad \frac{4}{\sin 30°} = \frac{c}{\sin 111.318°}$$

$$c = 1.732 \qquad c = 5.196$$

If that is not ambiguous enough, the SSA situation is still more so. There might not be two triangles. There might only be one--or even none.

EX 2: In ΔELP, e = 4, l = 5 and $m\angle L = 34°$. Find p.

$$\frac{\sin E}{4} = \frac{\sin 34°}{5}$$

$$\sin E = \frac{4\sin 34°}{5} = .447$$

$$m\angle E = \sin^{-1} .447 = \begin{Bmatrix} 26.574° \\ 153.426° \end{Bmatrix}$$

If $m\angle E = 26.574°$, then $m\angle P = 119.426°$. But if $m\angle E = 153.426°$, P would have to be a negative number to get a sum of 180°. Therefore,

$$m\angle P = 119.426°$$

$$\frac{5}{\sin\ 34°} = \frac{p}{\sin\ 119.426°}$$

$$\boxed{c = 7.788}$$

If we had used the Law of Cosines, one of the sides would have been negative.

EX 3: In ΔBTF, b = 2, t = 5 and $m\angle B = 63°$. Find o.

$$\frac{\sin\ T}{5} = \frac{\sin\ 63°}{2}$$

$$\sin\ T = \frac{5 \sin\ 63°}{2} = 2.228$$

$$T \ = \sin^{-1}\ 2.228 = error$$

There is no such triangle because side b is too short to reach side *f* with the given $m\angle B$.

0-4 Homework

Find all the possible missing sides and angles of these triangles.

1. In ΔKRD, k = 8, r = 15 and $m\angle K = 31°$.

2. In ΔDEF, $m\angle D = 35°$, d = 11 and f = 8.

3. In ΔMYK, $m\angle K = 163°$, m = 5 and k = 8.

4. In ΔPDQ, $m\angle P = 64°$, d = 17 and p = 6.

5. In ΔBFC, $m\angle B = 35°°$, c = 6 and b = 5.

6. In ΔRCQ, $m\angle R = 30°$, r = 5 and q = 10.

7. In ΔJAT, j = 9, a = 17 and t = 11.

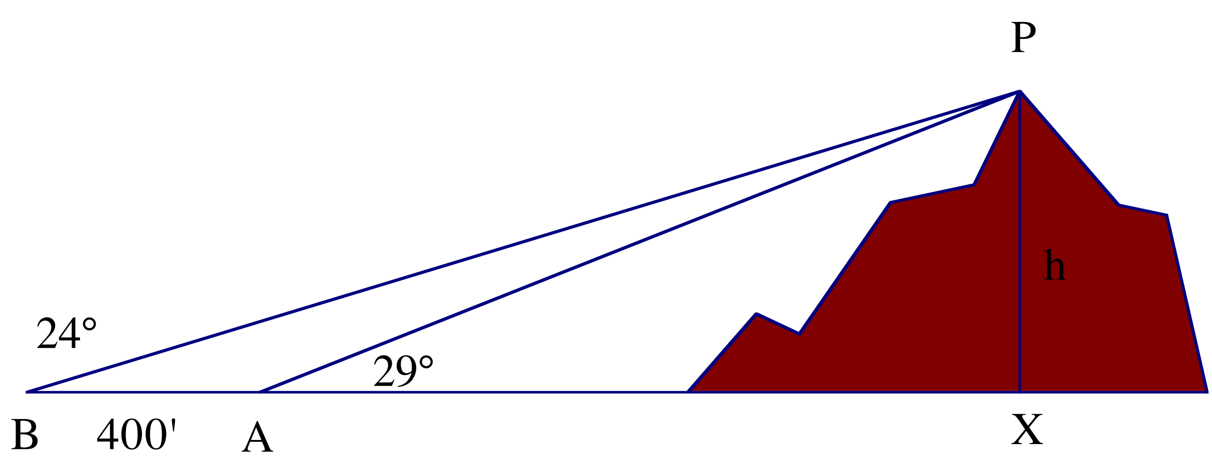
8. In ΔKRC, k = 8, r = 15 and $m\angle C = 76°$.

0-5: Mathematical Modeling with Triangles

Now that we have tools and techniques to deal with oblique as well as right triangles, we can apply them to real situations.

OBJECTIVE Use the Laws of Cosines and Sines to solve mathematical models involving triangles.

EX 1 Finding the height of a mountain is not as easy as finding the height of a building (section 1-1, EX 2) because you cannot directly measure your distance to a spot directly below the peak. What you can do is measure the angle to the peak from two different positions. At point A, the angle to the peak is 29°. At point B, which is 400’ further away, the angle of elevation is 24°. How tall is the mountain?



ΔAPX is a right triangle. If we knew AX or AP, we could use SOHCAHTOA to find PX (the height h). AP is a side in ΔABP which we can find by the Law of Sines (since we have two angles).

$m\angle XAP = 29°$, therefore $m\angle BAP = 151°$.
$m\angle B = 24°$, so $m\angle BPA = 5°$.

By the Law of Sines,

$$\frac{a}{\sin\ 151^\circ} = \frac{400}{\sin\ 5^\circ}$$

$$a = \frac{400 \sin\ 151^\circ}{\sin\ 5^\circ} = 2225.027'$$

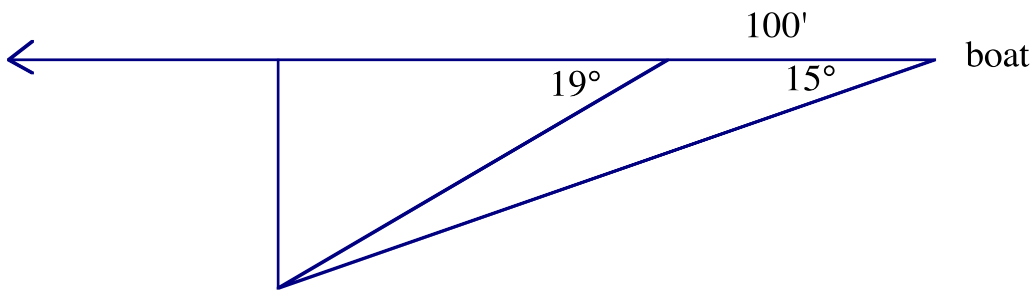
And

$$\sin\ 24^\circ = \frac{h}{2225.027}$$

$$\boxed{h = 2225.027 \sin\ 24^\circ = 905'}$$

0-5 Homework

1. A boat is sailing west toward the Golden Gate Bridge. At one time, the angle measured from due west to Fort Point is 15°. After sailing 100 yards, the angle is 19°. How close will the boat come to Fort Point?



2. Triangulation--often used in artillery practice--is a process that determines someone's (or something's) position by taking two sightings from positions that are a known distance apart. If sights that are 200 yards apart determine the target is at 53° and 48°, how far from each sighting position is the target?

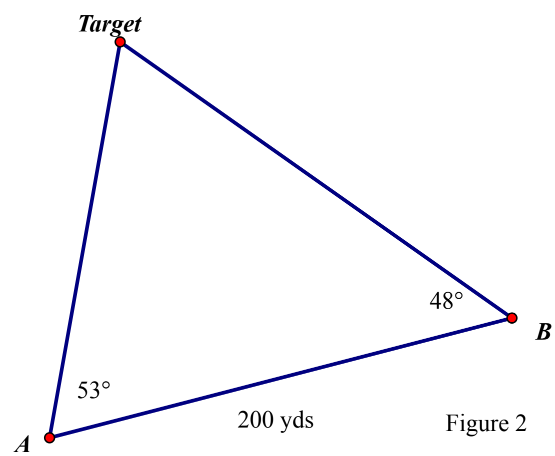


Figure 2

3. A parallelogram has sides of 18 and 26, and an angle of 39°. Find the length of the longer diagonal and the area of the parallelogram. [Draw a diagram first.]

4. If you participate in any of the marches from Justin Herman Plaza to City Hall, you walk 1.5 miles along Market street, then a quarter mile on Grove. Grove

meets Market at angle of 140°. How far, in a straight line, is your starting point from the end?

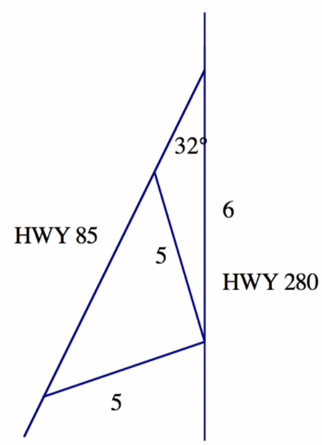
5. A swimmer sees two alligators. He can tell that the distance from him to one alligator is 30’, the distance between the alligators is 20’, and the angle where the swimmer is equals 58°.

a) Show that he is wrong about the measure of the angle.

b) Find the two possible distances between the swimmer and the second alligator using the correct angle, 28°.

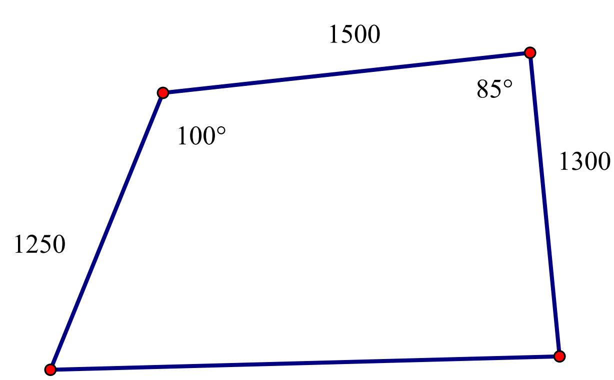
6. A surveyor measures three sides of a triangular field and finds them to be 102', 176' and 247'. What is the area of the field?

7. A trucker on HWY 280 has a radio with a range of 5 miles. HWY 280 intersects HWY 85 at a 32° angle. When the trucker is 6 miles south of the intersection, between what two distances from the intersection can another trucker on HWY 85 receive the signal?



Problem 7

8. The surface of the Sunset Reservoir at 24th and Quintara is a quadrilateral with sides 1500’, 1250’, and 1300’ and angles 100° and 85°. Find the other sides and angles and the surface area of the Reservoir.



Chapter 0 Test

Directions: Round at 3 decimal places. Show all setups and all work (and make sure it's both *legible* and *well organized*. Draw sketches for each problem.

1. Solve for all sides and angles in ΔABC if $m\angle A = 90°$, $a = 12$, and $c = 9$.

2. You and your best friend are at Bayfront Park watching planes land at San Francisco International airport runways 28L and 28R, which are at sea level. You use your clinometer to discover that the next plane is coming in at an angle of inclination of 12°. If Bayfront Park is at sea level and 1500m from the runway, and your eye height is 1.5m, find the plane's altitude.

3. Explain why you get an error message for $\sin^{-1}\left(\frac{140}{100}\right)$.

4. Find d in ΔDEF if $e = 45$, $f = 71$, and $m\angle D = 29°$.

5. Find the largest angle in a triangle whose side lengths are 20cm, 40cm, and 50cm.

6. In ΔGHI, $m\angle G = 41°$, $m\angle H = 67°$, and $i = 21$. Find g and h.

7. Find the area of the triangle in the previous problem.

8. Find all possible values of r in ΔPQR if $m\angle P = 24°$, $q = 34$, and $p = 40$.

9. A building is of unknown height. At a distance of 100 feet away from the building, an observer notices that the angle of elevation to the top of the building is 41º and that the angle of elevation to a poster on the side of the building is 21º. How far is the poster from the roof of the building?

10. A large helium balloon is tethered to the ground by two taut lines. One line is 100 feet long and makes an angle of 80° with the ground. The other line makes an angle of 40° with the ground. How far apart are the two tethers?

0-1 Homework

1. b = 11, A = 79.611°, $m\angle$B = 10.389°

2. t = $5\sqrt{17} = 28.615$, $m\angle$B = 50.906°, $m\angle$F = 39.094°

3. e = $\sqrt{21} = 4.583$, $m\angle$J = 23.578°, $m\angle$E = 66.442°

4. No such triangle

5. 27.881 *ft*

6. 1997.763 *ft*

7. 10.724°

8. Yes, d = 44.944 km; 20.854°

9. 15.524°; h = 30.682’

0-2 Homework

1. c = 4.024

2. d = 16.335

3. k = 56.139

4. p = 54.188

5. $m\angle$B = 31.586°

6. $m\angle$D = 38.942°

7. $m\angle$A = 116.065°

8. No such triangle

0-3 Homework

1. a = 5.098, c = 4.065, Area = 10.234

2. d = 4.634, e = 5.908, Area = 13.556

3. $k = 5.374$, $q = 5.275$, Area = 11.750

4. p = 54.471, d = 17.433, Area = 358.328

5. b = 8.751, f = 3.411, Area = 7.236

6. r = 7.143, c = 13.116, Area = 39.289

7. Area = 14.142

8. No such triangle

9. Area = 44.466

<u>0-4 Homework</u>

1. $m\angle R = \begin{cases} 74.949° \\ 105.051° \end{cases} \quad m\angle D = \begin{cases} 74.051° \\ 43.949° \end{cases}, \; d = \begin{cases} 14.934 \\ 10.780 \end{cases}$

2. $m\angle F = \begin{cases} 52.061° \\ 125.941° \end{cases}, \; m\angle E = \begin{cases} 91.939° \\ 19.061° \end{cases}, \; e = \begin{cases} 15.940 \\ 4.555 \end{cases}$

3. $m\angle Y = 10.529°, \; m\angle L = 6.471°, \; y = 3.083°$

4. no such triangle

5. $m\angle C = \begin{cases} 43.495° \\ 136.505° \end{cases} \quad m\angle F = \begin{cases} 101.505° \\ 8.495° \end{cases}, \; f = \begin{cases} 8.542 \\ 1.137 \end{cases}$

6. $m\angle Q = 90°, \; m\angle C = 60°, \; c = 5\sqrt{3}$

7. $m\angle J = 28.396$, $m\angle A = 116.065°$, $m\angle T = 35.539°$

8. $m\angle K = 30.717°$, $m\angle R = 73.283°$, $c = 15.197$

0-5 Homework

1. 120.796’

2. 162.717’ and 151.411’

3. 41.562

4. 1.699 miles

5a. no such triangle b. 12.289 or 40.689

6. 7515.815 ft^2

7. 8.947 or 1.229

8. 87.867°, 87.713, and 1605.056’, Area $= 1{,}965{,}417.397\ ft^2$

Chapter Test Key

1. $b = 7.937$; $m\angle B = 41.410°$; $m\angle C = 48.590°$
2. 320.335 m
3. Sine is defined as $\left(\frac{opp}{hyp}\right)$, and the hypotenuse must be the longest side of the triangle. However, $100 < 140$, so the result is an error message.
4. 38.434
5. 97.181°
6. $g = 14.486$; $h = 20.325$
7. 140.011 square units

8. $r = 68.594$

9. 48.542 feet

10. 134.730 feet apart