

10-1 Free Response Homework

1. $\frac{1}{3}\ln 27 + 2\ln 2 = \ln 27^{\frac{1}{3}} + \ln 2^2 = \ln 3 + \ln 4 = \ln(3 \cdot 4) = \ln 12$

3. $\frac{1}{2}\log_5 16 - 2\log_5 10 = \log_5 16^{\frac{1}{2}} - \log_5 10^2 = \log_5 4 - \log_5 100 = \log_5 \frac{4}{100} = \log_5 \frac{1}{25} = -2$

5. $13(5^x) = 1625 \rightarrow 5^x = 125 \rightarrow 5^x = 5^3 \rightarrow x = 3$

7. Graph $y_1 = 4e^{3x} + 1$ and $y_2 = 8^{1-x}$ on the calculator. Use 2nd Trace #5 to find the point of intersection. $x = 0.103$

9. $e^x = -3 \rightarrow \ln(e^x) = -3 \rightarrow x = \ln(-3) = dne \therefore$ no solution

11. $x = \log_{\sqrt{3}}\left(\frac{1}{9}\right) = \log_{\sqrt{3}}\left(\frac{1}{(\sqrt{3})^4}\right) = \log_{\sqrt{3}}\left(\left(\sqrt{3}\right)^{-4}\right) = -4$

13. $\log_4(x-1) + \log_4(3x+1) = 3 \rightarrow \log_4(x-1)(3x+1) = 3 \rightarrow (x-1)(3x+1) = 4^3 = 64$
 $3x^2 - 2x - 1 = 64 \rightarrow 3x^2 - 2x - 65 = 0 \rightarrow (3x-13)(x-5) = 0$

$x = 5, -\frac{13}{3}$. But $x = -\frac{13}{3}$ is not in the domain of the problem. Therefore, $x = 5$ is the answer.

15.

$$\frac{1}{2} \log_7 x = \log_7 20 - 2(\log_7 2 + \log_7 5)$$

$$\frac{1}{2} \log_7 x = \log_7 20 - 2 \log_7 2 - 2 \log_7 5$$

$$\log_7 x^{\frac{1}{2}} = \log_7 20 - \log_7 2^2 - \log_7 5^2$$

$$\log_7 x^{\frac{1}{2}} = \log_7 20 - \log_7 4 - \log_7 25$$

$$\log_7 x^{\frac{1}{2}} = \log_7 \frac{20}{4 \cdot 25}$$

$$\log_7 x^{\frac{1}{2}} = \log_7 \frac{1}{5}$$

$$7^{\log_7 x^{\frac{1}{2}}} = 7^{\log_7 \frac{1}{5}}$$

$$x^{\frac{1}{2}} = \frac{1}{5}$$

$$x = \frac{1}{25}$$

$$17. \quad 17(10^x) = 51 \rightarrow 5^x = 125 \rightarrow 10^x = 3 \rightarrow x = \log 3 \approx 0.477$$

19. Method 1

$$2^{3x-1} = 8^{1-x}$$

$$2^{3x-1} = (2^3)^{1-x}$$

$$2^{3x-1} = 2^{3-3x}$$

$$\log_2 2^{3x-1} = \log_2 2^{3-3x}$$

$$3x - 1 = 3 - 3x$$

$$6x = 4$$

$$x = \cancel{2}/3$$

Method 2

$$2^{3x-1} = 8^{1-x}$$

$$\ln 2^{3x-1} = \ln 8^{1-x}$$

$$(3x-1)\ln 2 = (1-x)\ln 8$$

$$3\ln 2 \cdot x - \ln 2 = \ln 8 - \ln 8 \cdot x$$

$$3\ln 2 \cdot x + \ln 8 \cdot x = \ln 8 + \ln 2$$

$$(3\ln 2 + \ln 8)x = \ln 8 + \ln 2$$

$$x = \frac{\ln 8 + \ln 2}{3\ln 2 + \ln 8}$$

$$x = \cancel{2}/3$$

10-1 Multiple Choice Homework

1. D

$$\begin{aligned}
 \frac{ab^{-1}}{a^{-1}-b^{-1}} &= \frac{a}{b} \div \left(\frac{1}{a} - \frac{1}{b} \right) \\
 &= \frac{a}{b} \div \left(\frac{b-a}{ab} \right) \\
 &= \frac{a}{b} \cdot \frac{ab}{b-a} \\
 &= \frac{a}{b} \cdot \frac{a\cancel{b}}{b-a} \\
 &= \frac{a^2}{b-a}
 \end{aligned}$$

3. B

$$\begin{aligned}
 f\left(\frac{2}{x}\right) + f(x) &= \log_2\left(\frac{2}{x}\right) + \log_2 x \\
 &= \log_2 2 - \log_2 x + \log_2 x \\
 &= \log_2 2 \\
 &= \log_2 2^1 \\
 &= 1
 \end{aligned}$$

5. A

$$y_1 = \frac{\ln x}{\ln 5}, y_2 = \ln(0.5x)$$

2nd Trace (CALC)

→ 5 (Intersect)

→ @x = 6.237

7. C

$$g(f(2)) = 10^{2\log 2} = 10^{\log 2^2} = 10^{\log 4} = 4$$

9. C

$$\begin{aligned}
 \log_a\left(\frac{x}{y^3z^2}\right) &= \log_a x - \log_a y^3 - \log_a z^2 \\
 &= \log_a x - 3\log_a y - 2\log_a z
 \end{aligned}$$

11. B

$$\log_{12} 3 + \log_{12} 48 = \log_{12}(3 \cdot 48) = \log_{12}(144) = 2$$

13. D

$$\ln[3^{1-x} = 7]$$

$$\ln 3^{1-x} = \ln 7$$

$$(1-x)\ln 3 = \ln 7$$

$$(1-x) = \frac{\ln 7}{\ln 3} = 1.771$$

$$x = -0.771$$

10-2 Free Response Homework

1. $r = 6.5\%, n = 12, S = 10,000, t = 10$

$$10,000 = P \left(1 + \frac{0.065}{12} \right)^{12 \cdot 10}$$

$$P = \$5229.62$$

$$3. \quad r = 3.6\%, n = 12, P = 300, t = 10$$

$$S = 300 \left(1 + \frac{0.036}{12} \right)^{12 \cdot 10}$$

$$S = \$429.77$$

$$r = 4.1\%, n = 4, P = 300, t = 10$$

$$S = 300 \left(1 + \frac{0.041}{4} \right)^{4 \cdot 10}$$

$$S = \$451.10$$

$\Rightarrow 4.1\%$ Quarterly

$$5. \quad r = 13\%, n = 12, S = 60,000, t = 10$$

$$60,000 = P \frac{\left[\left(1 + \frac{0.13}{12} \right)^{12 \cdot 10} - 1 \right]}{\frac{0.13}{12}}$$

$$650 = P \left[\left(1 + \frac{0.13}{12} \right)^{12 \cdot 10} - 1 \right]$$

$$P = \$245.86$$

$$7. \quad r = 2.9\%, n = 12, A = 40,000, t = 5$$

$$40,000 = P \frac{\left[1 - \left(1 + \frac{0.029}{12} \right)^{-12 \cdot 5} \right]}{\frac{0.029}{12}}$$

$$96.667 = P \left[1 - \left(1 + \frac{0.029}{12} \right)^{-12 \cdot 5} \right]$$

$$P = \$716.97$$

$$9. \quad r = 4.9\%, n = 12, A = 200,000, t = 10$$

$$200,000 = P \frac{\left[1 - \left(1 + \frac{0.049}{12} \right)^{-12 \cdot 10} \right]}{0.029}$$

$$816.667 = P \left[1 - \left(1 + \frac{0.049}{12} \right)^{-12 \cdot 10} \right]$$

$$P = \$2111.55 \text{ per month}$$

$$\text{Total Cost} = \$2111.55 \cdot 12 \frac{\text{months}}{\text{year}} \cdot 10 \text{ years}$$

$$= \$253,385.75$$

$$11. \quad r = 4.5\%, n = 12, A = 250,000, P = 2,000$$

$$250,000 = 2000 \frac{\left[1 - \left(1 + \frac{.045}{12} \right)^{-12t} \right]}{.045}$$

$$0.53125 = 1.00375^{-12t}$$

$$\ln 0.53125 = \ln 1.00375^{-12t}$$

$$\ln 0.53125 = -12t \ln 1.00375$$

$$t = \frac{\ln 0.53125}{-12 \ln 1.00375} = 14.082 \text{ years}$$

10-2 Multiple Choice Homework

15. B

The loan formula is the only one with a negative exponent.

10-3 Free Response Homework

1. Domain: $x \in \text{All Reals}$

Zeros: None

$$e^{3x^2} = 0 \Rightarrow \ln e^{3x^2} = \ln 0 = \text{DNE}$$

y-int: (0, 1)

VAs: None

3. Domain: $x \in \text{All Reals}$

Zeros: None

VAs: None

5. Domain: $x \in (-\infty, 7]$

$$\sqrt{7-x} = 0 \Rightarrow 7-x = 0 \Rightarrow x = 7$$

$$\begin{array}{c} 7-x \\ x \end{array} \leftarrow \begin{array}{c} + \\ 0 \\ - \end{array} \rightarrow$$

Zeros: (7, 0)

$$\begin{aligned}
 2^x \sqrt{7-x} &= 0 \\
 \Rightarrow 2^x &= 0 \text{ or } \sqrt{7-x} = 0 \\
 \Rightarrow \ln 2^x &= \ln 0 = \text{DNE} \text{ or } 7-x = 0 \\
 \Rightarrow x &= 7 \\
 \text{VAs: None}
 \end{aligned}$$

7. Domain: $x \in [-2, 2]$

$$\sqrt{4-x^2} = 0 \Rightarrow 4-x^2 = 0 \Rightarrow x = \pm 2$$

$$\begin{array}{c}
 4-x^2 \quad \leftarrow \begin{matrix} - & 0 & + & 0 & - \end{matrix} \\
 x \qquad \qquad \qquad \begin{matrix} -2 & & 2 \end{matrix}
 \end{array}$$

Zeros: None

$$e^{\sqrt{4-x^2}} = 0 \Rightarrow \ln e^{\sqrt{4-x^2}} = \ln 0 = \text{DNE}$$

VAs: None

$$\begin{aligned}
 x^2 \sqrt{25-x^2} &= 0 \Rightarrow x^2 = 0 \text{ or } \sqrt{25-x^2} = 0 \\
 \Rightarrow x &= 0 \text{ or } 25-x^2 = 0 \\
 \Rightarrow x &= 0, \pm 5
 \end{aligned}$$

VAs: None

9. Domain: $x \in \text{All Reals} [\text{or } x \in (-\infty, \infty)]$

$$(x-x^3)e^x = 0 \Rightarrow x-x^3 = 0 \text{ or } e^x = 0$$

Zeros: $(0, 0), (\pm 1, 0) \Rightarrow x(1-x^2) = 0 \text{ or } \ln e^x = \ln 0 = \text{DNE}$

$$\Rightarrow x = 0, \pm 1$$

VAs: None

11. Domain: $x \in (-\infty, \infty)$

Zeros: $(4, 0), (-2, 0)$

$$(x^2 - 2x - 8)e^x = 0 \Rightarrow x^2 - 2x - 8 = 0 \text{ or } e^x = 0$$

$$\Rightarrow (x-4)(x+2) = 0 \text{ or } \ln e^x = \ln 0 = \text{DNE}$$

$$\Rightarrow x = 4, -2$$

VAs: None

13. $y = 4(1-x^2)e^{-0.5x^2}$ Domain: $x \in (-\infty, \infty)$

Zeros: $(\pm 1, 0)$

$$4(1-x^2)e^{-0.5x^2} = 0 \rightarrow 1-x^2 = 0 \text{ or } e^{-0.5x^2} = 0$$

$$1-x^2 = 0 \rightarrow x = \pm 1 \quad e^{-0.5x^2} = 0 \rightarrow \text{no solution}$$

VAs: None because there is no denominator of \ln

15. Domain: $x \in (0, 1)$

$$x - x^2 = 0 \Rightarrow x(1-x) = 0$$

$$\Rightarrow x = 0, 1$$

$$\begin{array}{c} x - x^2 \\ \hline x \end{array} \leftarrow \begin{array}{ccccc} - & 0 & + & 0 & - \end{array} \rightarrow \begin{array}{c} 0 \\ | \\ 1 \end{array}$$

Zeros: None

$$\ln(x - x^2) = 0 \Rightarrow x - x^2 = 1$$

$$\Rightarrow x^2 - x + 1 = 0$$

No real solution

VA: $x = 0, x = 1$

17. Domain: $x \in (-2, 0), (2, \infty)$

$$x^3 - 4x = 0 \Rightarrow x(x-2)(x+2) = 0$$

$$\Rightarrow x = 0, \pm 2$$

$$\begin{array}{c} x^3 - 4x \\ \hline x \end{array} \leftarrow \begin{array}{ccccc} - & 0 & + & 0 & - \end{array} \rightarrow \begin{array}{c} -2 \\ | \\ 0 \\ | \\ 2 \end{array}$$

Zeros: $(-1.861, 0),$

$(-0.254, 0), (2.115, 0)$

$$\ln(x^3 - 4x) = 0 \Rightarrow x^3 - 4x = 1$$

$$\Rightarrow x^3 - 4x - 1 = 0$$

$$\Rightarrow x = -1.861, -0.254, 2.115$$

VA: $x = -2, x = 0, x = 2$

19. Domain: $x \in (-\infty, -3) \cup (0, 3)$

$$\frac{-3x}{x^2 - 9} = 0, \text{DNE} \Rightarrow -3x = 0 \text{ or } x^2 - 9 = 0$$

$$\Rightarrow x = 0, \pm 3$$

$$\frac{-3x}{x^2 - 9} \xleftarrow[x]{+ \text{ DNE } -3} \xrightarrow[0]{0 + \text{ DNE } 3}$$

Zeros: $(1.854, 0), (-4.854, 0)$

$$\ln\left(\frac{-3x}{x^2 - 9}\right) = 0 \Rightarrow \frac{-3x}{x^2 - 9} = 1$$

$$\Rightarrow x^2 + 3x - 9 = 0$$

$$\Rightarrow x = -4.854, 1.854$$

VA: $x = -3, x = 0, x = 3$

21. Domain: $x \in (0, \infty)$

Zeros: $(1, 0)$

$$x^2 - 1 = 0 \text{ or } \ln x = 0$$

$$\Rightarrow x = 1, \cancel{x=0}$$

VA: $x = 0$

23. Domain: $x \in \text{All Reals}$

$$e^x - x = 0 \Rightarrow \text{no solution}$$

$$\frac{e^x - x}{x} \xleftarrow[+]{x}$$

Zeros: $(0, 0)$

$$\ln(e^x - x) = 0 \Rightarrow e^x - x = 1$$

$$\Rightarrow e^x - x - 1 = 0$$

$$\Rightarrow x = 0$$

VA: None

25. Domain: $x \in (-\infty, -5) \cup (4, 5)$

$$\begin{array}{c} -x^3 + 4x^2 + 25x - 100 = 0 \Rightarrow x = 4, \pm 5 \\ \begin{array}{ccccccc} -x^3 + 4x^2 + 25x - 100 & & & & & & \\ \text{x} & \leftarrow & + & 0 & - & 0 & + & 0 & - \\ & & -5 & & 4 & & & 5 \end{array} \end{array}$$

Zeros: $(-5.011, 0), (4.125, 0), (4.886, 0)$

$$\ln(-100 + 25x + 4x^2 - x^3) = 0 \Rightarrow -100 + 25x + 4x^2 - x^3 = 1$$

$$\Rightarrow x^3 - 4x^2 - 25x + 101 = 0$$

VA:

$$\Rightarrow x = -5.011, 4.125, 4.886$$

$$x = \pm 5, -4$$

27. Domain: $x \in (-5, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

$$\begin{array}{c} x^3 + 5x^2 - 3x - 15 = 0 \Rightarrow x = -5, \pm \sqrt{3} \\ \begin{array}{ccccccc} x^3 + 5x^2 - 3x - 15 & & & & & & \\ \text{x} & \leftarrow & - & 0 & + & 0 & - & 0 & + \\ & & -5 & & -\sqrt{3} & & & \sqrt{3} \end{array} \end{array}$$

Zeros: $(-4.954, 0), (-1.821, 0), (1.774, 0)$

$$\ln(x^3 + 5x^2 - 3x - 15) = 0 \Rightarrow x^3 + 5x^2 - 3x - 15 = 1$$

$$\Rightarrow x^3 + 5x^2 - 3x - 16 = 0$$

$$\Rightarrow x = -4.954, -1.821, 1.774$$

VA: $x = \pm \sqrt{3}, x = -5$

10-3 Multiple Choice Homework

1. E $g(x)$ has a vertical asymptote where $e^x - 1 = 0$

$$e^x - 1 = 0 \rightarrow e^x = 1 \rightarrow x = \ln 1 = 0$$

3. D

The domain of $y = \ln(x^3 - 9x)$ is where $x^3 - 9x > 0$.

$$(x^2 - 9)x = x(x-3)(x+3) > 0$$

$$\begin{array}{c} x^3 - 9x \\ x \end{array} \leftarrow \begin{array}{cccccc} - & 0 & + & 0 & - & 0 & + \\ & -3 & & 0 & & 3 & \end{array} \rightarrow$$

$$x \in (-3, 0) \cup (3, \infty)$$

5. B

I. The domain of $y = e^{x^2-1}$ is All Reals, so FALSE.

II. The domain of $y = \ln(x^2 - 1)$ is where $x^2 - 1 > 0$.

$$\begin{array}{c} x^2 - 1 \\ x \end{array} \leftarrow \begin{array}{cccc} + & 0 & - & 0 & + \\ & -1 & & 1 & \end{array} \rightarrow$$

$$x \in (-\infty, -1) \cup (1, \infty), \text{ so TRUE.}$$

III. The domain of $y = \ln(1 - x^2)$ is where $1 - x^2 > 0$.

$$\begin{array}{c} 1 - x^2 \\ x \end{array} \leftarrow \begin{array}{cccc} - & 0 & + & 0 & - \\ & -1 & & 1 & \end{array} \rightarrow$$

$$x \in (-1, 1), \text{ so FALSE}$$

b) II only

10-4 Free Response Homework

1.

$$\begin{aligned}\frac{d}{dx} \left[e^{\sqrt{x}} \right] &= e^{\sqrt{x}} \cdot \frac{1}{2} x^{-\frac{1}{2}} \\ &= \frac{e^{\sqrt{x}}}{2\sqrt{x}}\end{aligned}$$

3.

$$\begin{aligned}\frac{d}{dx} \left[\ln(e^x - x^e) \right] &= \frac{1}{e^x - x^e} \cdot (e^x - ex^{e-1}) \\ &= \frac{e^x - ex^{e-1}}{e^x - x^e}\end{aligned}$$

5.

$$\begin{aligned}\frac{d}{dx} \left[\ln(x^2 - 4x + 5) \right] &= \frac{1}{x^2 - 4x + 5} \cdot (2x - 4) \\ &= \frac{2x - 4}{x^2 - 4x + 5}\end{aligned}$$

7.

$$y = \frac{e^x + e^{-x}}{2} = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$$

$$y' = \frac{1}{2}e^x - \frac{1}{2}e^{-x} = 0$$

$$\frac{1}{2}e^x = \frac{1}{2}e^{-x}$$

$$e^x = e^{-x}$$

$$\ln e^x = \ln e^{-x}$$

$$x = -x$$

$$2x = 0$$

$$\begin{aligned}x = 0 \Rightarrow y &= \frac{1}{2}e^0 + \frac{1}{2}e^{-0} = \frac{1}{2} + \frac{1}{2} = 1 \\ &\Rightarrow (0, 1)\end{aligned}$$

9.

$$y = e^{-x^2}$$

$$y' = e^{-x^2} \cdot -2x = 0$$

$$e^{-x^2} = 0 \text{ or } x = 0$$

$$\ln e^{-x^2} = \ln 0 \text{ or } x = 0$$

$$-x^2 = \text{DNE} \text{ or } x = 0$$

$$\ln e^x = \ln e^{-x}$$

$$x = 0 \Rightarrow y = e^{-0^2} = 1$$

$$\Rightarrow (0, 1)$$

11. $y = e^{\sqrt{4-x^2}}$

$$y' = e^{\sqrt{4-x^2}} \cdot \frac{1}{2} (4-x^2)^{-\frac{1}{2}} \cdot -2x$$

$$= \frac{-xe^{\sqrt{4-x^2}}}{(4-x^2)^{\frac{1}{2}}} = 0, \text{DNE, EoAASD}$$

$$\Rightarrow -x = 0 \text{ or } e^{\sqrt{4-x^2}} = 0 \text{ or } (4-x^2)^{\frac{1}{2}} = 0$$

c.v.s: $x = 0, \pm 2$

e.v.s: $y = 7.389, 1$

13. $y = \ln(-100 + 25x + 4x^2 - x^3)$

Extreme Point: $(4.513, 0.866)$

$$y' = \frac{25+8x-3x^2}{-100+25x+4x^2-x^3} = 0, \text{DNE, EoAASD}$$

c.v.s: $x = \cancel{4}, \cancel{-5}, \cancel{0.206}, 4.513$

e.v.s: $y = 0.866$

15.

$$y = \ln x$$

$$\text{zero} \Rightarrow \ln x = 0 \Rightarrow x = 1 \Rightarrow (1, 0)$$

$$y' = \frac{1}{x}$$

$$y'|_{x=1} = \frac{1}{1} = 1$$

$$y - 0 = 1(x - 1) = x - 1$$

$$y(1.1) = \ln(1.1) \approx 1.1 - 1 = 0.1$$

$$y(0.9) = \ln(0.9) \approx 0.9 - 1 = -0.1$$

17.

$$y = x + \ln x \quad \text{parallel to } 3x - y = 7$$

$$y' = 1 + \frac{1}{x} \qquad \qquad y = 3x - 7$$

$$3 = 1 + \frac{1}{x}$$

$$2 = \frac{1}{x} \Rightarrow x = \frac{1}{2} \Rightarrow y = \frac{1}{2} + \ln \frac{1}{2}$$

$$y - \frac{1}{2} - \ln \frac{1}{2} = 3\left(x - \frac{1}{2}\right)$$

19a)

$$(x(t), y(t)) = (e^{-2t}, t - e^{2t})$$

$$(x'(t), y'(t)) = (-2e^{-2t}, 1 - 2e^{2t})$$

$$(x'(2), y'(2)) = (-2e^{-2 \cdot 2}, 1 - 2e^{2 \cdot 2})$$

$$= (-2e^{-4}, 1 - 2e^4)$$

$$= (-0.037, -108.196)$$

19b)

$$\begin{aligned}(x''(t), y''(t)) &= (4e^{-2t}, -4e^{2t}) \\(x''(2), y''(2)) &= (4e^{-2 \cdot 2}, -4e^{2 \cdot 2}) \\&= (4e^{-4}, -4e^4) \\&= (0.073, -218.393)\end{aligned}$$

19c) $s(2) = \sqrt{[x'(2)]^2 + [y'(2)]^2}$

$$s(2) = \sqrt{[-0.037]^2 + [-108.196]^2} = 108.197$$

21. Moving right would mean $x'(t) > 0$ and moving up would mean $y'(t) > 0$

$$\begin{array}{ccc}x'(t) & \xleftarrow[t]{\begin{matrix} dne \\ 0 \end{matrix}} & \xrightarrow[e]{-\quad+\quad} \\ & & \end{array} \qquad \begin{array}{ccc}y'(t) & \xleftarrow[t]{\begin{matrix} -\quad0\quad+ \\ 0 \end{matrix}} & \xrightarrow{\quad} \\ & & \end{array}$$

Therefore, $t \in (0, e)$

10-4 Multiple Choice Homework

1. C

$$\begin{aligned}m_{\text{avg}} &= \frac{f(4) - f(1)}{4 - 1} = \frac{4 + \ln 4 - (1 + \ln 1)}{3} \\&= 1.462\end{aligned}$$

$$m_{\tan} = f'(x) = 1 + \frac{1}{x}$$

$$1 + \frac{1}{x} = 1.462 \Rightarrow x = 2.164$$

3. A

$$\begin{aligned}
 f'(x) &= 2 \cdot \ln x \cdot \frac{1}{x} = \frac{2 \ln x}{x} \\
 f''(x) &= \frac{x \cdot 2 \cdot \frac{1}{x} - 2 \ln x \cdot 1}{x^2} \\
 f''(\sqrt{e}) &= \frac{\sqrt{e} \cdot 2 \cdot \frac{1}{\sqrt{e}} - 2 \ln \sqrt{e} \cdot 1}{(\sqrt{e})^2} \\
 &= \frac{2 - 2 \ln e^{\frac{1}{2}}}{e} = \frac{2 - 2 \cdot \frac{1}{2} \cdot \ln e}{e} \\
 &= \frac{2 - \ln e}{e} = \frac{2 - 1}{e} = \frac{1}{e}
 \end{aligned}$$

5. A

$$g(x) = \sqrt[3]{x-1}$$

$$x = \sqrt[3]{g^{-1}(x)-1}$$

$$x = \sqrt[3]{f(x)-1}$$

$$x^3 = f(x)-1$$

$$x^3 + 1 = f(x)$$

$$[f(x)]' = 3x^2$$

7. C

$$f(x) = \ln(x^2 - e^{2x}) \rightarrow f'(x) = \frac{1}{x^2 - e^{2x}} (2x - 2e^{2x})$$

$$f'(1) = \frac{2 - 2e^1}{1 - e^1} = \frac{2(1 - e^1)}{1 - e^1} = 2$$

10-5 Free Response Homework

$$1. \quad \frac{d}{dx} [xe^{-x}] = x[e^{-x}(-1)] + e^{-x}(1) = -xe^{-x} + e^{-x} = e^{-x}(1-x)$$

$$3. \quad \begin{aligned} \frac{d}{dx} [(x^2 - 2x - 8)e^x] &= (x^2 - 2x - 8)e^x + e^x(2x - 2) \\ &= e^x(x^2 - 2x - 8 + 2x - 2) = e^x(x^2 - 10) \end{aligned}$$

$$5. \quad \begin{aligned} \frac{d}{dx} [(x^2 - 1)e^{-\frac{1}{2}x}] &= (x^2 - 1) \left[e^{-\frac{1}{2}x} \left(-\frac{1}{2} \right) \right] + e^{-\frac{1}{2}x}(2x) \\ &= -\frac{1}{2}e^{-\frac{1}{2}x}(x^2 - 1 + 4x) = -\frac{1}{2}e^{-\frac{1}{2}x}(x^2 - 4x + 1) \end{aligned}$$

$$7. \quad \frac{d}{dx} [x^2 e^{-4x}] = x^2 e^{-4x}(-4) + e^{-4x}(2x) = -2x e^{-4x}[x^2(2) - 1] = -2x e^{-4x}(2x - 1)$$

$$9. \quad \begin{aligned} \frac{dy}{dx} &= 4(1-x^2) \left[e^{-0.5x^2}(-x) \right] + e^{-0.5x^2}(-8x) = -x e^{-0.5x^2}[1-x^2+2] \\ &= -4x e^{-0.5x^2}[x^2 - 3] \end{aligned}$$

$$11. \quad \begin{aligned} \frac{d}{dx} [e^x \sqrt{7-x}] &= \frac{d}{dx} [e^x (7-x)^{\frac{1}{2}}] = e^x \left[\frac{1}{2}(7-x)^{-\frac{1}{2}}(-1) \right] + (7-x)^{\frac{1}{2}} e^x \\ &= e^x \left[\frac{-1}{2(7-x)^{\frac{1}{2}}} + (7-x)^{\frac{1}{2}} \right] = e^x \left[\frac{-1}{2(7-x)^{\frac{1}{2}}} + \frac{2(7-x)}{2(7-x)^{\frac{1}{2}}} \right] = e^x \left[\frac{-1+14-2x}{2(7-x)^{\frac{1}{2}}} \right] = e^x \left(\frac{13-2x}{2(7-x)^{\frac{1}{2}}} \right) \end{aligned}$$

$$13. \quad \begin{aligned} \frac{d}{dx} [x \sqrt{4-x^2}] &= \frac{d}{dx} \left[(x)(4-x^2)^{\frac{1}{2}} \right] = (x) \left[\frac{1}{2}(4-x^2)^{-\frac{1}{2}}(-2x) \right] + (4-x^2)^{\frac{1}{2}}(1) \\ &= (x) \left[\frac{-x}{(4-x^2)^{\frac{1}{2}}} \right] + (4-x^2)^{\frac{1}{2}}(1) = \frac{-x^2}{(4-x^2)^{\frac{1}{2}}} + \frac{(4-x^2)}{(4-x^2)^{\frac{1}{2}}} = \frac{4-2x^2}{(4-x^2)^{\frac{1}{2}}} \end{aligned}$$

$$\begin{aligned}
15. \frac{d}{dx} \left[(x^2) \sqrt{9-x^2} \right] &= \frac{d}{dx} \left[(x^2)(9-x^2)^{\frac{1}{2}} \right] = (x^2) \left[\frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x) \right] + (9-x^2)^{\frac{1}{2}}(2x) \\
&= (x^2) \left[\frac{-2x}{(9-x^2)^{\frac{1}{2}}} \right] + (9-x^2)^{\frac{1}{2}}(2x) = (x^2) \left[\frac{-2x}{(9-x^2)^{\frac{1}{2}}} + \frac{(9-x^2)(2x)}{(9-x^2)^{\frac{1}{2}}} \right] = \\
&\frac{-x^3}{(9-x^2)^{\frac{1}{2}}} + \frac{(9-x^2)(2x)}{(9-x^2)^{\frac{1}{2}}} = \frac{-x^3 + 18x - 2x^3}{(9-x^2)^{\frac{1}{2}}} = \frac{18x - 3x^3}{(9-x^2)^{\frac{1}{2}}}
\end{aligned}$$

$$\begin{aligned}
17. \quad \frac{d}{dx} \left[(3-x) \sqrt{9x-x^2} \right] &= \frac{d}{dx} \left[(3-x)(9x-x^2)^{\frac{1}{2}} \right] \\
&= (3-x) \left[\frac{1}{2}(9x-x^2)^{-\frac{1}{2}}(9-2x) \right] + (9x-x^2)^{\frac{1}{2}}(-1) = \frac{(3-x)(9-2x)}{2(9x-x^2)^{-\frac{1}{2}}} - (9x-x^2)^{\frac{1}{2}} \\
&= \frac{(3-x)(9-2x)}{2(9x-x^2)^{-\frac{1}{2}}} - (9x-x^2)^{\frac{1}{2}} = \frac{2x^2 - 15x + 27}{2(9x-x^2)^{\frac{1}{2}}} + \frac{-18x + 2x^2}{2(9x-x^2)^{\frac{1}{2}}} = \frac{4x^2 - 33x + 27}{2(9x-x^2)^{\frac{1}{2}}}
\end{aligned}$$

$$\begin{aligned}
19. \quad \frac{d}{dx} \left[(x^2+1) \sqrt{4-x^2} \right] \\
&\frac{d}{dx} \left[(x^2+1) \sqrt{25-x^2} \right] \\
&= (x^2+1) \cdot \frac{1}{2}(25-x^2)^{-\frac{1}{2}} \cdot -2x + (25-x^2)^{\frac{1}{2}} \cdot 2x \\
&= \frac{-x(x^2+1)}{\sqrt{25-x^2}} + 2x\sqrt{25-x^2} \\
&= \frac{-x(x^2+1)}{\sqrt{25-x^2}} + 2x\sqrt{25-x^2} \cdot \frac{\sqrt{25-x^2}}{\sqrt{25-x^2}} \\
&= \frac{-x(x^2+1) + 2x(25-x^2)}{\sqrt{25-x^2}} \\
&= \frac{-x^3 - x + 50x - 2x^3}{\sqrt{25-x^2}} = \frac{-3x^3 + 49x}{\sqrt{25-x^2}}
\end{aligned}$$

$$21. \quad \frac{d}{dx} \left[(x^2 + x + 13) \sqrt{x^2 + x + 13} \right]$$

$$\frac{d}{dx} \left[(x^2 + x + 13) \sqrt{x^2 + x + 13} \right]$$

$$= \frac{d}{dx} \left[(x^2 + x + 13)^{\frac{3}{2}} \right]$$

$$= \frac{3}{2} (x^2 + x + 13)^{\frac{1}{2}} \cdot (2x + 1)$$

$$= \frac{3}{2} (2x + 1) (x^2 + x + 13)^{\frac{1}{2}}$$

$$23. \quad \frac{d}{dx} \left[(2x^2 + 4)^5 (5x^3 - 1)^4 \right]$$

$$= (2x^2 + 4)^5 4(5x^3 - 1)^3 (15x^2) + (5x^3 - 1)^4 5(2x^2 + 4)^4 (4x)$$

$$= 20x(2x^2 + 4)^3 (5x^3 - 1)^3 [3x(2x^2 + 4) + (5x^3 - 1)]$$

$$= 20x(2x^2 + 4)^3 (5x^3 - 1)^3 [6x^3 + 12x + 5x^3 - 1]$$

$$= 20x(2x^2 + 4)^3 (5x^3 - 1)^3 [11x^3 + 12x - 1]$$

$$25. \quad \frac{d}{dx} \left[(3x^2 + 2)^{\frac{1}{2}} (6x - 1)^{\frac{2}{3}} \right]$$

$$= (3x^2 + 2)^{\frac{1}{2}} \left[\frac{2}{3} (6x - 1)^{-\frac{1}{3}} (6) \right] + (6x - 1)^{\frac{2}{3}} \left[\frac{1}{2} (3x^2 + 4)^{-\frac{1}{2}} (6x) \right]$$

$$= \frac{4(3x^2 + 2)^{\frac{1}{2}}}{(6x - 1)^{\frac{1}{3}}} + \frac{3x(6x - 1)^{\frac{2}{3}}}{(3x^2 + 4)^{\frac{1}{2}}}$$

$$= \frac{4(3x^2 + 2) + 3x(6x - 1)}{(6x - 1)^{\frac{1}{3}} (3x^2 + 4)^{\frac{1}{2}}} = \frac{(12x^2 + 8) + (18x^2 - 3x)}{(6x - 1)^{\frac{1}{3}} (3x^2 + 4)^{\frac{1}{2}}}$$

$$= \frac{(30x^2 - 3x + 8)}{(6x-1)^{1/3} (3x^2 + 4)^{1/2}}$$

27. $y = \pm \sqrt{9 - \frac{9}{4}x^2}$

$$A = (2x)(2y) = 4xy = (4x)\sqrt{9 - \frac{9}{4}x^2}$$

$$\frac{dA}{dx} = (4x) \left[\frac{1}{2} \frac{1}{\left(9 - \frac{9}{4}x^2\right)^{1/2}} \left(-\frac{9}{2}x\right) \right] + \left(9 - \frac{9}{4}x^2\right)^{1/2}(4) = \frac{-9x^2}{\left(9 - \frac{9}{4}x^2\right)^{1/2}} + \left(9 - \frac{9}{4}x^2\right)^{1/2}(4) = 0$$

$$4\left(9 - \frac{9}{4}x^2\right)^{1/2} = \frac{9x^2}{\left(9 - \frac{9}{4}x^2\right)^{1/2}} \Rightarrow 4\left(9 - \frac{9}{4}x^2\right) = 9x^2 \Rightarrow 36 - 9x^2 = 9x^2 \Rightarrow 36 = 18x^2$$

$$\Rightarrow x = \sqrt{2} \Rightarrow y = \frac{3}{\sqrt{2}} \Rightarrow A = 4xy = 12$$

29. $\frac{d}{dx}[x^2 + xy - 4y - 1 = 0]$

$$2x + x\frac{dy}{dx} + y(1) - 4\frac{dy}{dx} = 0$$

$$(x-4)\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x-4}$$

31. $\frac{dy}{dx}[x^2y + xy^2 = 6] \Rightarrow x^2\frac{dy}{dx} + y(2x) + x\left(2y\frac{dy}{dx}\right) + y^2(1) = 0$

$$(x^2 - 2xy)\frac{dy}{dx} = -2xy - y^2$$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 - 2xy}$$

33. $\frac{3x^2 - 2xy - 1}{x^2 + 1}$

$$\frac{d}{dx} \left[x^2(x-y)^2 = (x-y)(x+y) \right] \Rightarrow$$

$$\frac{d}{dx} \left[x^2(x-y) \cancel{x} = \cancel{(x-y)}(x+y) \right] \Rightarrow$$

$$\frac{d}{dx} \left[x^3 - x^2y = x + y \right] \Rightarrow$$

$$3x^2 - \left(x^2 \frac{dy}{dx} + y \cdot 2x \right) = 1 + \frac{dy}{dx}$$

$$3x^2 - x^2 \frac{dy}{dx} - 2xy = 1 + \frac{dy}{dx}$$

$$(x^2 + 1) \frac{dy}{dx} = 3x^2 - 2xy - 1$$

$$\frac{dy}{dx} = \frac{3x^2 - 2xy - 1}{x^2 + 1}$$

35. $\frac{d}{dx} [ye^{3x} + xe^{4y} = 17]$

$$\frac{d}{dx} [ye^{3x} + xe^{4y} = 17]$$

$$y(e^{3x}(3)) + e^{3x} \left(\frac{dy}{dx} \right) + xe^{4y} \left(4 \frac{dy}{dx} \right) + e^{4y}(1) = 0$$

$$(e^{3x} + 4xe^{4y}) \frac{dy}{dx} = -3ye^{3x} - xe^{4y}$$

$$\frac{dy}{dx} = \frac{-3ye^{3x} - xe^{4y}}{e^{3x} + 4xe^{4y}}$$

10-5 Multiple Choice Homework

1. D

$$f(x) = (x-1)(x^2+2)^3$$

$$f'(x) = (x-1) \cdot 3(x^2+2) \cdot 2x + (x^2+2)^3 \cdot 1$$

$$= (x^2+2)^2 [6x(x-1) + x^2 + 2]$$

$$= (x^2+2)^2 [6x^2 - 6x + x^2 + 2]$$

$$= (x^2+2)^2 [7x^2 - 6x + 2]$$

3. B

$$f'(x) = h'(x^2 - 3) \cdot 2x$$

$$f'(2) = h'(2^2 - 3) \cdot 2 \cdot 2$$

$$f'(2) = h'(1) \cdot 2 \cdot 2$$

$$f'(2) = 4h'(1)$$

5. A

$$h(x) = g(x) \cdot f(x) \rightarrow h'(x) = g(x)f'(x) + f(x)g'(x)$$

$$h'(2) = g(2)f'(2) + f(2)g'(2)$$

$$= 8(-2) + (4)(1)$$

$$= -12$$

7. E

$$\frac{d}{dx} [5x^3 - 4xy - 2y^2 = 1]$$

$$15x^2 + (-4x)\frac{dy}{dx} + y(-4) - 4y\frac{dy}{dx} = 0$$

$$(-4x - 4y)\frac{dy}{dx} = -15x^2 + 4y$$

$$\frac{dy}{dx} = \frac{-15x^2 + 4y}{-4x - 4y} = \frac{15x^2 - 4y}{4x + 4y}$$

10-6 Free Response Homework

1.

$$\lim_{x \rightarrow 3} \frac{x^2 + 7x + 12}{x^2 - 9} = \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow 3} \frac{2x + 7}{2x} = \frac{1}{-6} = -\frac{1}{6}$$

3.

$$\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{3x - 1} = \frac{0}{5} = 0$$

5.

$$\lim_{x \rightarrow 1} \frac{x^2 - x}{e^x - 1} = \frac{0}{e - 1} = 0$$

7.

$$\lim_{x \rightarrow 4} \frac{\ln(x/4)}{\sqrt{x-2}} = \frac{0}{0}$$

$$\begin{aligned} & \stackrel{LH}{=} \lim_{x \rightarrow 4} \frac{\frac{1}{x/4} \cdot \frac{1}{4}}{\frac{1}{2}x^{-\frac{1}{2}}} = \lim_{x \rightarrow 4} \frac{\frac{4}{x} \cdot \frac{1}{4}}{\frac{1}{2}x^{-\frac{1}{2}}} \\ &= \lim_{x \rightarrow 4} \frac{\frac{1}{x}}{\frac{1}{2}x^{-\frac{1}{2}}} = \frac{\frac{1}{4}}{\frac{1}{2} \cdot 4^{-\frac{1}{2}}} = 1 \end{aligned}$$

9.

$$\begin{aligned} & \lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{x^2 - 81} = \frac{0}{0} \\ & \stackrel{LH}{=} \lim_{x \rightarrow 9} \frac{-\frac{1}{2}x^{-\frac{1}{2}}}{2x} = \frac{-\frac{1}{2} \cdot 9^{-\frac{1}{2}}}{18} = -\frac{1}{108} \end{aligned}$$

11.

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x} = \frac{0}{0} \\ & \stackrel{LH}{=} \lim_{x \rightarrow 1} \frac{2x}{\frac{1}{x}} = \frac{2}{1} = 2 \end{aligned}$$

13.

$$\lim_{x \rightarrow -1} \frac{x+1}{e^{x+1}} = \frac{0}{e^0} = \frac{0}{1} = 0$$

15.

$$\begin{aligned} & \lim_{x \rightarrow -1} \frac{(1-x^2)^2}{x^3 - 2x^2 - 3x} = \frac{0}{0} \\ & \stackrel{LH}{=} \lim_{x \rightarrow -1} \frac{2(1-x^2) \cdot -2x}{3x^2 - 4x - 3} = \frac{0}{4} = 0 \end{aligned}$$

10-6 Multiple Choice Homework

1. A

$$\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 3} \frac{4x^3}{1} = 4(3)^3 = 108$$

3. E

$$\lim_{x \rightarrow 3} \frac{f(x)}{(x-3)^3} \stackrel{L'H}{=} \lim_{x \rightarrow 3} \frac{f'(x)}{3(x-3)^2} \stackrel{L'H}{=} \lim_{x \rightarrow 3} \frac{f''(x)}{6(x-3)} = \frac{7}{0}$$

5. E

$$\lim_{x \rightarrow 1} \frac{x}{\ln x} = \frac{1}{0} = dne$$

7. B

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{\ln\left(\frac{x-1}{2}\right)}{3-x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 3} \frac{\frac{1}{(x-1)/2} \cdot \frac{1}{2}}{-1} \\ & = \frac{\frac{1}{(3-1)/2} \cdot \frac{1}{2}}{-1} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{-1} = -\frac{1}{2} \end{aligned}$$

10-7 Free Response Homework

1. $y = e^{3x^2}$; Domain: $x \in (-\infty, \infty)$

Left end: $\lim_{x \rightarrow -\infty} e^{3x^2} = \infty \therefore \text{up}$

Right end: $\lim_{x \rightarrow \infty} e^{3x^2} = \infty \therefore \text{up}$

Both ends up

3. $y = e^{\sqrt{4-x^2}}$; Domain: $x \in [-2, 2]$

No end behavior on either side because of the domain.

5. $y = xe^{-x}$; Domain: $x \in (-\infty, \infty)$

Left end: $\lim_{x \rightarrow -\infty} xe^{-x} = -\infty \therefore \text{down}$

Right end: $\lim_{x \rightarrow \infty} xe^{-x} = 0$

Left end = down; Right $y = 0$

7. $y = (x^2 - 2x - 8)e^x$; Domain: $x \in (-\infty, \infty)$

Left end: $\lim_{x \rightarrow -\infty} (x^2 - 2x - 8)e^x = 0$

Right end: $\lim_{x \rightarrow \infty} (x^2 - 2x - 8)e^x = \infty \therefore \text{up}$

Left end $y = 0$; Right up

9. $y = (x^2 - 1)e^{-\frac{1}{2}x}$; Domain: $x \in (-\infty, \infty)$

Left end: $\lim_{x \rightarrow -\infty} (x^2 - 1)e^{-\frac{1}{2}x} = \infty \therefore \text{up}$

Right end: $\lim_{x \rightarrow \infty} (x^2 - 1)e^{-\frac{1}{2}x} = 0$

Left end $y = 0$; Right up

11. $y = x^2 e^{-4x}$; Domain: $x \in (-\infty, \infty)$

Left end: $\lim_{x \rightarrow -\infty} x^2 e^{-4x} = \infty \therefore \text{up}$

Right end: $\lim_{x \rightarrow \infty} x^2 e^{-4x} = 0$

Left end $y=0$; Right up

13. $y = 4(1-x^2)e^{-0.5x^2}$

Left end: $\lim_{x \rightarrow -\infty} 4(1-x^2)e^{-0.5x^2} = \lim_{x \rightarrow -\infty} e^{-0.5x^2} = 0$

Right end: $\lim_{x \rightarrow \infty} 4(1-x^2)e^{-0.5x^2} = \lim_{x \rightarrow \infty} e^{-0.5x^2} = 0$

Left end $y=0$; Right end $y=0$

15. $y = e^x \sqrt{7-x}$; Domain: $x \in (-\infty, 7]$

Left end: $\lim_{x \rightarrow -\infty} e^x \sqrt{7-x} = 0 \therefore y=0$

Right end: None because of the domain

Left end $y=0$; Right none

17. $y = \ln(x-x^2)$; Domain: $x \in (0, 1)$

No end behavior on either side because of the domain.

19. $y = \ln(x^3 - 4x)$

$x^3 - 4x = 0 \Rightarrow x = 0, \pm 2$

$$\begin{array}{c} x^3 - 4x \\ x \end{array} \leftarrow \begin{array}{ccccc} - & 0 & + & 0 & - \\ -2 & & 0 & & 2 \end{array} \rightarrow$$

Domain: $x \in (-2, 0) \cup (2, \infty)$

Left end = none; right end = up

21. $y = \ln(-100 + 25x + 4x^2 - x^3)$

$$-x^3 + 4x^2 + 25x - 100 = 0 \Rightarrow x = 4, \pm 5$$

$$\begin{array}{c} -x^3 + 4x^2 + 25x - 100 \\ x \end{array} \leftarrow \begin{array}{ccccc} + & 0 & - & 0 & + \\ -5 & & 4 & & 5 \end{array} \rightarrow$$

Domain: $x \in (-\infty, -5) \cup (4, 5)$

Left end: $\lim_{x \rightarrow -\infty} \ln(-100 + 25x + x^2 - x^3) = +\infty \therefore up$

Right end: None because of the domain

$$23. \quad y = \ln(x^3 + 5x^2 - 3x - 15)$$

$$x^3 + 5x^2 - 3x - 15 = 0 \Rightarrow x = -5, \pm \sqrt{3}$$

$$\begin{array}{c} x^3 + 5x^2 - 3x - 15 \\ x \end{array} \leftarrow \begin{array}{ccccc} - & 0 & + & 0 & - \\ -5 & & -\sqrt{3} & & \sqrt{3} \end{array} \rightarrow$$

Domain: $x \in (-5, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

Left end: None because of the domain

Right end: $\lim_{x \rightarrow \infty} \ln(x^3 + 5x^2 - 3x - 15) = +\infty \therefore up$

$$25. \quad y = \ln\left(\frac{-3x}{x^2 - 9}\right); \text{ Domain: } x \in (-\infty, -3) \cup (0, 3)$$

$$\frac{-3x}{x^2 - 9} = 0, \text{DNE} \Rightarrow -3x = 0 \text{ or } x^2 - 9 = 0$$

$$\Rightarrow x = 0, \pm 3$$

$$\begin{array}{c} \frac{-3x}{x^2 - 9} \\ x \end{array} \leftarrow \begin{array}{ccccc} + & \text{DNE} & - & 0 & + \\ -3 & & 0 & & 3 \end{array} \rightarrow$$

Left end: $\lim_{x \rightarrow -\infty} \ln\left(\frac{-3x}{x^2 - 9}\right) = \ln\left(\lim_{x \rightarrow \infty} \frac{-3x}{x^2 - 9}\right) = \ln(0) = -\infty \therefore down$

Right end: None because of the domain

Left end = down; right end = none

27. $y = (x^2 - 1) \ln x$; Domain: $x \in (0, \infty)$

Left end: None because of the domain

Right end: $\lim_{x \rightarrow \infty} (x^2 - 1) \ln x = (\infty)(\infty) = \infty \therefore \text{up}$

Left end = none; right end up

29. $y = \ln(e^x - x)$; Domain: All Reals

$$e^x - x = 0 \Rightarrow \text{no solution}$$

$$\frac{e^x - x}{x} \xleftarrow{+}$$

Left end: $\lim_{x \rightarrow -\infty} \ln(e^x - x) = \ln(0 - (-\infty)) = \infty \therefore \text{up}$

Right end: $\lim_{x \rightarrow \infty} \ln(e^x - x) = \ln(\lim_{x \rightarrow \infty} (e^x - x)) = \ln(\infty) = \infty \therefore \text{up}$

Both ends up

10-7 Multiple Choice Homework

1. A

$$\lim_{x \rightarrow \infty} \frac{4x^4 + 2x^3 + x^2 + 1}{3x^5 - 9x^4 + 4x^3 + 15} = 0 \text{ because the denominator power is higher.}$$

3. A

$$\lim_{x \rightarrow \infty} \frac{2x^2 - 2}{4x^3 - 3} = \frac{\infty}{\infty}$$

$$\stackrel{LH}{=} \lim_{x \rightarrow \infty} \frac{4x}{12x^2} = \frac{\infty}{\infty}$$

$$\stackrel{LH}{=} \lim_{x \rightarrow \infty} \frac{4}{24x} = \frac{4}{\infty} = 0$$

HA: $y = 0$ on right

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{2x^2 - 2}{4x^3 - 3} &= \frac{\infty}{-\infty} \\ \stackrel{LH}{=} \lim_{x \rightarrow -\infty} \frac{4x}{12x^2} &= \frac{-\infty}{\infty} \\ \stackrel{LH}{=} \lim_{x \rightarrow -\infty} \frac{4}{24x} &= \frac{4}{-\infty} = 0 \end{aligned}$$

HA: $y = 0$ on left

5. A

Domain: $x \in (0, \infty)$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\ln x}{x} &= \frac{\infty}{\infty} \\ \stackrel{LH}{=} \lim_{x \rightarrow \infty} \frac{1/x}{1} &= \lim_{x \rightarrow \infty} \frac{1}{x} = 0 \end{aligned}$$

HA: $y = 0$ on right

7. $y = (x^2 - 4x)e^x$

Left end: $\lim_{x \rightarrow -\infty} (x^2 - 4x)e^x = ((\infty)(0)) = 0$

Right end: $\lim_{x \rightarrow \infty} (x^2 - 4x)e^x = (+\infty)(+\infty) = +\infty \therefore up$

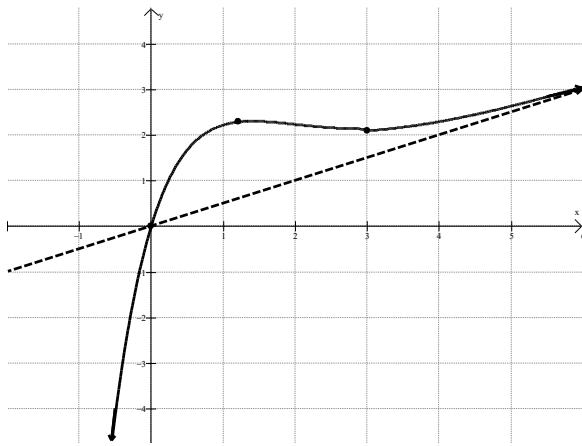
9. $y = (x^2 - 4x)e^{-x}$

Left end: $\lim_{x \rightarrow -\infty} (x^2 - 4x)e^{-x} = ((+\infty)(+\infty)) = +\infty \therefore up$

Right end: $\lim_{x \rightarrow \infty} (x^2 - 4x)e^{-x} = (+\infty)(0) = 0$

10-8 Free Response Homework

1.



3. $y = e^{3x^2}$

Domain: $x \in \text{All Reals}$

Range: $y \in [1, \infty)$

Zeros: None

$$e^{3x^2} = 0 \Rightarrow \ln e^{3x^2} = \ln 0 = \text{DNE}$$

y -int: $(0, 1)$

EB: Both ends up

$$\lim_{x \rightarrow \infty} e^{3x^2} = \infty$$

Right end up \nearrow

$$\lim_{x \rightarrow -\infty} e^{3x^2} = \infty$$

Left end up \nwarrow

VAs: None

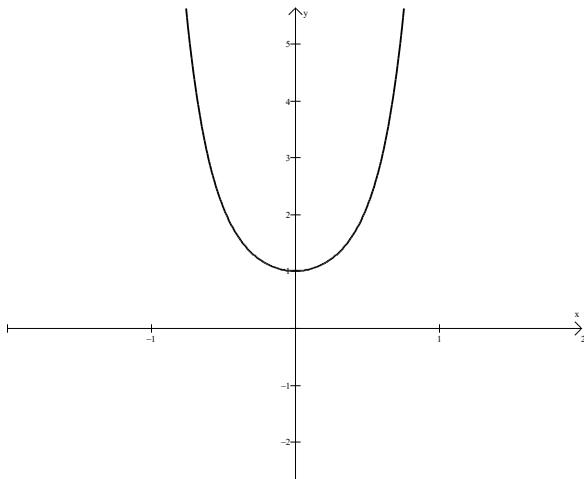
POEs: None

Extreme Point: $(0, 1)$

$$y' = e^{3x^2} \cdot 6x = 0, \text{DNE, EoAASD}$$

c.v.s: $x = 0$

e.v.s: $y = 1$



5. $y = e^{\sqrt{4-x^2}}$

Domain: $x \in [-2, 2]$ [See 10-2 #7]

Zeros: None [See 10-2 #7]

$y\text{-int: } y(0) = e^{\sqrt{4-(0)^2}} = e^2 = 7.389 \rightarrow (0, 7.389)$

VAs & POEs: None because there is no denominator

EB: None because of the domain.

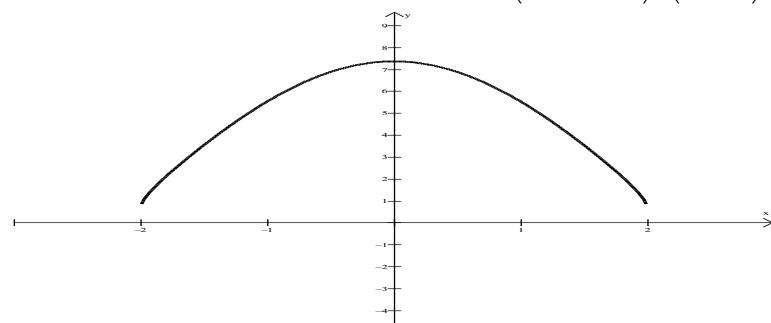
Extreme Point: $\frac{dy}{dx} = e^{\sqrt{4-x^2}} \left(\frac{1}{2}(4-x^2)^{-1/2}(-2x) \right) = e^{\sqrt{4-x^2}} \left(\frac{-x}{(4-x^2)^{1/2}} \right)$

i. $\frac{dy}{dx} = 0 \rightarrow e^{\sqrt{4-x^2}} = 0 \text{ or } -x = 0 \rightarrow x = 0$

ii. $\frac{dy}{dx} = dne \rightarrow 4-x^2 = 0 \rightarrow x = \pm 2$

iii. no restriction given

$$(0, 7.389), (\pm 2, 1)$$



Range: $y \in [1, 7.389]$

7. $y = xe^{-x}$

Domain: $x \in \text{All Reals}$, because there is no denominator, radical or logarithm.

Zeros: $y = xe^{-x} = 0 \rightarrow x = 0 \text{ or } e^{-x} = 0 \rightarrow x = 0 \rightarrow (0, 0)$

y-int: $y(0) = (0)e^{-(0)} = 0 \rightarrow (0, 0)$

VAs & POEs: None because there is no denominator

EB: Left end down; HA $y = 0$ on right [See 10-6 #5]

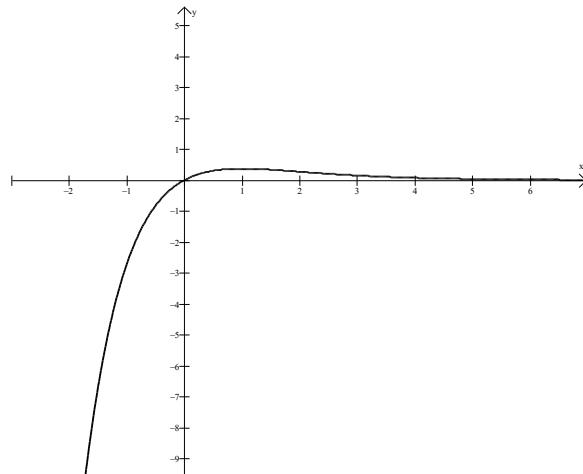
Extreme Point: $\frac{dy}{dx} = e^{-x}(1-x)$ [See 10-4 #1]

i. $\frac{dy}{dx} = 0 \rightarrow e^{-x}(1-x) = 0 \rightarrow e^{-x} = 0 \text{ or } 1-x = 0 \rightarrow x = 1$

ii. $\frac{dy}{dx} = dne \rightarrow \text{no solution}$

iii. no restriction given

$(1, 0.368)$



Range: $y \in (-\infty, 0.368]$

9. $y = (x^2 - 2x - 8)e^x$

Domain: $x \in (-\infty, \infty)$, because there is no denominator, radical or logarithm.

Zeros: $(4, 0), (-2, 0)$

$$(x^2 - 2x - 8)e^x = 0 \Rightarrow x^2 - 2x - 8 = 0 \text{ or } e^x = 0$$

$$\Rightarrow (x-4)(x+2) = 0 \text{ or } \ln e^x = \ln 0 = \text{DNE}$$

$$\Rightarrow x = 4, -2$$

y -int: $(0, -8)$

VAs & POEs: None because there is no denominator

EB: HA $y = 0$ on left; right end up

$$\lim_{x \rightarrow \infty} (x^2 - 2x - 8)e^x = \infty \cdot \infty = \infty$$

Right end up \nearrow

$$\lim_{x \rightarrow -\infty} (x^2 - 2x - 8)e^x = \infty \cdot e^{-\infty} = \infty \cdot 0$$

$$= \underset{\substack{\text{Algebraic} \\ = \\ \infty}}{\infty} \cdot \underset{\substack{\text{Exponential} \\ = \\ 0}}{0} = 0$$

Left end - HA: $y = 0$

VAs: None

POEs: None

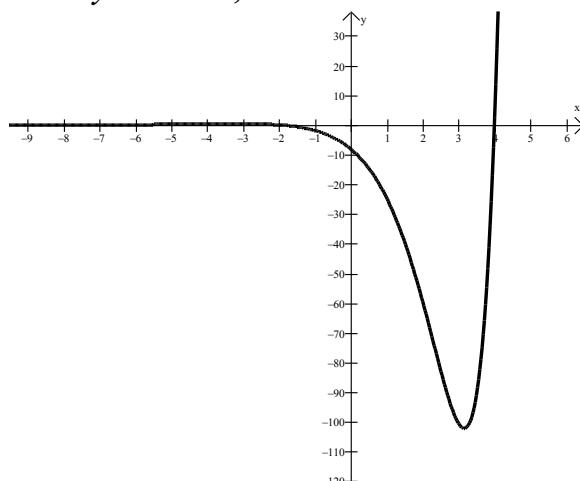
Extreme Points: $(-3.162, 0.352), (3.162, -102.165)$

$$y' = (x^2 - 2x - 8) \cdot e^x + e^x \cdot (2x - 2)$$

$$= e^x [x^2 - 10] = 0, \text{DNE, EoAASD}$$

$$\text{c.v.s: } x = \pm\sqrt{10}$$

$$\text{e.v.s: } y = 0.352, -102.165$$



Range: $y \in [-102.165, \infty)$

11. $y = (x^2 - 1)e^{-\frac{1}{2}x}$

Domain: $x \in (-\infty, \infty)$ because there is no denominator, radical or log.

Zeros: $y = (x^2 - 1)e^{-\frac{1}{2}x} \rightarrow (x^2 - 1) = 0 \text{ or } e^{-\frac{1}{2}x} = 0 \rightarrow x = \pm 1 \text{ or no solution}$
 $(\pm 1, 0)$

y -int: $y = (0^2 - 1)e^{-\frac{1}{2}(0)} = -1 (0, -1)$

VAs & POEs: None because there is no denominator

EB: Left end up; HA $y = 0$ on right {See 10-6 #11]

Extreme Points: $\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}x}(x^2 - 4x + 1)$ [See 10-4 #11]

i. $\frac{dy}{dx} = 0 \rightarrow -\frac{1}{2}e^{-\frac{1}{2}x} = 0 \text{ or } x^2 - 4x + 1 = 0$

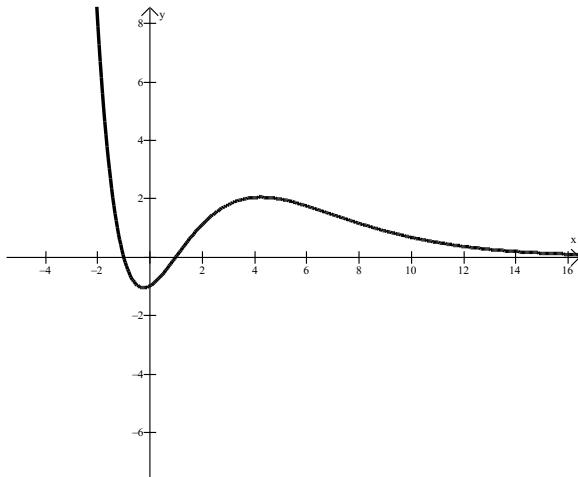
$-\frac{1}{2}e^{-\frac{1}{2}x} = 0 \rightarrow \text{no solution}$

$$x^2 - 4x + 1 = 0 \rightarrow x = \frac{4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} = 4.236 \text{ or } -0.236$$

ii. $\frac{dy}{dx} = \text{dne} \rightarrow \text{no solution}$

iii. no restriction given

$(4.236, 2.038), (-0.236, -1.063)$



Range: $y \in [-1.063, \infty)$

13. $y = x^2 e^{-4x}$

Domain: $x \in (-\infty, \infty)$ because there is not denominator, radical or log.

Zeros: $y = x^2 e^{-4x} \rightarrow x^2 = 0$ or $e^{-4x} = 0 \rightarrow x = 0$ or no solution

$$(0, 0)$$

y -int: $y = (0^2)e^{-4(0)} = 0 \rightarrow (0, 0)$

VAs & POEs: None because there is no denominator

EB: Left end up; HA $y = 0$ on right [See 10-6 #13]

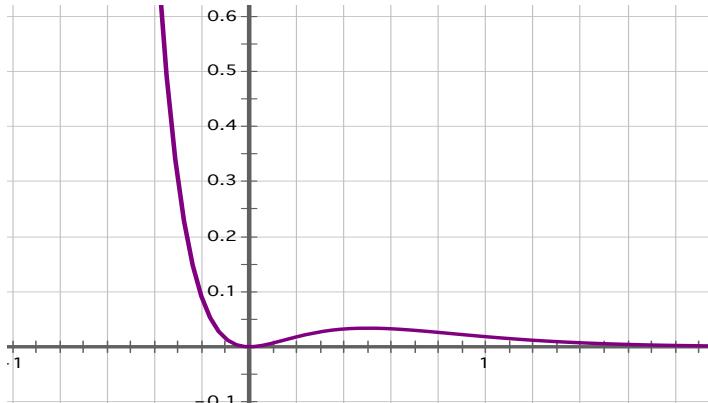
Extreme Points: $\frac{d}{dx}[x^2 e^{-4x}] = -2x e^{-4x}(2x-1)$ [See 10-4 #7]

i. $\frac{dy}{dx} = 0 \rightarrow -2x = 0$ or $e^{-4x} = 0$ or $2x-1 = 0 \rightarrow x = 0$ or $\frac{1}{2}$

ii. $\frac{dy}{dx} = dne \rightarrow \text{no solution}$

iii. no restriction given

$$(0, 0), (0.5, 0.034)$$



Range: $y \in [0, \infty)$

15. $y = 4(1-x^2)e^{-0.5x^2}$

Domain: $x \in (-\infty, \infty)$ because there is not denominator, radical or log.

Zeros: $y = 4(1-x^2)e^{-0.5x^2} \rightarrow 1-x^2=0 \text{ or } e^{-0.5x^2}=0 \rightarrow x=0 \text{ or no solution}$

$$(\pm 1, 0)$$

y -int: $y = 4(1-(0)^2)e^{-0.5(0)^2} = 4 \rightarrow (0, 4)$

EB:

HA $y=0$ on left; HA $y=0$ on the right

VAs: None POEs: None

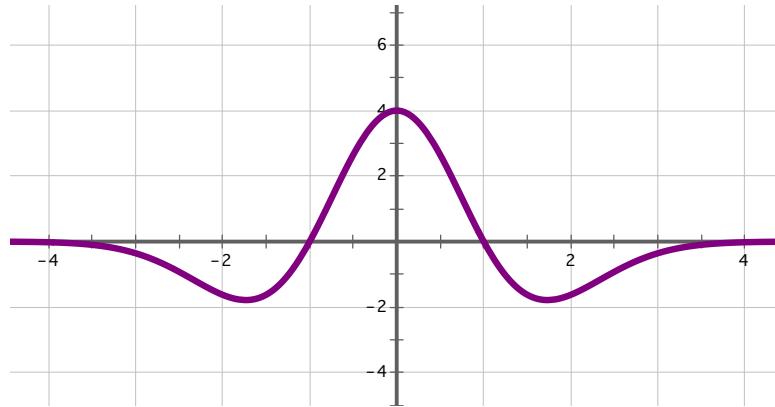
Extreme Points: $\frac{dy}{dx} = -4xe^{-0.5x^2}[x^2 - 3]$ See 10-6 #13

i. $\frac{dy}{dx} = 0 \rightarrow x=0 \text{ or } x^2-3=0 \text{ or } e^{-4x}=0 \rightarrow x=0 \text{ or } \pm\sqrt{3}$

ii. $\frac{dy}{dx} = dne \rightarrow \text{no solution}$

iii. no restriction given

$$(-\sqrt{3}, -1.785), (\sqrt{3}, -1.785), (0, 4)$$



Range: $y \in [-1.785, 4]$ See graph.

17. $y = e^x \sqrt{7-x}$

Domain: $x \in (-\infty, 7]$ [See 10-3 #5]

Zeros: $y = e^x \sqrt{7-x} = 0 \rightarrow e^x = 0 \text{ or } \sqrt{7-x} = 0 \rightarrow x = 7 \therefore (7, 0)$

y -int: $y(0) = e^{(0)} \sqrt{7-(0)} = \sqrt{7} \therefore (0, \sqrt{7})$

EB: HA $y = 0$ on left; right end none [See 10-6 #15]

VAs & POEs: None because there is no denominator

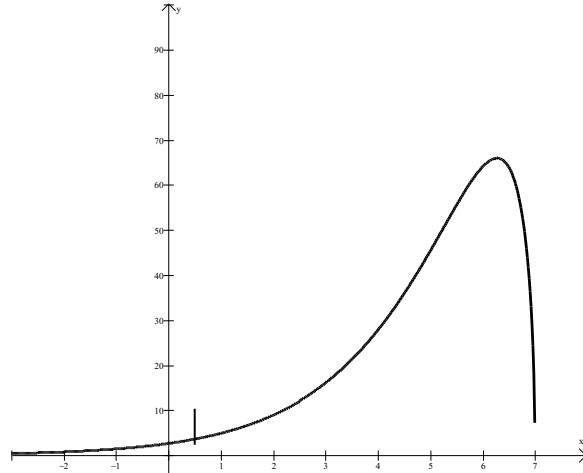
Extreme Points: $\frac{dy}{dx} = e^x \left(\frac{13-2x}{2(7-x)^{1/2}} \right)$ [See 10-4 #9]

i. $\frac{dy}{dx} = 0 \rightarrow 13-2x=0 \text{ or } e^x=0 \rightarrow x=6.5$

ii. $\frac{dy}{dx} = dne \rightarrow 7-x=0 \rightarrow x=7$

iii. no restriction given

$$(6.5, 65.938), (7, 0)$$



Range: $y \in [0, 65.938]$

19. $y = x\sqrt{4-x^2}$

Domain: $4-x^2 \geq 0 \rightarrow x^2 \leq 4 \rightarrow -2 \leq x \leq 2 \rightarrow x \in [-2, 2]$

Zeros: $y = x\sqrt{4-x^2} = 0 \rightarrow x = 0 \text{ or } 4-x^2 = 0 \rightarrow x = 0, \pm 2 \therefore (0, 0), (\pm 2, 0)$

y -int: $y(0) = (0)\sqrt{4-(0)^2} = 0 \therefore (0, 0)$

EB: None because of the domain

VAs & POEs: None because there is no denominator

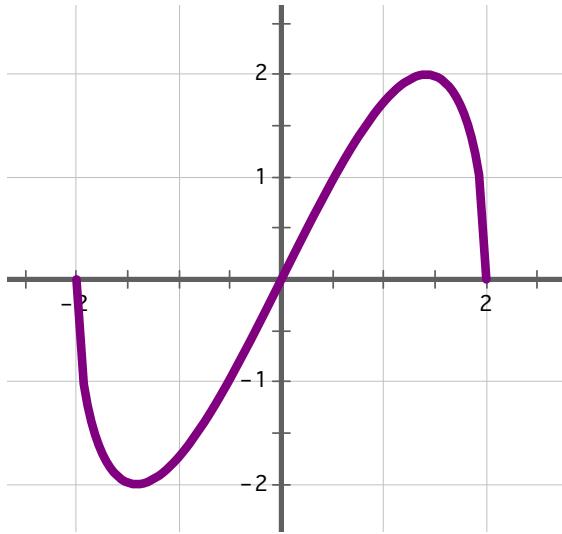
Extreme Points: $\frac{dy}{dx} = \frac{4-2x^2}{(4-x^2)^{-1/2}}$ [see 10-4 #1]

i. $\frac{dy}{dx} = 0 \rightarrow 4-2x^2 = 0 \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$

ii. $\frac{dy}{dx} = dne \rightarrow 4-x^2 = 0 \rightarrow x = \pm 2$

iii. no restriction given

$$(\pm 2, 0), (-\sqrt{2}, -2), (\sqrt{2}, 2)$$



Range: $y \in [-2, 2]$

21. $y = (x^2)\sqrt{25 - x^2}$

Domain: $x \in [-5, 5]$

$$\sqrt{25 - x^2} = 0 \Rightarrow 25 - x^2 = 0 \Rightarrow x = \pm 5$$

$$\frac{25 - x^2}{x} \leftarrow \begin{array}{c} - \\ 0 \\ -5 \\ 5 \end{array} \quad \begin{array}{c} + \\ 0 \\ - \end{array}$$

Zeros: $(\pm 5, 0)$, $(0, 0)$

$$x^2 \sqrt{25 - x^2} = 0 \Rightarrow x^2 = 0 \text{ or } \sqrt{25 - x^2} = 0$$

$$\Rightarrow x = 0 \text{ or } 25 - x^2 = 0$$

$$\Rightarrow x = 0, \pm 5$$

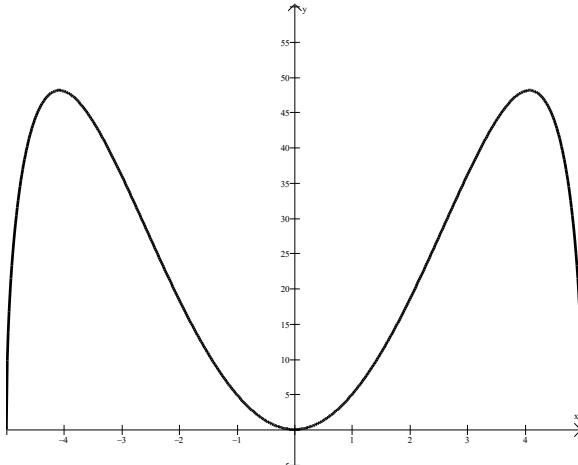
y -int: $(0, 0)$

EB: None because of the domain

VAs & POEs: None because there is no denominator

Extreme Points: $(0, 0)$, $(\pm 5, 0)$, $(\pm 4.082, 48.113)$

$$\begin{aligned}
y' &= x^2 \cdot \frac{1}{2} (25-x^2)^{-\frac{1}{2}} \cdot -2x + (25-x^2)^{\frac{1}{2}} \cdot 2x \\
&= \frac{-x^3}{(25-x^2)^{\frac{1}{2}}} + 2x(25-x^2)^{\frac{1}{2}} \\
&= \frac{-x^3}{(25-x^2)^{\frac{1}{2}}} + 2x(25-x^2)^{\frac{1}{2}} \cdot \frac{(25-x^2)^{\frac{1}{2}}}{(25-x^2)^{\frac{1}{2}}} \\
&= \frac{-x^3 + 2x(25-x^2)}{(25-x^2)^{\frac{1}{2}}} \\
&= \frac{-3x^3 + 50x}{(25-x^2)^{\frac{1}{2}}} = \frac{-x(3x^2 - 50)}{(25-x^2)^{\frac{1}{2}}} = 0, \text{DNE, EoAASD} \\
&\Rightarrow -x(3x^2 - 50) = 0 \text{ or } (25-x^2)^{\frac{1}{2}} = 0 \\
&\text{c.v.s: } x = 0, \pm \sqrt{\frac{50}{3}} \approx \pm 4.082, \pm 5 \\
&\text{e.v.s: } y = 0, 48.113, 0
\end{aligned}$$



Range: $y \in [0, 48.113]$

23. $y = (3-x)\sqrt{9x-x^2}$

Domain: $9x-x^2 \geq 0 \rightarrow x(9-x) \leq 0 \rightarrow 0 \leq x \leq 9 \rightarrow x \in [0, 9]$

Zeros: $y = (3-x)\sqrt{9x-x^2} = 0 \rightarrow 3-x = 0 \text{ or } 9x-x^2 = 0 \rightarrow x = 3, 0, 9$

$\therefore (0, 0), (3, 0), (9, 0)$

$y\text{-int: } y(0) = (3-(0))\sqrt{9(0)-(0)^2} = 0 \therefore (0, 0)$

EB: None because of the domain

VAs & POEs: None because there is no denominator

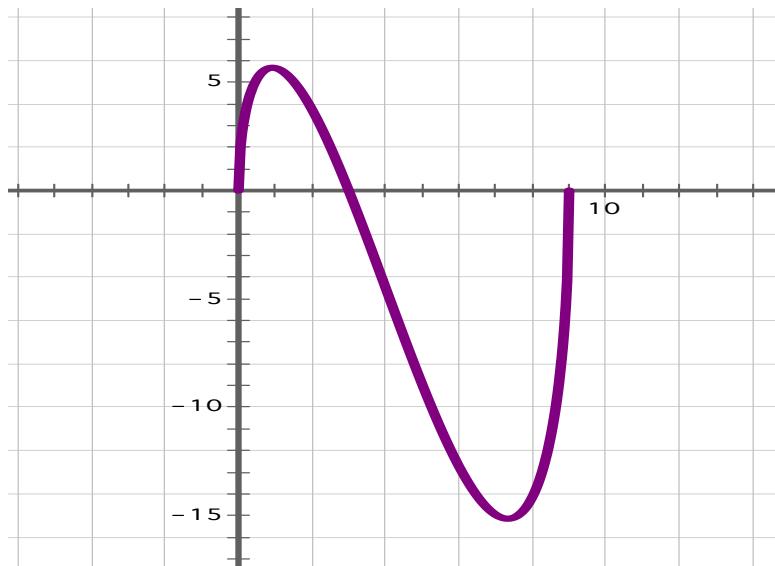
Extreme Points: $\frac{dy}{dx} = \frac{4x^2 - 33x + 27}{2(9x - x^2)^{1/2}}$ [see 10-4 #15]

i. $\frac{dy}{dx} = 0 \rightarrow 4x^2 - 33x + 27 = 0 \rightarrow x = \frac{33 \pm \sqrt{33^2 - 4(4)(27)}}{2(4)} = 7.329, 0.921$

ii. $\frac{dy}{dx} = dne \rightarrow 9x - x^2 = 0 \rightarrow x = 0, 9$

iii. no restriction given

$$(0, 0), (9, 0), (0.921, 5.671), (7.329, -15.135)$$



Range: $y \in [-15.135, 5.671]$

25. $y = (x-2)\sqrt{4x-x^2}$

Domain: $4x - x^2 \geq 0 \rightarrow x(4-x) \geq 0 \rightarrow 0 \leq x \leq 4 \rightarrow x \in [0, 4] x \in [0, 4]$

Zeros: $(2, 0), (0, 0), (4, 0)$

$y\text{-int: } y(0) = ((0)-2)\sqrt{4(0)-(0)^2} = 0 \rightarrow (0, 0)$

EB: None because of the domain

VAs & POE: None because there is no denominator

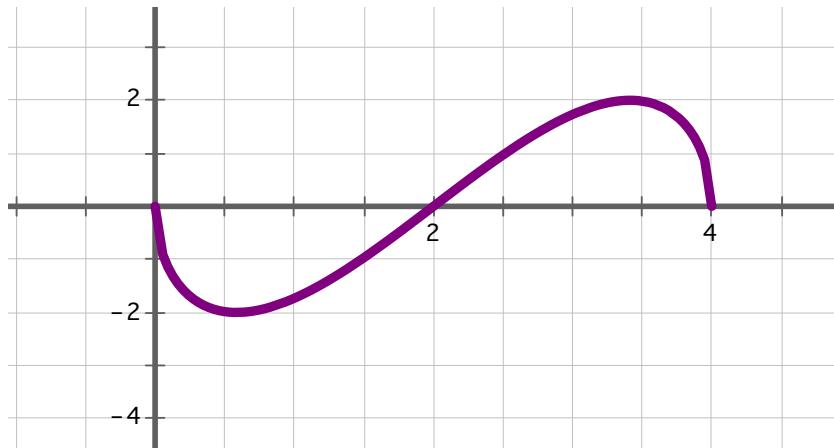
Extreme Points:

$$\begin{aligned} \frac{d}{dx}[(x-2)\sqrt{4x-x^2}] &= \frac{d}{dx}[(x-2)(4x-x^2)^{1/2}] \\ &= (x-2)\left[\frac{1}{2}(4x-x^2)^{-1/2}(4-2x)\right] + (4x-x^2)^{1/2}(1) = \frac{(x-2)(2-x)}{(4x-x^2)^{1/2}} + (4x-x^2)^{1/2} \\ &= \frac{(-x^2+4x-4)+(4x-x^2)}{(4x-x^2)^{1/2}} = \frac{-2x^2+8x-4}{(4x-x^2)^{1/2}} = \frac{-2(x^2-4x+2)}{(4x-x^2)^{1/2}} \\ i. \quad \frac{dy}{dx} = 0 \rightarrow x^2 - 4x + 2 = 0 \rightarrow x = \frac{4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)} &= 3.414, -0.586 \end{aligned}$$

ii. $\frac{dy}{dx} = dne \rightarrow 4x - x^2 = 0 \rightarrow x = 0, 4$

iii. no restriction given

$$(0, 0), (4, 0), (-0.586, -2), (3.414, 2)$$



Range: $y \in [-2, 2]$

27. $y = e^{-x/4} \sqrt{16x^2 - x^4}$

Domain: $16x^2 - x^4 \geq 0 \rightarrow x^2(16 - x^2) \geq 0$

$$\begin{array}{ccccccc} x^2(16 - x^2) & - & 0 & + & 0 & + & 0 \\ x & \xleftarrow[-4]{} & 0 & \xrightarrow[0]{} & 4 & \xrightarrow[4]{} & - \end{array}$$

$x \in [-4, 4]$

Zeros: $(0, 0), (\pm 4, 0)$ y-int: $(0, 0)$

EB: None because of the domain

VAs & POE: None because there is no denominator

$$\begin{aligned}
 \text{Extreme Points: } & \frac{dy}{dx} = e^{-x/4} \left[\frac{1}{2} (16x^2 - x^4)^{-1/2} (32x - 4x^3) \right] + (16x^2 - x^4)^{-1/2} \left[e^{-x/4} \left(-\frac{1}{4} \right) \right] \\
 &= e^{-x/4} \left(\frac{16x - 2x^3}{(16x^2 - x^4)^{1/2}} \right) + (16x^2 - x^4)^{1/2} \left[e^{-x/4} \left(-\frac{1}{4} \right) \right] \\
 &= e^{-x/4} \left[\frac{16x - 2x^3}{(16x^2 - x^4)^{1/2}} - \frac{(16x^2 - x^4)^{1/2}}{4} \right] \\
 &= e^{-x/4} \left[\frac{4(16x - 2x^3) - (16x^2 - x^4)}{4(16x^2 - x^4)^{1/2}} \right] \\
 &= e^{-x/4} \left[\frac{4(16x - 2x^3) - (16x^2 - x^4)}{4(16x^2 - x^4)^{1/2}} \right] \\
 &= e^{-x/4} \left[\frac{64x - 8x^3 - 16x^2 + x^4}{4(16x^2 - x^4)^{1/2}} \right] \\
 &= e^{-x/4} \left[\frac{x(x^3 - 8x^2 - 16x + 64)}{4(16x^2 - x^4)^{1/2}} \right]
 \end{aligned}$$

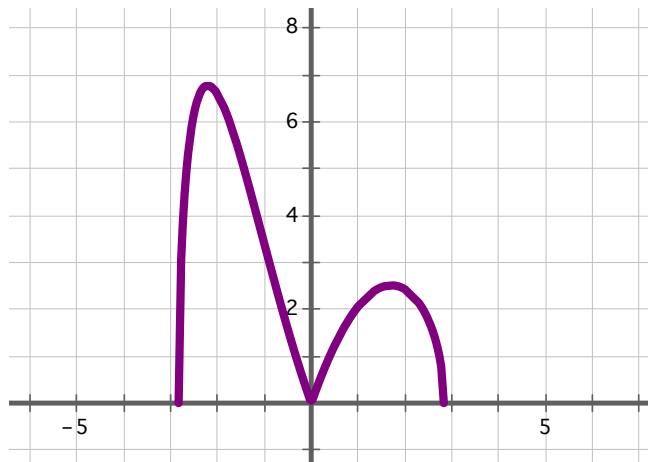
i. $\frac{dy}{dx} = 0 \rightarrow x(x^3 - 8x^2 - 16x + 64) = 0 \rightarrow x = 0 \text{ or } x^3 - 8x^2 - 16x + 64 = 0 \rightarrow$

By grapher, $x = -3.208, 2.220$, or 8.988 . But $x = 8.988$ is not in the domain.

ii. $\frac{dy}{dx} = dne \rightarrow 16x^2 - x^4 = 0 \rightarrow x = 0, \pm 4$

iii. no restriction given

$$(\pm 4, 0), (0, 0), (-3.208, 17.093), (2.220, 4.241)$$



Range: $y \in [0, 17.093]$

10-8 Multiple Choice Homework

1. A

Domain: $x \in (-\infty, \infty)$

$$f'(x) = x^2 \cdot e^{-2x} \cdot -2 + e^{-2x} \cdot 2x = 0, \text{ DNE, EoAASD}$$

c.v.s: $x = 0, 1$

$$\begin{array}{c} f'(x) \\ \hline x \end{array} \leftarrow \begin{matrix} - & 0 & + & 0 & - \end{matrix}$$

Increasing: $x \in (0, 1)$

3. D

$$y(1+x) = 2x$$

$$y = \frac{2x}{1+x}$$

VA: $x = -1$

HA: $y = 2$

5. D

The right end behavior is $y=0$, so the power on e is negative. The answer must be C or D. The left end behavior is “up,” so the coefficient must be positive for negative values of x . The answer is $y = (x^2 - 1)e^{-x/2}$.

7. C

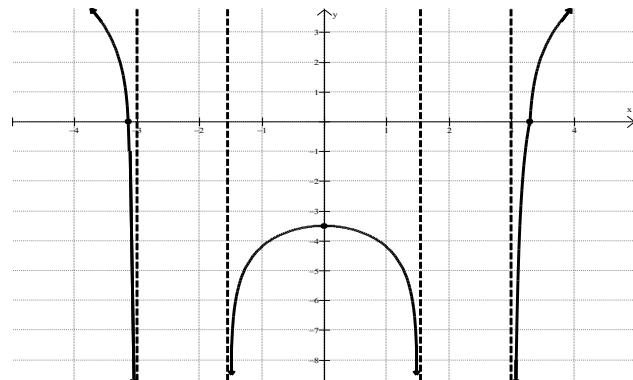
The polynomial end behavior is that the right end goes up and the left end goes down, therefore, only A and C could be correct. A does not have the correct zeros. Therefore, C is correct.

9. A

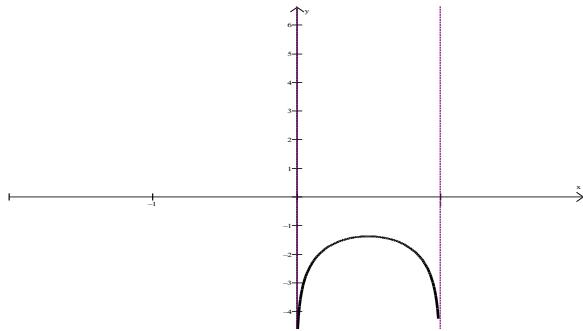
B is a sine wave, C is upside down, and D would have a square on the x outside the radical.

10-9 Free Response Homework

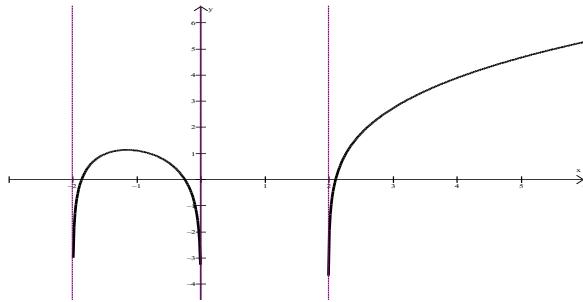
1.



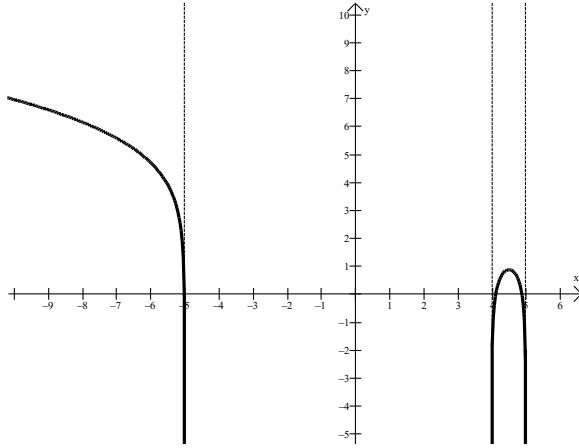
3. Domain: $x \in (0, 1)$ Range: $y \in (-\infty, -1.386]$
 Zeros: None y -int: None EB: None
 VA: $x = 0, x = 1$ POEs: None Extreme Point: $(0.5, -1.386)$



5. Domain: $x \in (-2, 0) \cup (2, \infty)$ Range: $y \in \text{All Reals}$
 Zeros: $(-1.861, 0), (-0.254, 0), (2.115, 0)$ $y\text{-int: None}$
 EB: Left end none; right end up VA: $x = -2, x = 0, x = 2$
 POEs: None Extreme Point: $(-1.155, 1.125)$



7. $y = \ln(-100 + 25x + 4x^2 - x^3)$
- Domain: $x \in (-\infty, -5) \cup (4, 5)$ Range: $y \in (-\infty, \infty)$
 Zeros: $(-5.011, 0), (4.125, 0), (4.886, 0)$ $y\text{-int: None}$
 EB: Left end up; right end none Extreme Point: $(4.513, 0.866)$
 VA: $x = \pm 5, -4$ POEs: None



9. $y = \ln(x^3 + 5x^2 - 3x - 15)$

Domain: $x \in (-5, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

Range: $y \in (-\infty, \infty)$

Zeros: $(-4.954, 0), (-1.821, 0), (1.774, 0)$

y -int: None

EB: Left end none; right end up

VA: $x = \pm\sqrt{3}, x = -5$

POEs: None

Extreme Point: $(-3.610, 2.635)$

19. $y = \ln\left(\frac{-3x}{x^2 - 9}\right)$

Domain: $x \in (-\infty, -3) \cup (0, 3)$

Range: $y \in \text{All Reals}$

Zeros: $(1.854, 0), (-4.854, 0)$

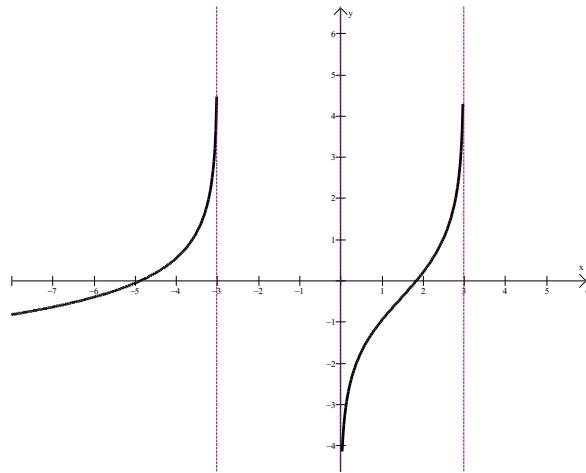
y -int: None

EB: Left end down; right end none

VA: $x = -3, x = 0, x = 3$

POEs: None

Extreme Points: None



21. $y = (x^2 - 1) \ln x$

Domain: $x \in (0, \infty)$

Range: $y \in [0, \infty)$

Zeros: $(1, 0)$

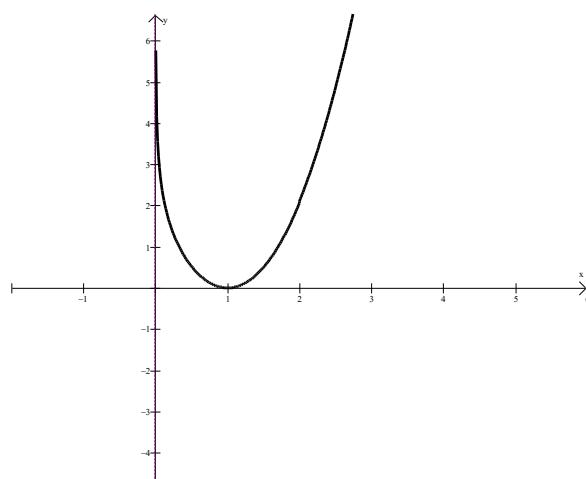
y -int: None

VA: $x = 0$

POEs: None

EB: Left end none; right end up

Extreme Point: $(1, 0)$



23. $y = \ln(e^x - x)$

Domain: $x \in \text{All Reals}$

Range: $y \in [0, \infty)$

Zeros: $(0, 0)$

y -int: $(0, 0)$

EB: Both ends up

VA: None

POEs: None

Extreme Point: $(0, 0)$

