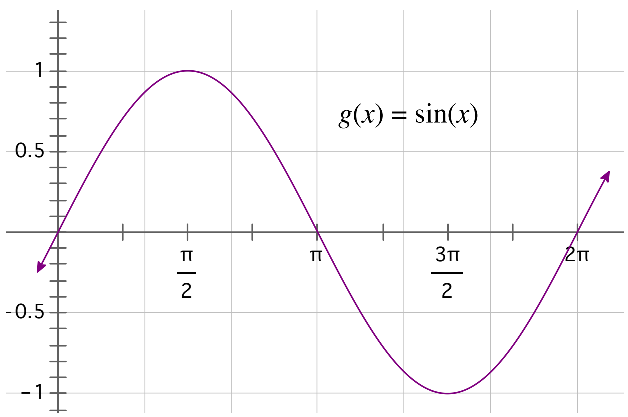
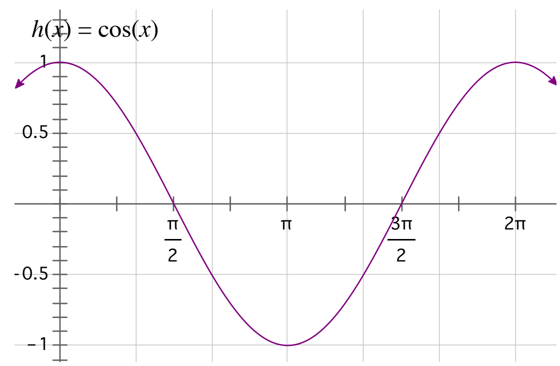
Chapter 2:

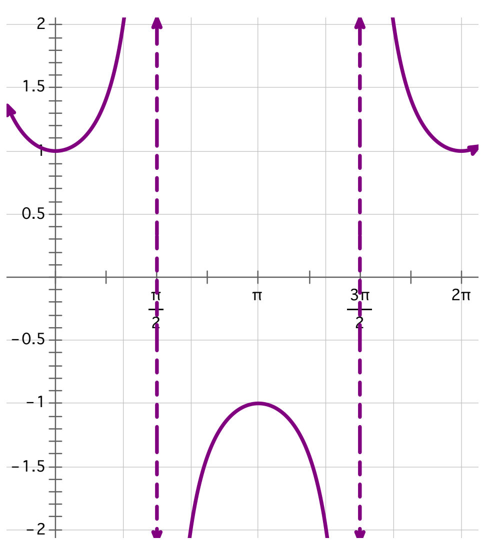
Parent Trigonometric Functions

Chapter 2 Overview: Traits of Parent Trigonometric Functions

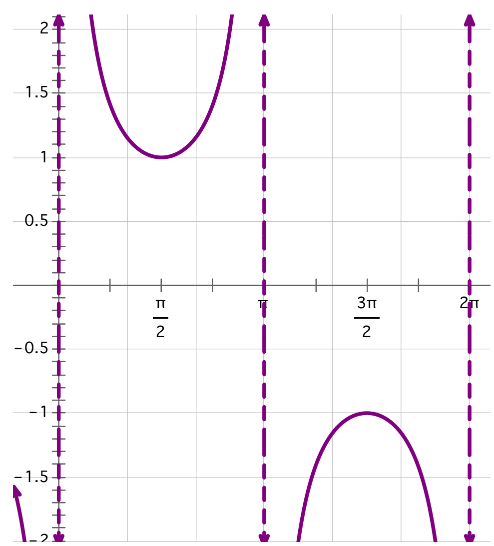
In many Precalculus courses (and in Physics), a great deal of time is spent looking at the trigonometric graphs as transformations of a *parent function.* The six trigonometric parent functions look like this:



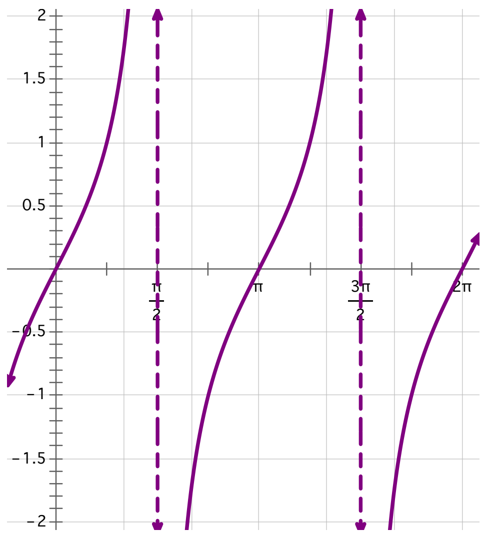




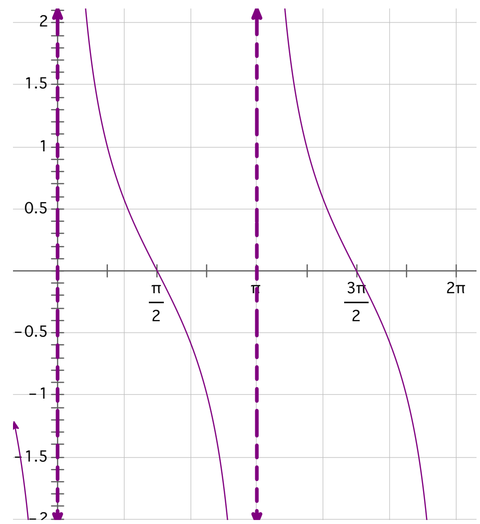
$y = \sec x$



$y = \csc x$



$y = \tan x$



$y = \cot x$

The transformations are shifts (translations) or stretching/shrinking (dilations), which are caused by introducing numbers into the parent function by multiplication or addition, either inside or outside the functions. These translation and dilation numbers are:

- Amplitude
- Period
- Vertical Shift
- Horizontal Shift

(Note: Since this book consistently uses the word *traits* for the collective characteristics which overarch the families of functions, the same word will not be used to refer to these translations and dilations. They will just be referred to as *characteristics*.)

The sine and cosine waves, in particular, can be used to model many real world problems, especially those that are periodic in nature (i.e., those where the y-values repeat with constancy). The analysis of trigonometric functions in order to use them to predict behavior will be explored.

2-1: Graphing Trigonometric Functions by Calculator

Now that geometric trigonometry has been fully reviewed and used to build a base of analytic trigonometry, it is time to leave triangles behind and explore the nature of the six trigonometric functions as *functions* (like polynomials, etc.) rather than as *operations*. A common vocabulary must be established. Much of it comes directly from Algebra 1 and 2.

Vocabulary:

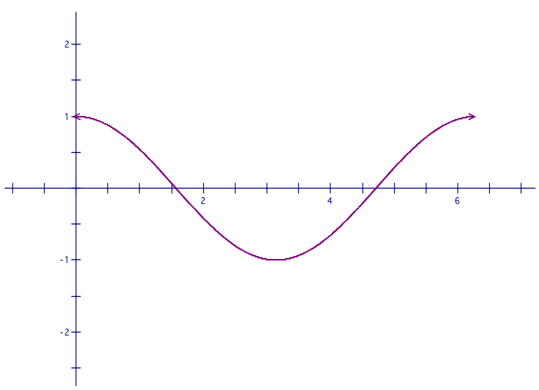
1. ***Domain*** – Defn: the set of values of the independent variable
   Means: the set of x-values that can be substituted into the equation to get a real y value (i.e., no zero denominator, no negative under an even radical, and no non-positive number in a logarithm)
2. ***Range*** – Defn: the set of values of the dependent variable
   Means: the set of y-values that can come from the equation
3. ***Relation*** – Defn: a set of ordered pairs
   Means: the equation that creates/defines the pairs
4. ***Function*** – Defn: a relation for which there is exactly one value of the dependent variable for each value of the independent variable
   Means: an equation where every x gets only one y
5. ***Critical Value*** – the x-coordinate of the maximum or minimum points
6. ***Extreme Value*** – the y-coordinate of the maximum or minimum points

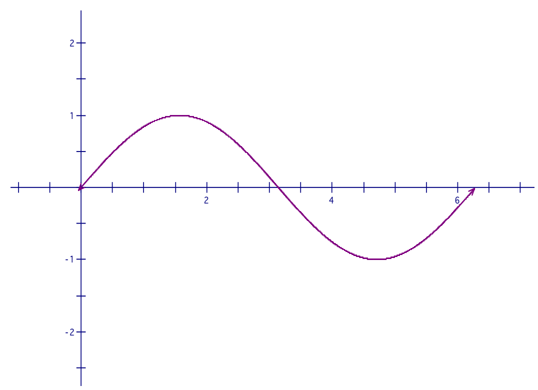
One of the main differences between trigonometric functions and the algebraic functions (polynomials, rationals, etc.) is that trigonometric functions are periodic. This means the curve repeats itself just as trigonometric values repeat for coterminal angles.

Vocabulary:

7. ***Parent Function*** – a non-transformed version indicative of a family of functions; the most basic version
8. ***Trigonometric Function*** – a relation in degrees between x and y based on six trigonometric operations
9. ***Sinusoidal Function*** – functions involving sine or cosine
10. ***Cycle*** – one section of curve without repeating values
11. ***Primary Cycle*** – the cycle beginning with the horizontal (a.k.a. phase) shift
12. ***Period*** – the width of one cycle
13. ***Sinusoidal Axis*** – imaginary horizontal centerline of the curve
14. ***Axis Points*** – the points where the curve crosses the sinusoidal axis

The graphs of $y = \cos x$ and $y = \sin x$ look like this, respectively:





LEARNING OUTCOMES

Explore the relationship between the equation and the graph of a sinusoidal.
Use a graphing calculator to find the graph of a trigonometric equation.

In-class Assignment:

Graph each equation on your calculator and sketch it, indicating the window used. Be sure the mode is RADIANS.

1. $y = \cos x$

2. $y = 3\cos x$

3. $y = 2 + \cos x$

4. $y = 2 - \cos x$

5. $y = \cos(2x)$

6. $y = \cos\left(\frac{1}{2}x\right)$

7. $y = \cos(x - 2)$

8. $y = \cos(x + 2)$

9. $y = 1 + 4\cos\left(\frac{\pi}{3}(x - 2)\right)$

10. $y = \sin x$

11. $y = 3\sin x$

12. $y = 2 + \sin x$

13. $y = 2 - \sin x$

14. $y = \sin(2x)$

15. $y = \sin\left(\frac{1}{2}x\right)$

16. $y = \sin(x - 2)$

17. $y = \sin(x + 2)$

18. $y = 3 + 2\sin\left(\frac{\pi}{2}(x - 2)\right)$

19. If the general equation of a sinusoidal is $y = k + A \cdot f\left[B(x - h)\right]$, how does each of the constants—A, B, h, and k—affect the graph?

Analysis of these graphs reveals certain conclusions about the introduction of constants in the places of A, B, h, and k in the general equation.

A = stretches or shrinks the curve vertically
B = stretches or shrinks the curve horizontally
k = shifts the curve vertically
h = shifts the curve horizontally

General Equation: $y = k + A \cdot f\left[B(x-h)\right]$

where $|A| =$ amplitude – vertical distance from extreme value to the sinusoidal axis

$|B| =$ the number of cycles between 0 and 2π*

$\frac{2\pi}{|B|} =$ period**

$\frac{1}{|period|} =$ frequency

$k =$ vertical shift

$h =$ horizontal shift (a.k.a. phase shift)

and $f =$ the sinusoidal function

*This is also known as *angular velocity*.
**Tangent and cotangent functions have a different period.

Note that negative values of h will shift the curve left and negative values of k will shift the curve down. Negative values of A will mirror the curve vertically (and negative values of B will mirror the curve horizontally, with the exception of cosine and secant, although we will not consider this case).

LEARNING OUTCOMES

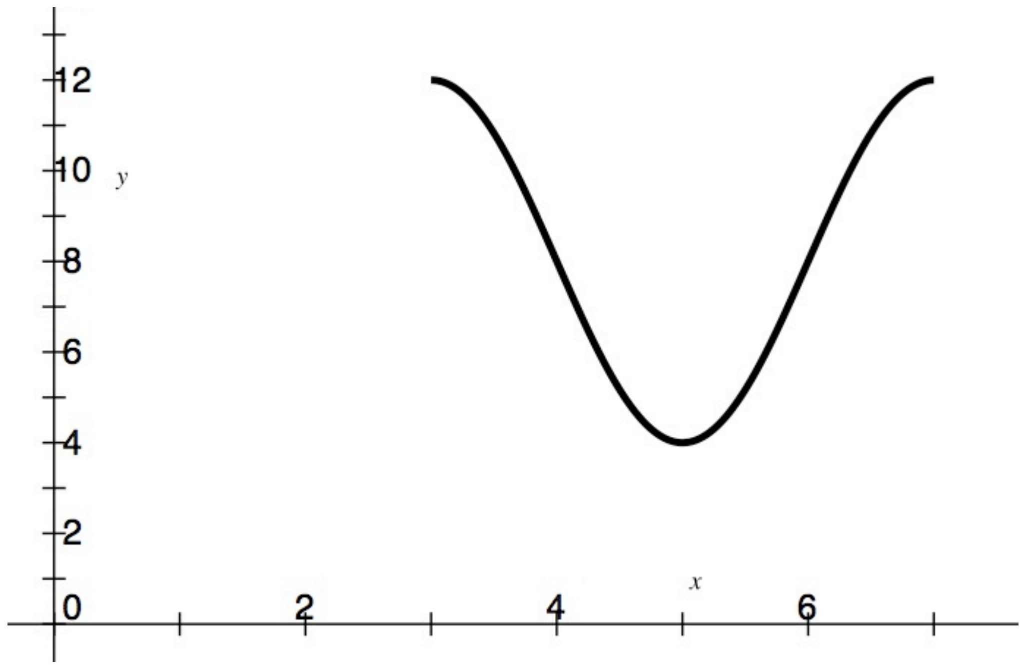
Find the graph from the equation of a sinusoidal without a graphing calculator.
Given a sinusoidal graph or its traits, find its equation.

EX 1 Sketch the graph of the sinusoidal with maximum value of 12 at $x = 3$ and $x = 7$, and minimum value of 4 at $x = 5$.

The easiest way to look at this is problem is to note that three ordered pairs are given:

$$(3,12),(5,4), \text{ and} (7,12)$$

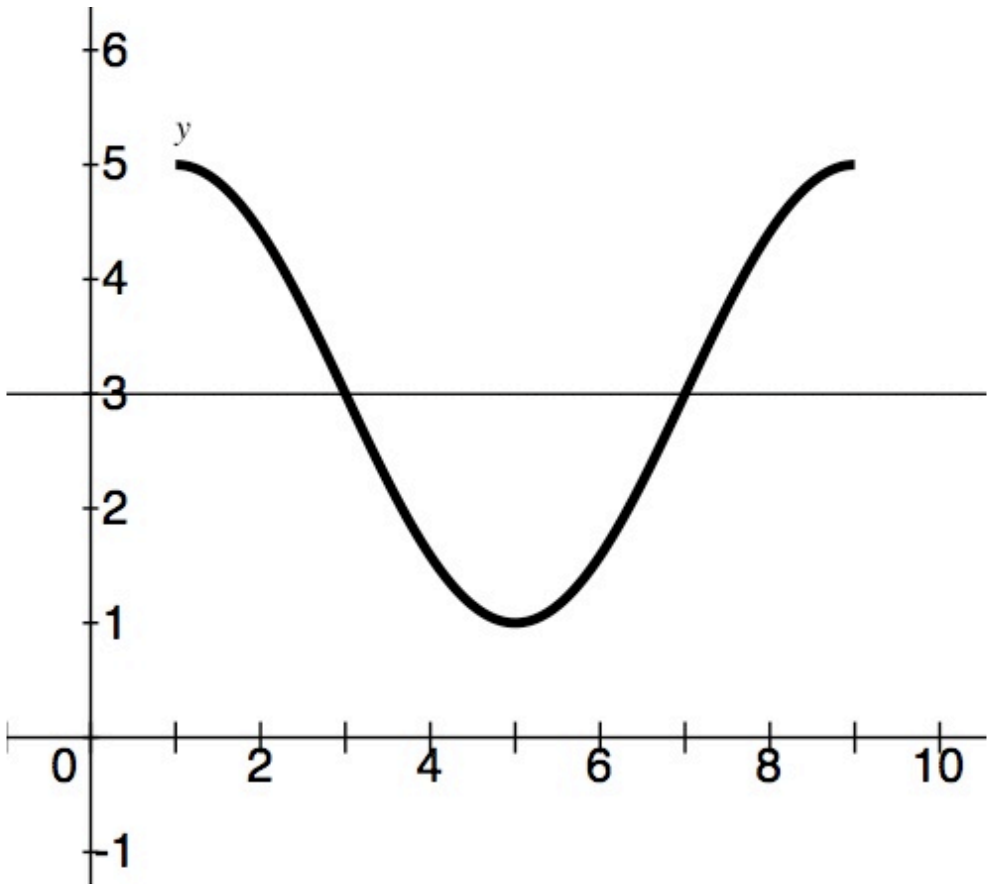
Plot the three points and connect them sinusoidally.



EX 2 Sketch the primary cycle of $y = 3 + 2\cos\left[\frac{\pi}{4}(x-1)\right]$.

$$\text{Amplitude} = 2 \text{Period} = \frac{2\pi}{\pi/4} = 8$$

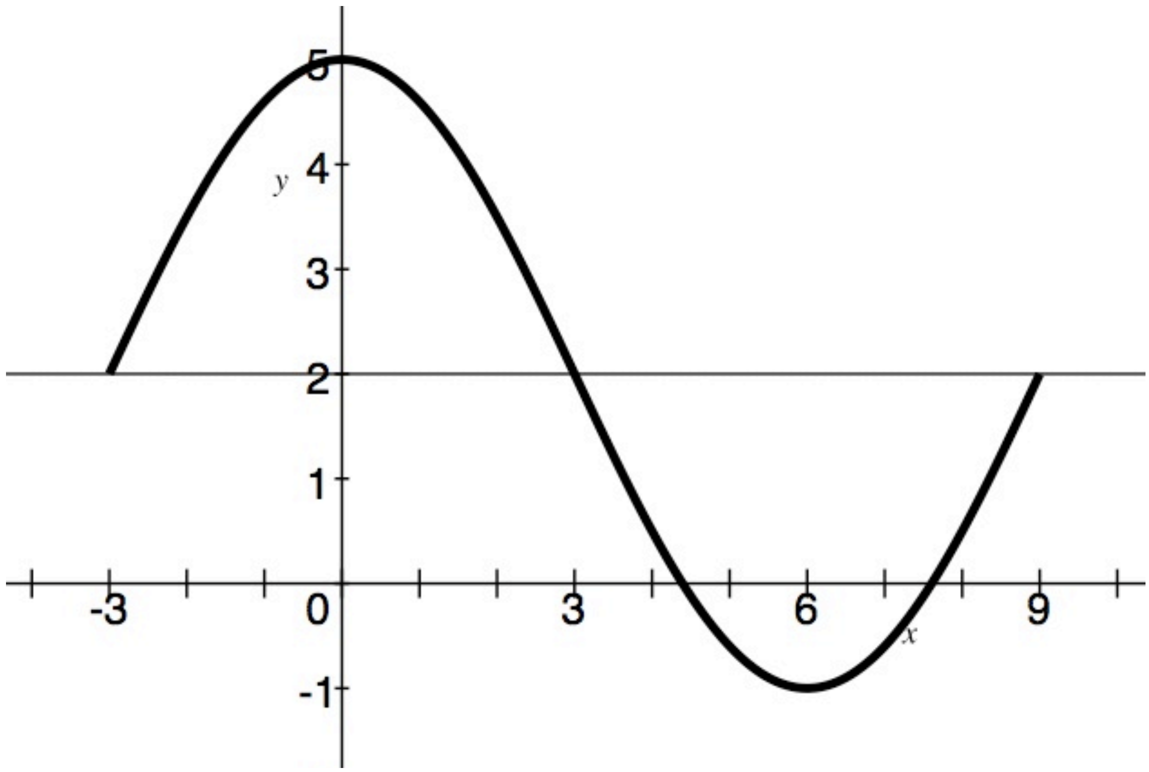
$$\text{Vertical Shift} = 3 \text{Horizontal shift} = 1$$



EX 3 Sketch the primary cycle of $y = 2 + 3\sin\left[\frac{\pi}{6}(x+3)\right]$.

$$\text{Amplitude} = 3 \quad \text{Period} = \frac{2\pi}{\pi/6} = 12$$

$$\text{Vertical Shift} = 2 \quad \text{Phase shift} = -3$$

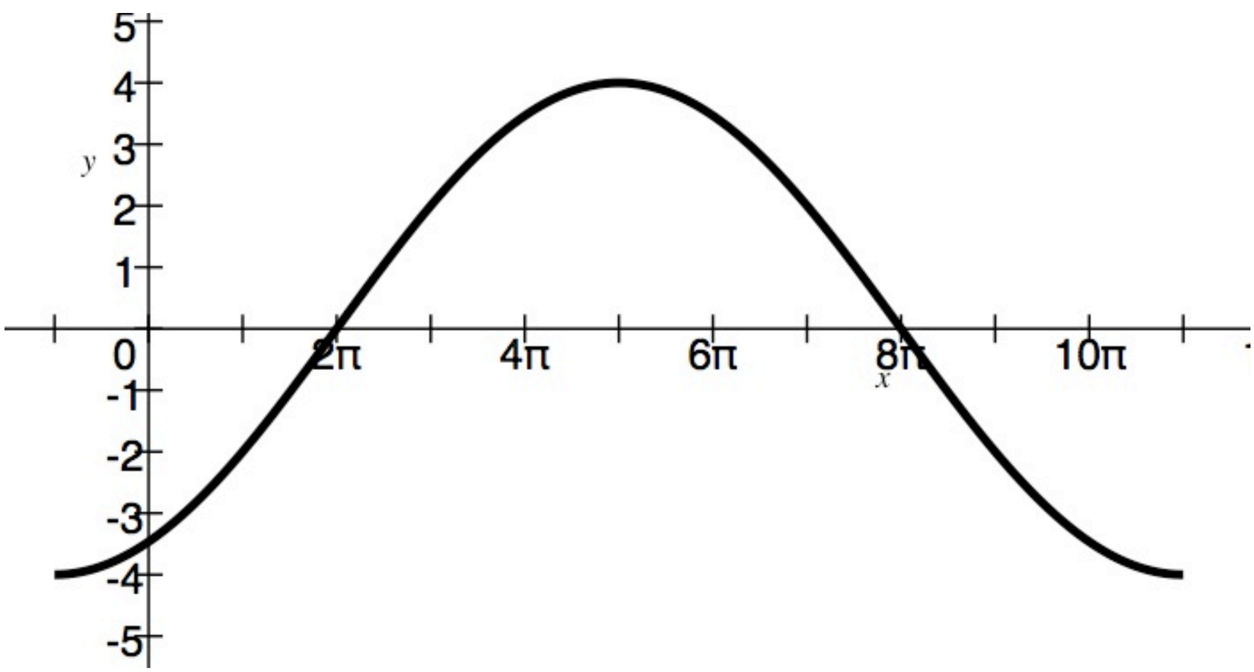


EX 4 Sketch the primary cycle of $y = -4\cos\left[\frac{1}{6}(x+\pi)\right]$.

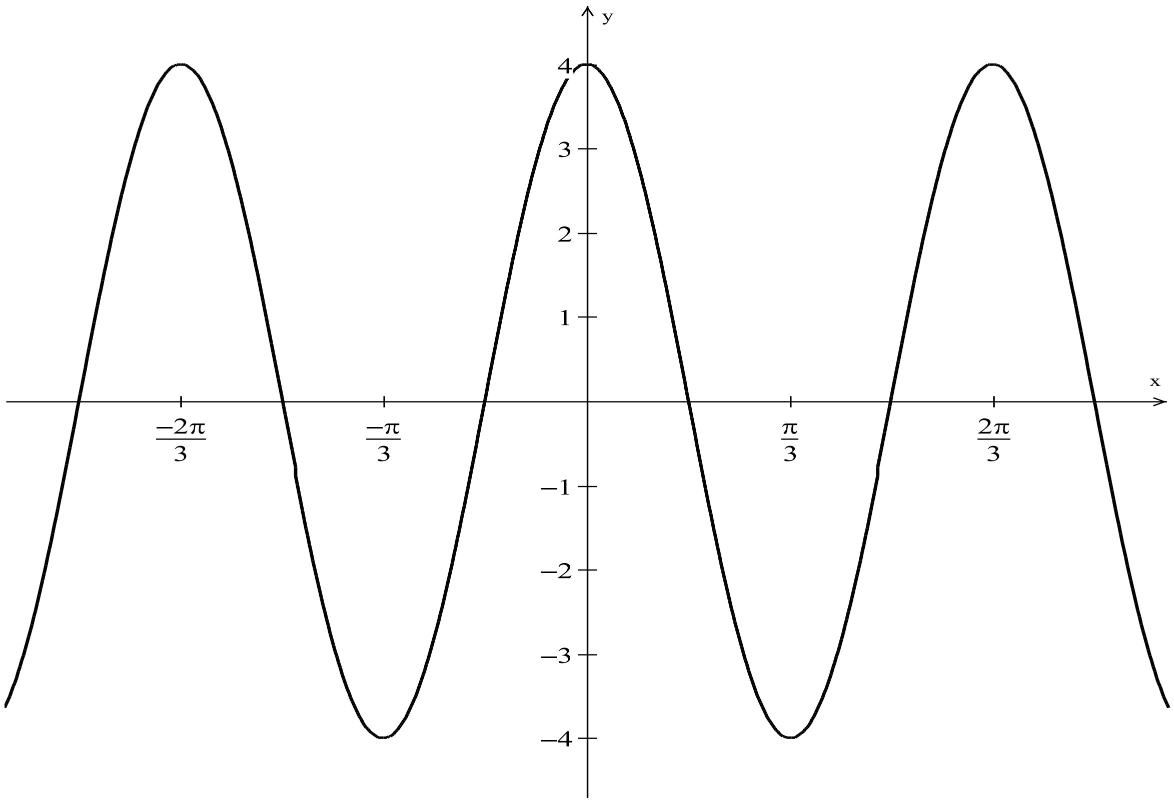
$$\text{Amplitude} = 4 \text{Period} = \frac{2\pi}{1/6} = 12\pi$$

$$\text{Vertical Shift} = 0 \text{Phase shift} = -\pi$$

Note that a negative *A* value means the graph will be mirrored vertically.



EX 5 Find one sine and one cosine equation for this graph.



Amplitude $= 4$ Period $= \frac{2\pi}{3}$, so $b = 3$ Vertical Shift $= 0$

Cosine crosses the y-axis at its maximum, so there is no phase shift for the cosine equation. The cosine equation for this graph is

$$y = 4\cos 3x.$$

Sine has a phase shift $\frac{1}{4}$ of the period to the left of cosine. Since the period is $\frac{2\pi}{3}$, the shift is $\frac{\pi}{6}$. The sine equation for this graph is

$$y = 4\sin 3\left(x + \frac{\pi}{6}\right).$$

2-1 Free Response Homework

Identify the period, amplitude, vertical shift and horizontal (phase) shift, and sketch the primary cycle of each of these equations.

1. $y = 4 + \sin\left[\frac{\pi}{8}(x-3)\right]$

2. $y = 2 + 3\cos\left[\frac{\pi}{4}(x)\right]$

3. $y = 1 + 5\sin\left[\frac{1}{2}(x-\pi)\right]$

4. $y = 1 - 3\sin[\pi(x)]$

5. $y = -3\cos\left[\frac{\pi}{6}(x+2)\right]$

6. $y = 2 + \cos\left[\frac{\pi}{2}(x-2)\right]$

7. $y = -2 - 4\sin\left[\frac{\pi}{5}(x-2)\right]$

8. $y = 2 - 2\cos\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right]$

9. $y = 3 + 5\sin\left[\frac{\pi}{12}(x-1)\right]$

10. $y = -3\cos\left[\frac{\pi}{4}x\right]$

11. $y = 1 - 7\cos\left[\frac{1}{6}(x+\pi)\right]$

12. $y = 3 - 5\sin\left[\frac{1}{4}\left(x - \frac{\pi}{2}\right)\right]$

13. $y = -2\sin\left[\frac{\pi}{8}(x-1)\right]$

14. $y = 2 - 5\cos\left[\frac{\pi}{6}(x+2)\right]$

15. $y = 5\cos\left[\frac{\pi}{5}(x+2)\right]$

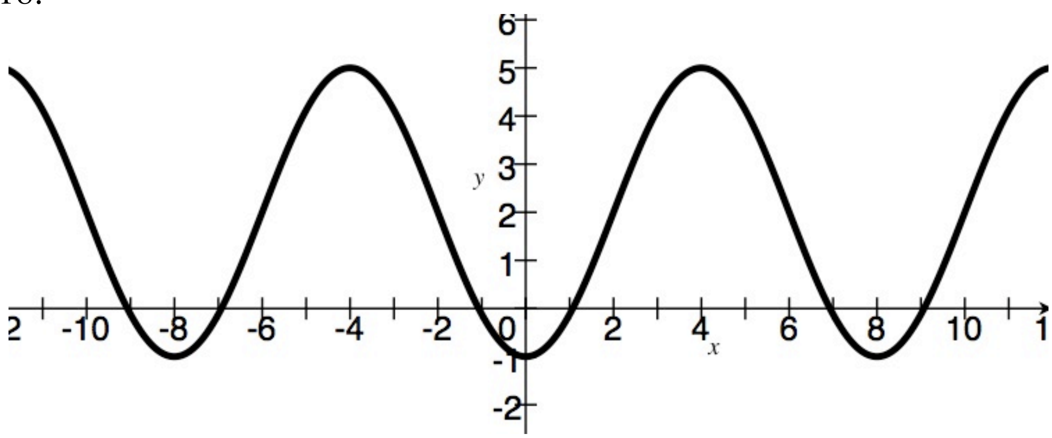
16. $y = 3 + 5\cos[4(x+\pi)]$

17. The temperature at a location on a distant planet is given by $T = -40 + 70\sin\left[\dfrac{\pi}{7}(x-5)\right]$ where T is temperature in Fahrenheit and x is time of day in hours. Determine whether each statement is true or false and **EXPLAIN** your choice. Reference the equation in your explanation.

a) The location is ALWAYS freezing (below 32ºF).
b) A day on this planet is 24 hours long.
c) At "midnight" $(x = 0)$, the location is at its coldest point.
d) The temperature will equal –10ºF twice every day.

Find one sine and one cosine equation for each of these graphs.

18.



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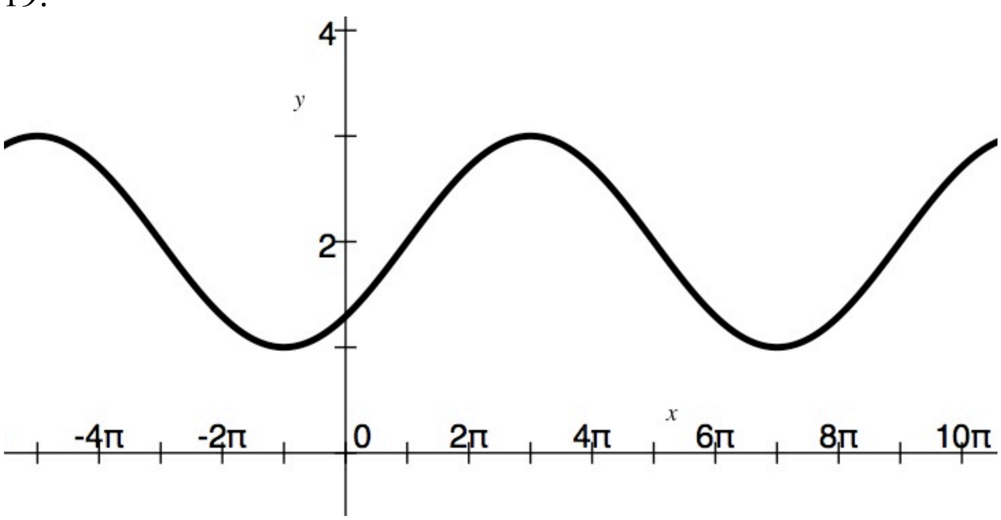

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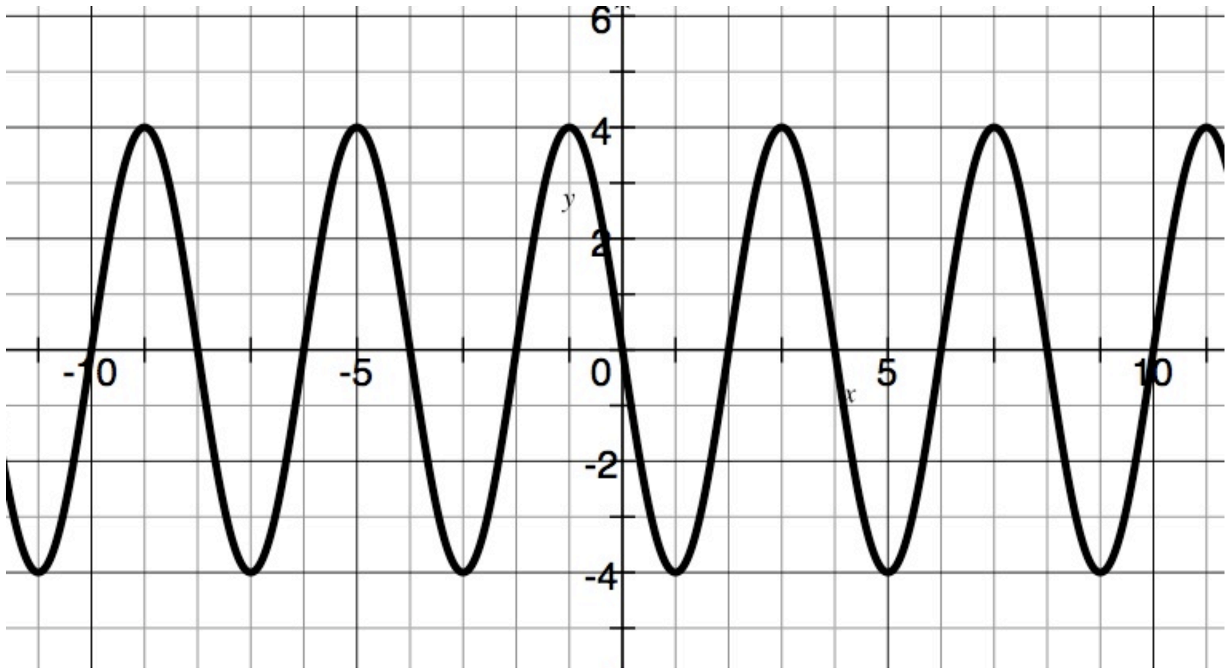

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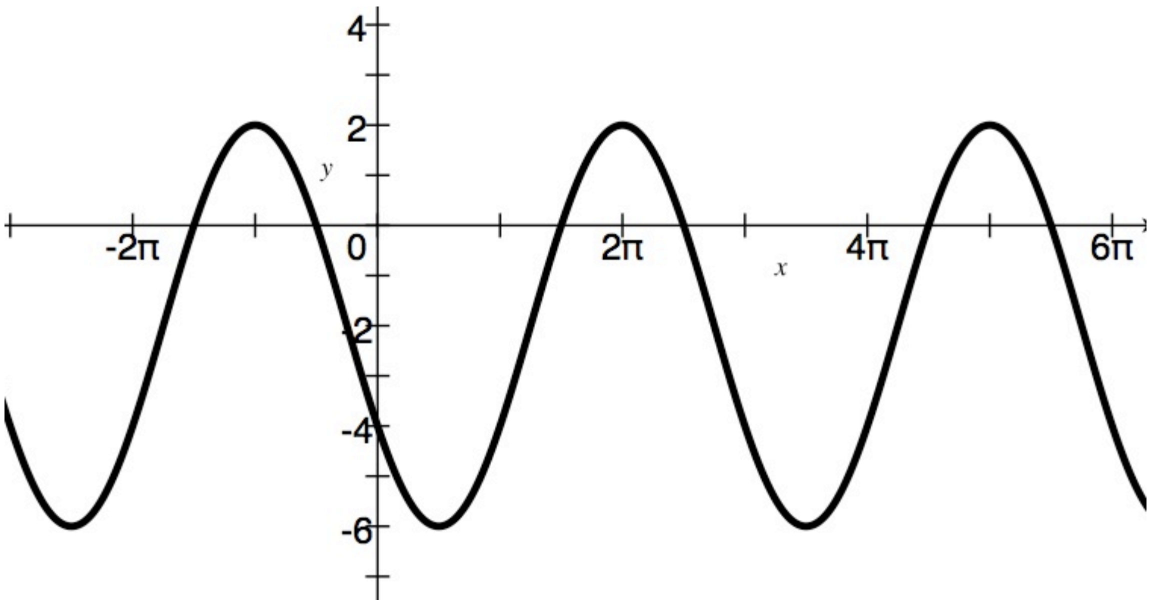

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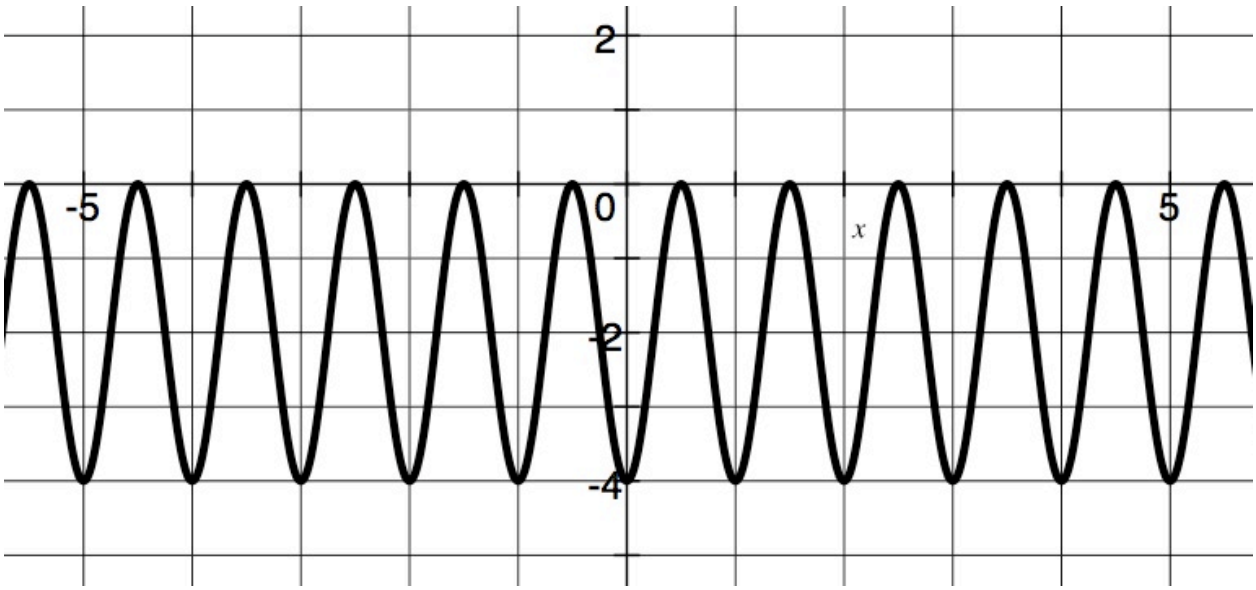

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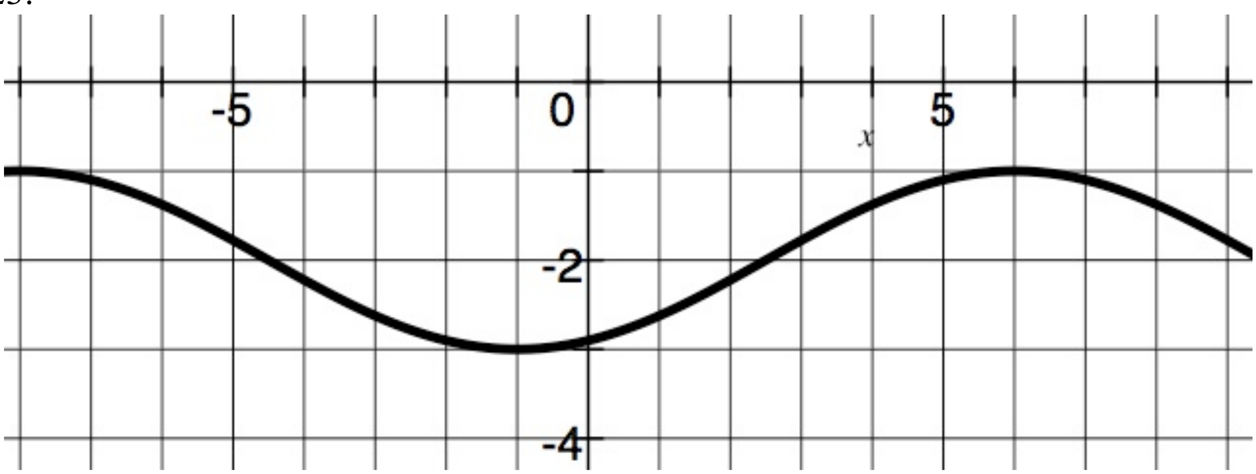

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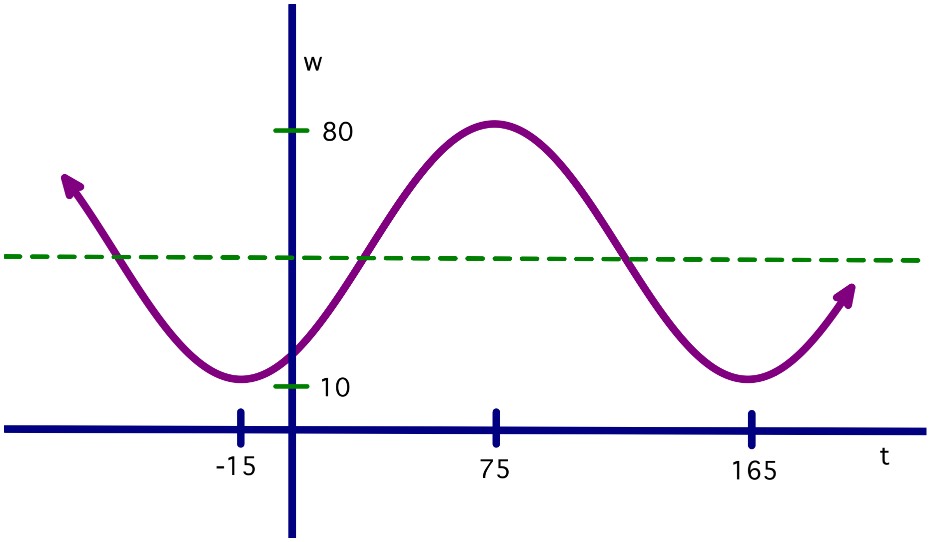

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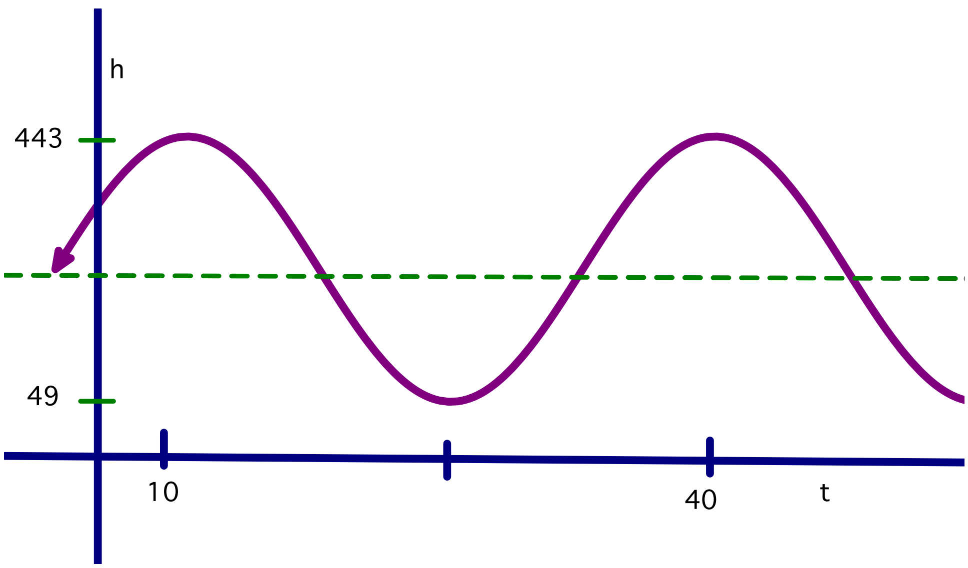
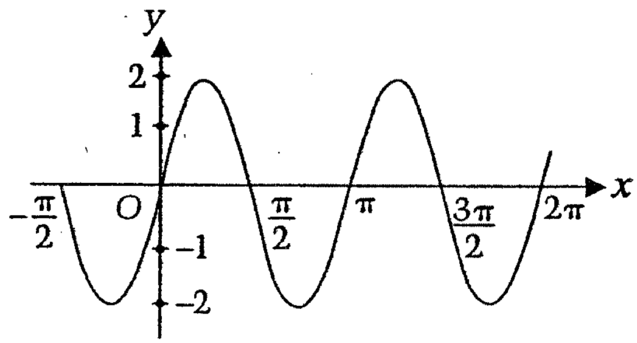


2-1 Multiple Choice Homework

1. Which of the following equations has the graph shown in the figure below?

a) $y = \sin\frac{x}{2} + 1$
b) $y = \sin 2x$
c) $y = 2\sin\frac{x}{2}$
d) $y = 2\sin x$
e) $y = 2\sin 2x$



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2. The vertical distance between the minimum and maximum values of the function $y = \left|-\sqrt{2}\sin\sqrt{3}x\right|$ is

a) 1.414   b) 2.828   c) 1.732

d) 3.464   e) 2.094

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3. On the graph of $y = \cos x$, as x increases on $x \in \left[-\frac{1}{4}, \frac{1}{4}\right]$, the function y

a) decreases
b) is constant
c) increases
d) decreases, then increases
e) increases, then decreases

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4. Which of the following must be true for $f(x) = 1 - 4\sin x$?

I. The minimum value is –3
II. The maximum value is 5
III. The amplitude is 4

a) I only b) II only c) III only d) I and III only e) I, II, and III

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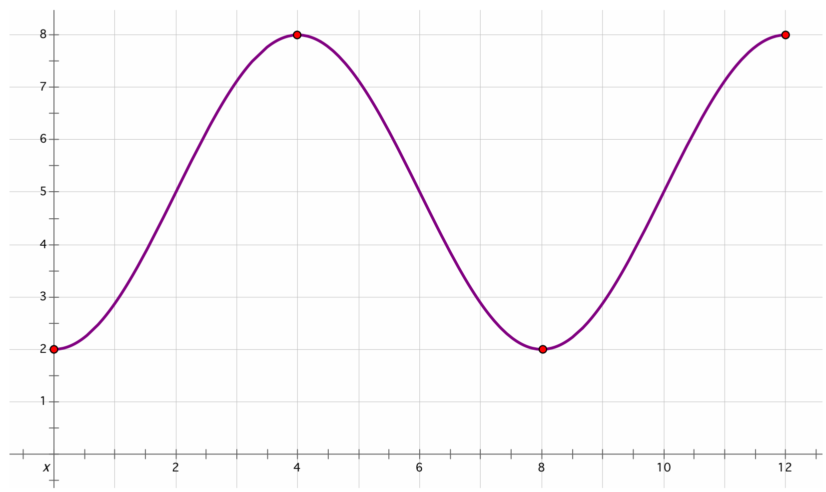
5. What is the smallest positive value of t for which $\cos(2t - 40°) = 1$?

a) 0° b) 20° c) 25° d) 65° e) 80°

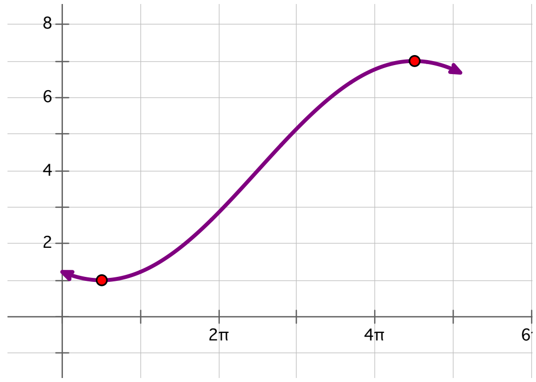


6. Assuming the graph above is sinusoidal, which of the following must be true?

I. The vertical shift is 2. II. The period is $\dfrac{\pi}{4}$

III. The amplitude is 3

a) I only b) II only c) III only d) II and III only e) I, II, and III



7. Which of the following is an equation for the graph above?

a) $y = 4 + 3\cos\left[\frac{1}{4}\left(x - \frac{\pi}{2}\right)\right]$

b) $y = 4 - 3\cos\left[\frac{1}{4}\left(x - \frac{\pi}{2}\right)\right]$

c) $y = 4 + 6\cos\left[\frac{\pi}{4}\left(x - \frac{1}{2}\right)\right]$

d) $y = 3 - 4\cos\left[\frac{\pi}{4}\left(x - \frac{1}{2}\right)\right]$

e) $y = 4 - 3\sin\left[\frac{1}{4}\left(x - \frac{\pi}{2}\right)\right]$

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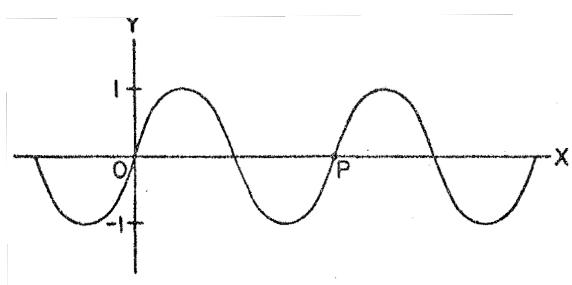
8. The figure below shows the graph of $y = \sin 3x$. What is the value of x at point P?



a) $\frac{\pi}{3}$

b) $\frac{2\pi}{3}$

c) 2π

d) 3π

e) 6π

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9. As x increases from $-\dfrac{\pi}{4}$ to $\dfrac{3\pi}{4}$, the value of $\sin x$

a) always increases
b) always decreases
c) increases then decreases
d) decreases then increases
e) does none of the above

2-2: Solving Sinusoidal Equations

LEARNING OUTCOME
Given a sinusoidal equation, find values of *y* from *x* and vice versa.

The algebraic process of solving a trig equation is not too difficult; it is just a matter of isolating the variable.

EX 1 If $f(x) = 2 + 3\cos\left(\frac{\pi}{4}(x-1)\right)$, find $f(2.3)$.

Since $f(x)$ is already isolated, this can be done on the calculator.

$$f(2.3) = 2 + 3\cos\left(\frac{\pi}{4}(2.3-1)\right)$$

$$f(2.3) = 3.567$$

Remember that these functions are periodic and that isolating for *x* will yield multiple answers.

> REMEMBER:
>
> $$\cos^{-1}\frac{x}{r} = \left\{\begin{array}{c} \text{calculator} \pm 2\pi n \\ -\text{calculator} \pm 2\pi n \end{array}\right\} \qquad \sin^{-1}\frac{y}{r} = \left\{\begin{array}{c} \text{calculator} \pm 2\pi n \\ \pi - \text{calculator} \pm 2\pi n \end{array}\right\}$$

So if, for example, $x = \left\{\begin{array}{c} 1 \pm 6n \\ -3 \pm 6n \end{array}\right\}$, this means there is an infinite number of answers, namely

$$x = \left\{\begin{array}{c} -11, -5, 1, 7, 13, \\ -15, -9, -3, 3, 9, \end{array}\right\}$$

It is likely specific solutions will be solicited.

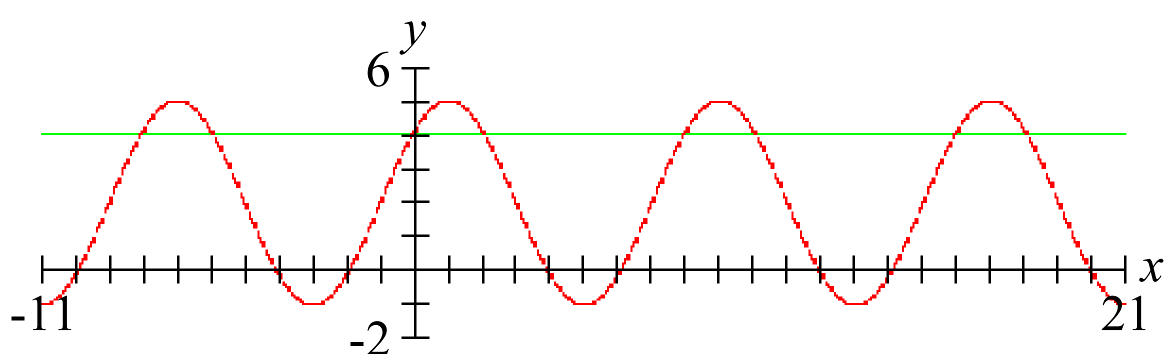
EX 2 If $f(x) = 2 + 3\cos\left(\frac{\pi}{4}(x-1)\right)$, find the first three positive values where $f(x) = 4$.

$$4 = 2 + 3\cos\left(\frac{\pi}{4}(x-1)\right)$$
$$2 = 3\cos\left(\frac{\pi}{4}(x-1)\right)$$
$$\frac{2}{3} = \cos\left(\frac{\pi}{4}(x-1)\right)$$
$$\cos^{-1}\left(\frac{2}{3}\right) = \frac{\pi}{4}(x-1)$$
$$\left\{\begin{matrix} 0.841 \pm 2\pi n \\ -0.841 \pm 2\pi n \end{matrix}\right\} = \frac{\pi}{4}(x-1)$$
$$\left\{\begin{matrix} 1.071 \pm 8n \\ -1.071 \pm 8n \end{matrix}\right\} = x - 1$$
$$\left\{\begin{matrix} 2.071 \pm 8n \\ -0.071 \pm 8n \end{matrix}\right\} = x$$

The first three positive x-values are

$$x = \{2.071, 7.929, 10.071\}$$

Notice that if the function in EX 1 and 2 are sketched, the points involved are:



The x-values of the points of intersection with the line $y = 4$ appear to be at the approximate location of the solutions to EX 2.

EX 3 For $f(x) = 1 - 4\sin\left[\frac{\pi}{2}(x+1)\right]$, find a) $f\left(\frac{2}{3}\right)$, and b) the first three negative values of x, closest to zero, where $f(x) = 2$.

a) $$f\left(\frac{2}{3}\right) = 1 - 4\sin\left[\frac{\pi}{2}\left(\frac{2}{3}+1\right)\right]$$

$$f\left(\frac{2}{3}\right) = -1$$

b) $$2 = 1 - 4\sin\left[\frac{\pi}{2}(x+1)\right]$$

$$-\frac{1}{4} = \sin\left[\frac{\pi}{2}(x+1)\right]$$

$$\sin^{-1}\left(-\frac{1}{4}\right) = \frac{\pi}{2}(x+1)$$

$$\left\{\begin{array}{c} -0.253 \pm 2\pi n \\ 3.394 \pm 2\pi n \end{array}\right\} = \frac{\pi}{2}(x+1)$$

$$\left\{\begin{array}{c} -0.161 \pm 4n \\ 2.161 \pm 4n \end{array}\right\} = x+1$$

$$\left\{\begin{array}{c} -1.161 \pm 4n \\ 1.161 \pm 4n \end{array}\right\} = x$$

The first three negative x-values, closest to zero, are

$$x = \{-1.161, -2.839, -5.161\}$$

2-2 Free Response Homework

For each of the following, find a) y if $x = -2.7$, and b) the smallest three positive values of x where $y = 0$.

1. $y = 2 + 3\cos\left[\frac{\pi}{4}(x)\right]$

2. $y = 4 - \cos\left[\frac{\pi}{4}(x - 12)\right]$

3. $y = 1 + 5\sin\left[\frac{1}{2}(x - \pi)\right]$

4. $y = 3 + 7\cos\left[\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right]$

For each of the following, find a) y if $x = 4.8$, and b) the first four negative values of x, closest to zero, where $y = 0$.

5. $y = -3\cos\left[\frac{\pi}{6}(x + 21)\right]$

6. $y = 2 + \cos\left[\frac{\pi}{2}(x - 5)\right]$

7. $y = 0.7 + \sin\left[\frac{\pi}{8}(x - 3)\right]$

8. $y = 1 - 3\sin[\pi(x)]$

For each of the following, find the solutions in the given domain.

9. If $f(x) = 3 + 7\cos\left[\frac{2\pi}{5}(x - 11)\right]$, find the values of x where $f(x) = 5$ and $0 \le x \le 25$.

10. If $g(x) = 1 + 4\cos\left[\frac{\pi}{8}(x - 13)\right]$, find the values of x where $g(x) = 1.3$ and $0 \le x \le 20$.

11. If $y = 2 - 5\cos\left[\frac{3\pi}{4}(x - 2)\right]$, find the values of x where $y = -1.2$ and $-5 \le x \le 0$.

12. If $H(x) = -1 + 4\cos\left[\frac{\pi}{3}(x - 2)\right]$, find the values of x where $H(x) = 2.3$ and $0 \le x \le 10$.

13. If $f(x) = 1 - 4\cos\left[\frac{1}{4}(x - \pi)\right]$, find the values of x where $f(x) = -1.8$ and $0 \le x \le 51$.

14. If $y = -1 - 4\cos\left[\frac{\pi}{12}(x - 2)\right]$, find values of x where $y = 2.3$ and $0 \le x \le 20$.

15. If $y = 1 + 3\sin\left[\frac{1}{8}\left(x + \frac{\pi}{2}\right)\right]$, find the values of x where $y = 2.1$ and $-100 \le x \le 0$.

16. If $y = -1 - 3\cos\left[\frac{\pi}{10}(x + 2)\right]$, find the values of x where $y = 1.3$ and $-20 \le x \le 20$.

2-2 Multiple Choice Homework

1. What is the smallest positive x-intercept of the graph of $y = 3\sin 2\left(x + \frac{2\pi}{3}\right)$?

a) 0 b) 2.09 c) 1.05

d) 0.52 e) 1.31

---

2. What is the smallest positive angle that will make $y = 5 - \sin\left(x + \frac{\pi}{6}\right)$ a maximum?

a) 1.05 b) 2.09 c) 1.57 d) 4.19 e) 5.24

---

3. The smallest positive value of x satisfying the equation $\tan 5x = -2$ is closest to

a) 13° b) 31° c) 23° d) 49° e) 63°

---

4. When the graph of $y = \sin 2x$ is drawn for all values of x between 10° and 350°, it crosses the x-axis

a) zero times
b) one time
c) two times
d) three times
e) six times

---

5. $x > \sin x$ for

a) all $x > 0$
b) all $x < 0$
c) all x for which $x \neq 0$
d) all x
e) all for which $-\frac{\pi}{2} < x < 0$

2-3: Mathematical Modeling with Sinusoidals

LEARNING OUTCOME
Model and solve sinusoidal situations.

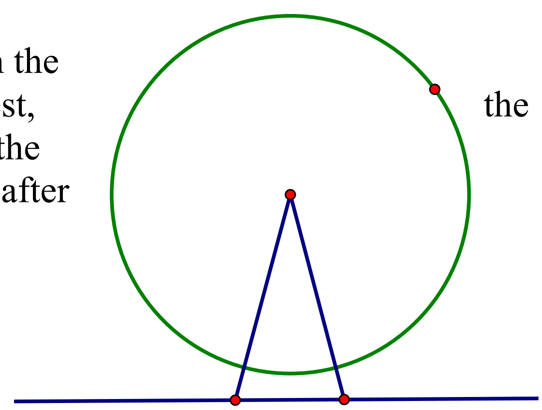
The nard thing about word problems has always been how to translate them into mathematical equations which can be solved. With word problems which involve variations, though, there are certain trigger phrases that indicate what equations should be used:

Vocabulary: (variation word problem phrases)

1. ***Dependent Variable*** – the unknown that varies
2. ***Independent Variable*** – the unknown that the dependent varies with
3. ***"y varies directly as x"*** – $y = kx$
4. ***"y varies inversely as x"*** – $y = \frac{k}{x}$
5. ***"f varies jointly with x and y"*** – $f = kxy$
6. ***"y varies sinusoidally with t"*** – $y = k + A\cos(B(t-h))$ or $y = k + A\sin(B(t-h))$

This will make the most sense in an example.

EX 1 The height of a seat on a Ferris wheel from the ground varies sinusoidally with time. At its lowest, the seat is 2 feet above the ground. The diameter of the wheel is 50 feet. The seat is at the top 3 seconds after you start timing, and the wheel goes around once every 120 seconds.



a) Sketch a graph of the movement.

b) Find a sinusoidal function to represent the graph.

c) How high is the seat at 35 seconds? at 85 seconds?

d) When is the third time the seat is at its lowest?

We might consider the following roadmap for these problems:

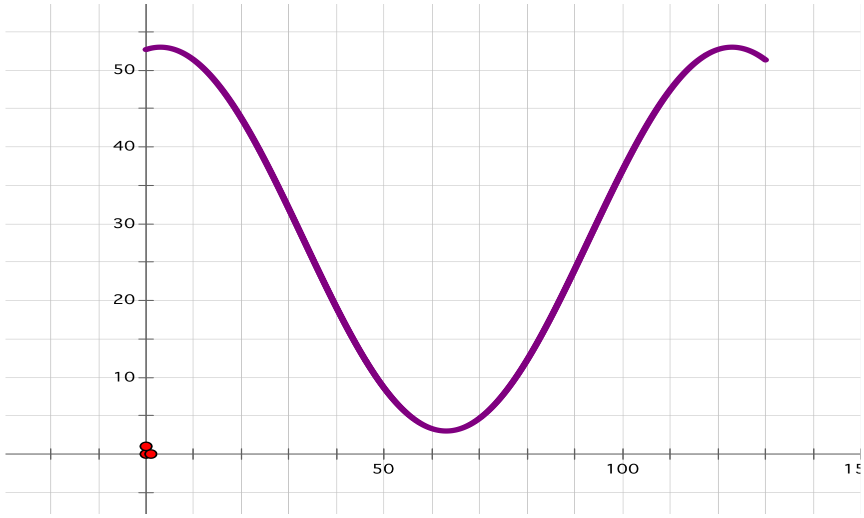
Steps to Setting Up Sinusoidal Modeling Problems

1. Read the problem at least twice.
2. Determine which variable is independent and which is dependent (i.e., which will be on the x-axis and which will be on the y-axis, respectively).
3. Read the problem again, looking for two ordered pairs. Consider where the high and low points are and what values of the independent variable put them there.
4. Plot those ordered pairs.
5. Connect the plotted points with a sinusoidal wave.
    - 5a. Extend the wave if needed.
    - 5b. Include the sinusoidal axis.
6. Find the equation from the graph.
7. Answer the questions asked.
8. Check that the answers make sense in context.

In terms of example 1, here would be the steps:

1. Read it again
2. "height from the ground varies sinusoidally with time" means that height is on the y-axis and time is on the x-axis.
3. The highest point is at 53 feet and occurs when t=3 and at t=123.

4/5.



6. $h(t) = 27 + 25\cos\left[\frac{\pi}{60}(t-3)\right]$

7. c) How high is the seat at 35 seconds? at 85 seconds?

$$h(35) = 27 + 25\cos\left[\frac{\pi}{60}(35-3)\right] = 24.387$$

$$h(85) = 27 + 25\cos\left[\frac{\pi}{60}(85-3)\right] = 16.832$$

d) When is the third time the seat is at its lowest?

$$h(t) = 2 = 27 + 25\cos\left[\frac{\pi}{60}(t-3)\right]$$

$$-1 = \cos\left[\frac{\pi}{60}(t-3)\right]$$

$$\pm\pi \pm 2\pi n = \frac{\pi}{60}(t-3)$$

$$\pm 60 \pm 60n = t-3$$

$$\left.\begin{array}{c} 63 \pm 60n \\ -57 \pm 60n \end{array}\right\} = t$$

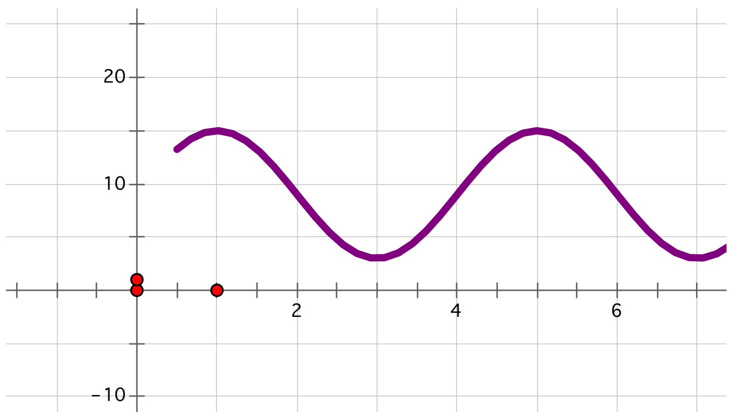
$$t = \{3,\ 63,\ 123, \ldots\}$$

$$t = 123$$

8. Yes, they make sense.

EX 2 A particular weight attached to a spring bounces up and down such that the height of the weight from the table varies sinusoidally with time. The furthest the weight gets from the table is 15 inches and the closest is 3 inches. The weight is at its highest point at 1 seconds and goes from highest to lowest point and back to the highest point in 4 seconds.

a) Sketch a graph of the movement.



b) Find a sinusoidal function to represent the graph.

$$h(t) = 9 + 6\cos\left[\frac{\pi}{2}(t-1)\right]$$

c) How high is the weight at 0 seconds? How about at 1.7 seconds?

$$h(0) = 9 + 6\cos\left[\frac{\pi}{2}(-1)\right] = 9$$

$$h(1.7) = 9 + 6\cos\left[\frac{\pi}{2}(1.7-1)\right] = 11.724$$

d) When are the first three times that the weight is at 8 inches?

$$8 = 9 + 6\cos\left[\frac{\pi}{2}(t-1)\right] = 9$$

$$-1 = 6\cos\left[\frac{\pi}{2}(t-1)\right]$$

$$\frac{-1}{6} = \cos\left[\frac{\pi}{2}(t-1)\right]$$

$$\pm 1.738 \pm 2\pi n = \frac{\pi}{2}(t-1)$$

$$\pm 1.107 \pm 4n = t - 1$$

$$\left.\begin{matrix} 2.107 \pm 4n \\ -0.107 \pm 4n \end{matrix}\right\} = t$$

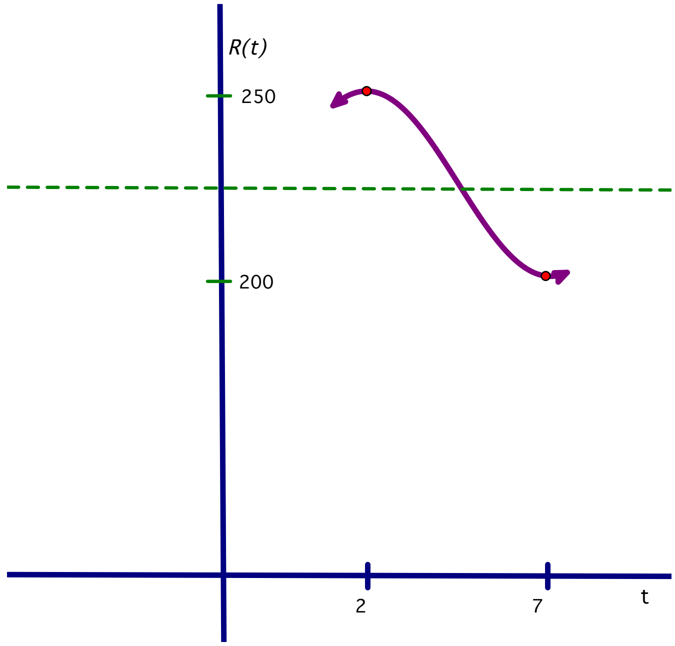
$$t = \{2.107,\ 3.893,\ 6.107\}$$

EX 3 A veterinarian studying a sick cat notices that the cat's heart rate is varying sinusoidally with time. Two minutes after she starts timing, the cat's heart rate is at its highest, 250 beats per minute. Five minutes later, it is at its lowest, 200 beats per minute. Find the first three times since the vet started timing that the cat had a heart rate of 220 beats per minute.

Notice that, though they did not explicitly say to so the steps in the process, we cannot answer the question without an equation to solve and we cannot find the equation with finding the points to plot and drawing a graph. And we cannot find the points to plot before first deciding what he variables are and which is on which axis.

"heart rate is varying sinusoidally with time" tells us that *R* is on the y-axis and t is on the *x*-axis.

$t = 2 \rightarrow R = 250$ and $t = 2 \rightarrow R = 250$, so



$$R(t) = 225 + 25\cos\left[\frac{\pi}{5}(t-2)\right]$$

Now we can answer the question, which is find the first three values of t when $R(t) = 220$.

$$220 = 225 + 50\cos\left[\frac{\pi}{5}(t-2)\right]$$

$$.2 = \cos\left[\frac{\pi}{5}(t-2)\right]$$

$$\pm 1.369 \pm 2\pi n = \frac{\pi}{5}(t-2)$$

$$\pm 2.180 \pm 10n = t - 2$$

$$\left.\begin{array}{r} 4.180 \pm 10n \\ -0.180 \pm 10n \end{array}\right\} = t$$

$$t = \{4.180,\ 9.820,\ 14.180\}$$

2-3 Free Response Homework

1. When the London Eye opened in 2000, it was the largest Ferris Wheel in the world. The highest point is 443 feet above the surface of the Thames, and the diameter is 394 feet. It makes one revolution every 30 minutes and moves slowly enough that it does not actually have to stop to let people get on or off. The queue (waiting line) can get long, but it moves quickly. As you wait in line, you notice that the height from the river's surface to a particular capsule varies sinusoidally with time. After 10 minutes of watching, you see that capsule reach the top.

   a) Sketch one cycle of the situation.

   b) Create an equation that models the height h of the capsule on the Eye in terms of time t.

   c) How high was the capsule when you started watching it?

   d) What are the first three positive times when the capsule is 100 feet off the ground?

2. In episode 218 of the Mythbusters, Jamie Hyneman bungee-jumped from a 100-foot-high lift to bob for apples in a pool. Like the Bouncing Spring Problem, Jamie's height from the water's surface varies sinusoidally with time. It took four seconds for Jamie to fall from the tower to the water and two feet under the surface.

   a) Sketch the graph of his drop.

   b) Create an equation that models Jamie's height h in terms of time t.

   c) How far above the water is Jamie at 1.4 seconds?

   d) At what time did Jamie come back out of the water?

3. Todd is meditating; he is concentrating on his breathing. His breathing varies sinusoidally with time. His lung capacity is 68 cubic centimeters at minimum capacity (exhale). When he inhales completely, his lung capacity is 128 cubic centimeters. At 2 seconds, his lung capacity is at its lowest, and he is back to his lowest 8 seconds later.

a) Sketch a graph of the situation and find a sinusoidal function to represent the situation.
b) Find his lung capacity at 0 seconds, 3 seconds, and 10 seconds.
c) Use your graph to predict the first three times that his lung capacity is at its highest.

4. In a Chemistry experiment, researchers find that the temperature of a compound varies sinusoidally with time. 17 minutes after they start timing, the temperature is its highest, which is 56° Celsius. 12 minutes after it has reached its maximum, the temperature hits its minimum, which is 40° Celsius.

a) Sketch a graph of the temperature as a function of time and write a sinusoidal equation that describes the temperature y in terms of time t.
b) What was the temperature at the start of the experiment?
c) What are the first three times that the temperature is 51°?

5. A particular teacher's ability to reach her students varies sinusoidally with time. One month after the start of school, in September, she is teaching her best– reaching all 100 of her students. Three months later she reaches her lowest, only 50 of her students.

a) Sketch a graph of the number of students reached as a function of months after school starts.
b) Write a sinusoidal equation that describes the number of students reached in terms of months after school starts.
c) How many students were reached at the start of school (in months)?
d) When are the first two times in terms of months she will reach 85 students?

6. While driving on HWY 280 from San Francisco to San Jose during rush hour, a driver notices that the speed of traffic seems to vary sinusoidally with the distance from San Francisco. There are bottlenecks at HWY 380, HWY 92, Stanford University and Cupertino where the speed of the traffic drops to 5 mph. Half way between each pair of sites, the speed gets up to 75 mph. The distance between each of these bottlenecks is 6 miles and HWY 380 is 3.5 miles south of San Francisco.

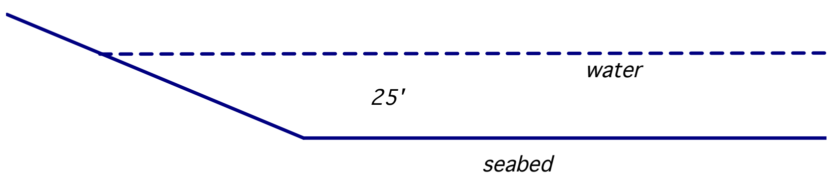
a) Sketch one cycle of the situation.
b) Find an equation that represents s in terms of d.
c) What is the speed of the traffic 7.2 miles south of San Francisco?

d) Between HWY 92 and Stanford, at what two distances from San Francisco is the speed of traffic 60 mph?

7. The height of the feet of a person jumping on a trampoline varies sinusoidally with time. The surface of the trampoline is 45 cm above the ground. At 0.20 seconds, she reaches her highest point of 110 cm above the surface of the trampoline. 0.12 seconds later she is at the lowest point, 20 cm below the surface of the trampoline. Sketch the graph of 4 cycles of this function. Find an equation that fits this model. Find the first four times she is at a height of 1 meter above the ground. [NB: Decide first if you want the *x*-axis to represent the ground or the level of the trampoline.]

a) Sketch a graph of the height $H(t)$ as a function of time.
b) Write a sinusoidal equation that describes the $H(t)$ in terms of time *t*.
c) What are the first four times when she is 1 meter above the ground?

8. Tsunamis are tidal waves in the Pacific Ocean caused by earthquakes in various places around the Pacific Rim. In 1946, 159 people were killed by a tsunami caused by an earthquake in the Aleutian Islands, Alaska. The sea level first dropped 55 feet, exposing the seabed–which is normally 25 feet below sea level–and then the water rose an equal distance above sea level.



Many school children were drowned when they ran out into the exposed area, not realizing the sea when would be rushing back in. The cycles took roughly 12 minutes. Assuming the waves conform to a sinusoidal graph (i.e., the height of the wave *y* varies sinusoidally with time), determine an equation to describe this phenomenon and predict how long the seabed would be exposed.

a) Sketch a graph of the height of the water $P(t)$ as a function of time.
b) Write a sinusoidal equation that describes the $P(t)$ in terms of time *t*.
c) For how long is the seabed exposed?

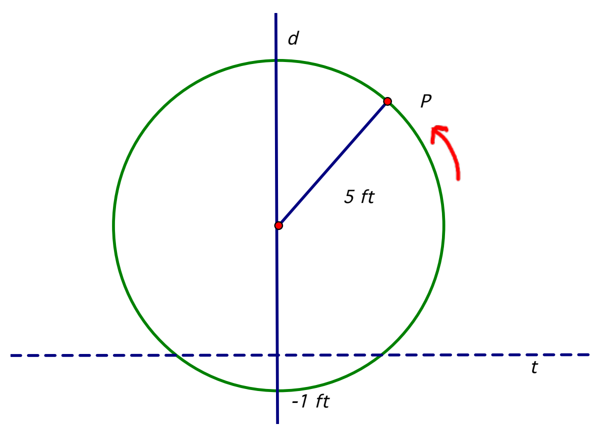
9. A patient in a hospital is experiencing fluctuations in his blood pressure. His pressure seems to be varying sinusoidally with time. Over 24 hours, it varies from 90 to 150 and back twice. At 9 am, the pressure is 120 and is on the rise. What would

you expect the pressure at 1:15 pm? What are the first three times of day when $P(t) = 130$?

10. The price of gasoline over the past year seems to have varied with time. On June 10th (day 161), the cost was its highest, which was \$3.97. On December 10th (day 344), the cost was its lowest, which was \$2.97. When will the price of gas first rise above \$3.10? What is the first day of the year when the price will drop below \$3.10?

11. One Saturday you and a friend are watching whales migrating along the coast. You immediately recognize that their depth below the water's surface varies sinusoidally with time. After four seconds of watching a particular whale, you observe that it is seven feet below the surface and that this is the lowest depth. Forty seconds later, it rises one foot above the water's surface to blow its spout and get a breath. The whale dives again. At what time *t* will it resurface?

12. Gold was discovered in California at Sutter's Mill in 1848 when James Marshal; was digging a millrace in the America River near Coloma to build a water-driven sawmill. A millrace is a trench for water to flow through so it can turn a wheel that drives the saws. Suppose the waterwheel has a five-foot radius and sits one foot into the millrace as pictured here. Assume that it rotates at 5 revolutions per minute and that the distance *d* from point P to the water's surface varies sinusoidally with time. Three seconds after you start timing the motion, point P is at its highest. What is the first time *t* when P ***emerges*** from the water?



13. A baseball player who bats .300 over a season has streaks and slumps and rarely bats exactly .300 at a particular time. Let us assume that a player's batting average varies sinusoidally with time and ranges from a high of .425 to a low of .175. Let us further assume that each cycle lasts 54 games, and he reaches his first high 10 games into the season. When are the first three times that his batting average is .375?

14. The stock market goes through cycles of growth and loss referred to as Bull and Bear Markets. A Bull market is characterized by increasing stock prices, while a Bear market has decreasing prices. A study of the gains and losses from 1873 to the present shows that highs generally reach 400% growth while the lows reach -60% as a loss. The last high was reached in the year 2000, while the last low was reached in 2009. Assume the trend is sinusoidal. One investment marketing strategy suggests selling stocks when the gains are above 370% and buy when the stocks are growing less than 20%. Between what two times after 2000 should stocks be bought?

15. A tropical flower closes and opens at different times of the day. The diameter of the petals varies sinusoidally with time. At midnight $(t=0)$, the petals of the flower completely open to an 8-inch diameter. At 6am, they completely close to a diameter of 2 inches. What are the times of the day when the petals are 3 inches open?

16. The new roller coaster, the Raging Behemoth, contains many loops and swoops. Its height on one of the loops varies sinusoidally with time. At its highest, the coaster is 85 feet off the ground 15 seconds after the start of the ride. After five seconds, it reaches its lowest point, 5 feet off the ground. Determine when the coaster is 78' off the ground.

2-3 Multiple Choice Homework

1. On a given day the maximum height of a tide is 25 m and the minimum height of the tide is 16m. What is the vertical displacement (median value) for the height of the tide?

a) 16 b) 11.5 c) 19.5 d) 20.5 e) 25

---

2. A weight, attached to the end of a very long spring, is bouncing up and down. For a small period of time, this motion can be modeled by a sinusoidal function. When your stopwatch reads 1.3 seconds, the weight is at a minimum height of 2.4 feet above the floor. When your stop watch reads 1.9 seconds, the weight reaches the next maximum height of 3.2 feet. Determine the equation modeling the height of the weight, *h*, in terms of time, *t*.

a) $h(t) = 2.8 - 0.4\cos\left[\frac{2\pi}{1.2}(t - 1.3)\right]$

b) $h(t) = 2.4 - 0.6\cos\left[\frac{2\pi}{1.2}(t - 1.3)\right]$

c) $h(t) = 2.4 - 0.4\cos\left[\frac{2\pi}{0.6}(t - 1.3)\right]$

d) $h(t) = 2.8 - 0.6\cos\left[\frac{2\pi}{0.6}(t - 1.9)\right]$

e) $h(t) = 2.8 - 0.4\cos\left[\frac{2\pi}{1.2}(t - 1.9)\right]$

3. The total increase in a penguin population (births minus deaths) can be approximated by the sinusoidal equation $f(x) = 1500 + 2000\sin(0.523x)$, where x is the number of months since April 1, 2017. After how many months do the number of deaths outnumber the births? Give your answer to the nearest month.

a) 5 b) 7 c) 9 d) 11 e) never

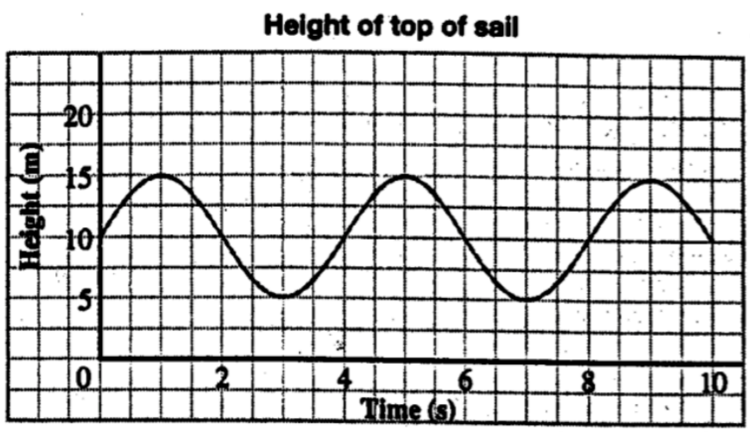
---

4. The primary period of the function $f(x) = 2\cos^2 3x$ is

a) 2π b) $\frac{\pi}{3}$ c) π d) $\frac{\pi}{2}$ e) 3π

---

5. The graph below shows the height h (meters) of the tip of a sail of a windmill at time t (seconds). How many rotations does the paddle make in 2 hours?



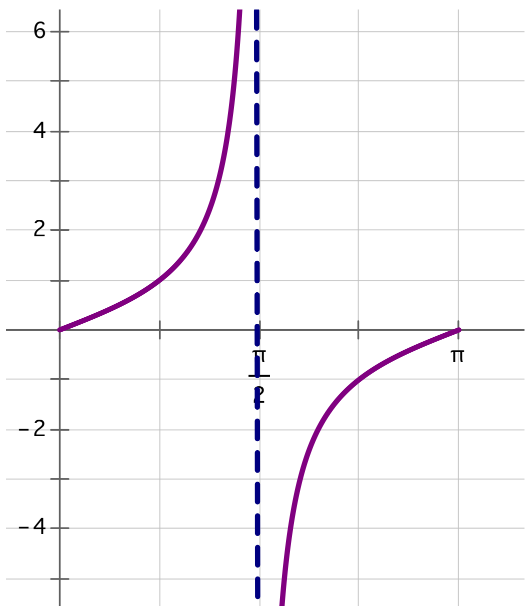
a) 900 b) 1200 c) 1600

d) 1800 e) 3600

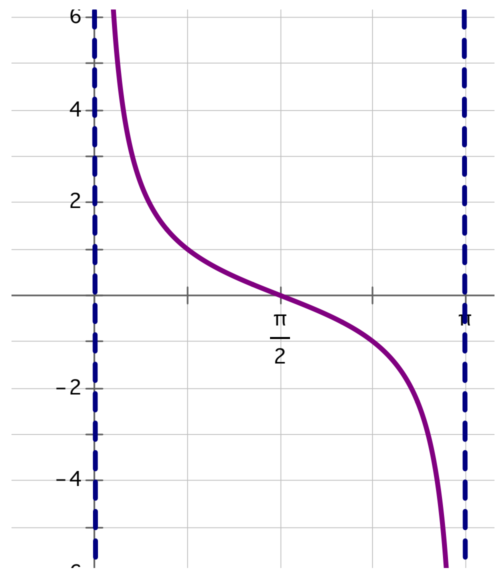
2-4: Reciprocal and Quotient Curves

As with the sinusoidals, the other four trig functions have graphs that are periodic.

The Quotient Curves:



$y = \tan x$

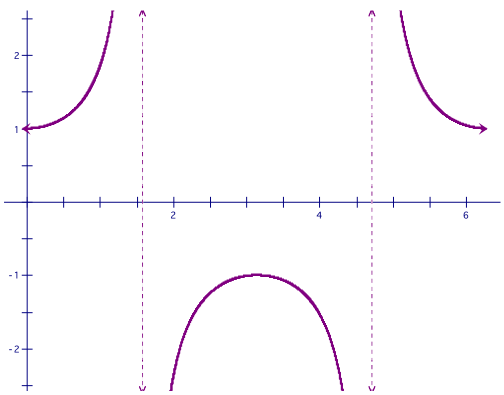


$y = \cot x$

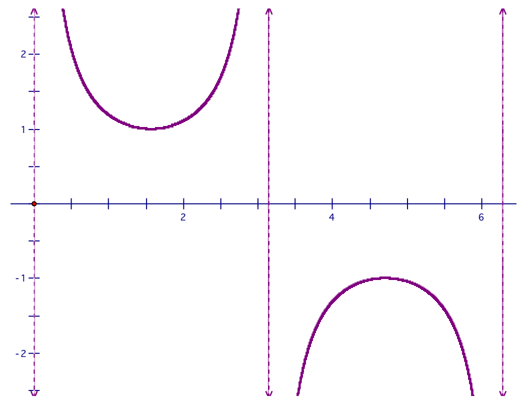
Notice the $\tan x$ and $\cot x$ curves repeat more often than the sinusoidals. **If the period is considered to be the width of one section of curve without repeated *y*-values, the period on these curves is $\frac{\pi}{|B|}$, instead of $\frac{2\pi}{|B|}$, as it is for the other four trig functions.**

Note that the curves drawn above show more than one cycle. The primary cycle is still considered to be the one starting with *h* and ending at *h* + the period, just as with the sinusoidals. Some authors (and teachers) prefer a more aesthetic graph and will start at an asymptote, regardless of *h*. This might be helpful when considering inverse functions later, but, for now, consistency with the sinusoidals will be maintained. However, if your teacher prefers the aesthetic approach, do as he or she requires.

The Reciprocal Curves:



$y = \sec x$



$y = \csc x$

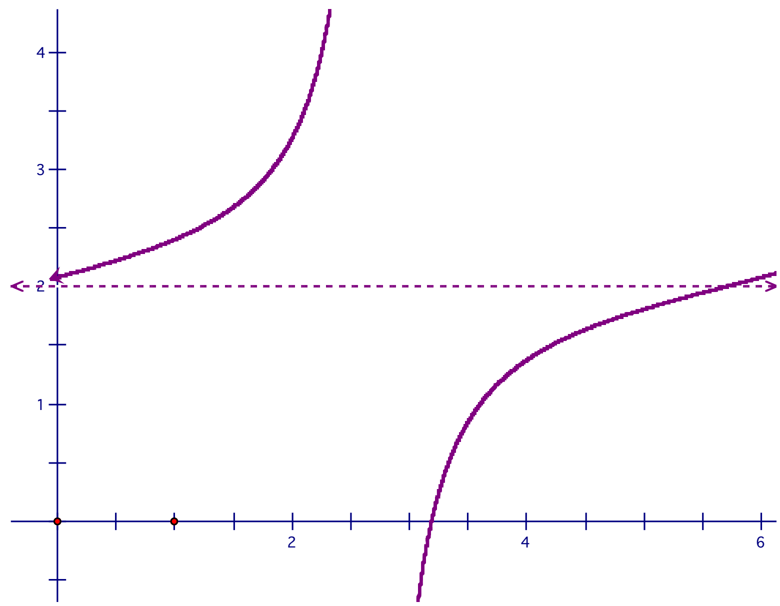
All the facts learned in a previous section about *A, B*, *h* and *k* will remain the same with the new graphs, except that the period for $y = \tan x$ and $y = \cot x$ is $\dfrac{\pi}{|B|}$.

LEARNING OUTCOME
Find the graph from the equation of tangent, cotangent, secant, and cosecant functions.

EX 1 Identify the period, amplitude, vertical shift and phase shift and sketch the primary cycle of $y = 2 + \dfrac{1}{2}\tan\left[\dfrac{\pi}{6}x\right]$.

Vertical shift $= 2$ Phase shift $= 0$

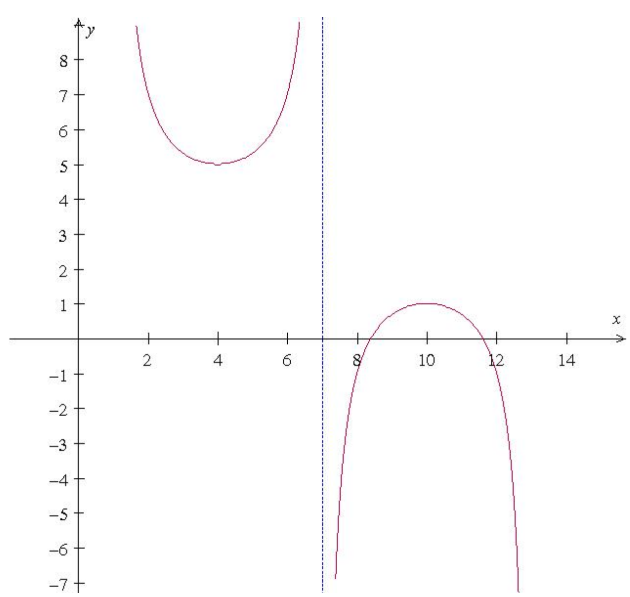
Amplitude $= \dfrac{1}{2}$ Period $= \dfrac{\pi}{\pi/6} = 6$



EX 2 Identify the period, amplitude, vertical shift and phase shift and sketch the primary cycle of $y = 3 + 2\csc\left[\dfrac{\pi}{6}(x-1)\right]$.

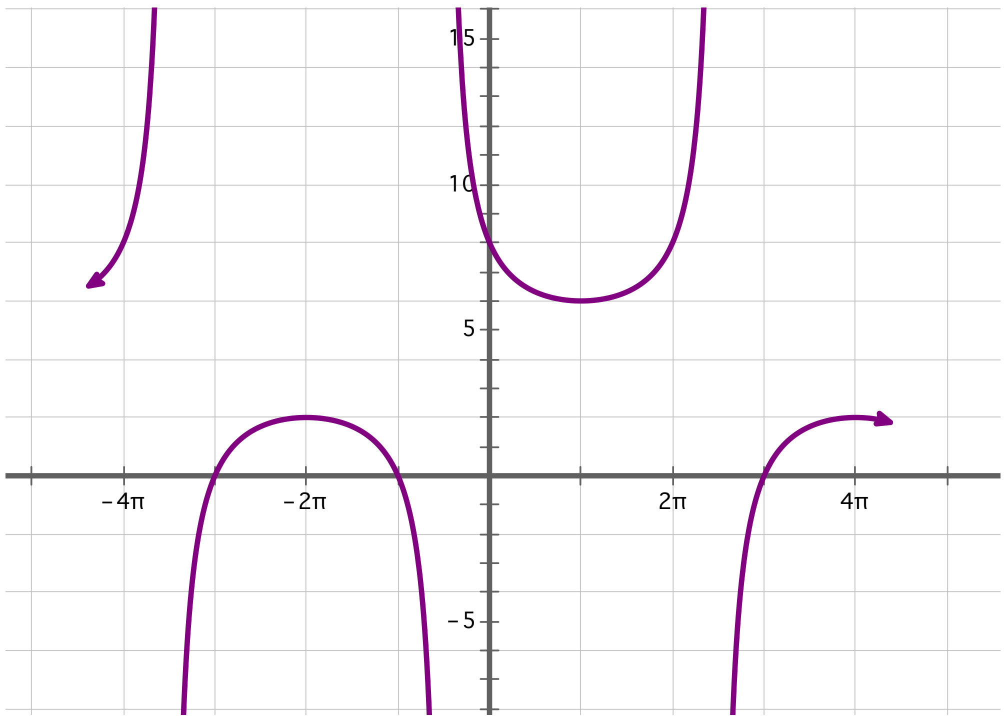
Vertical shift = 3 Horizontal shift = 1

Amplitude = 2 $\text{Period} = \dfrac{2\pi}{\pi/6} = 12$



As with sinusoidals, we should be able to reverse the process and find the equation from the graph:

EX 3 Find two cofunction equations for this graph:



$$\text{Vertical shift} = 4; \quad \text{Amplitude} = 2; \quad \text{Period} = 6\pi \rightarrow B = \frac{1}{3}$$

The *h*-value will depend on whether the equation is *sec* or *csc*:

$$y = 4 + 2\sec\left[\frac{1}{3}(x - \pi)\right] \quad \text{and} \quad y = 4 + 2\csc\left[\frac{1}{3}\left(x + \frac{\pi}{2}\right)\right]$$

2-4 Free Response Homework

Identify the period, amplitude, vertical shift and phase shift and sketch (without using a calculator) the primary cycle each of these equations.

1. $y = 2 + 3\sec\left[\frac{\pi}{6}(x-1)\right]$

2. $y = 2 + 3\csc\left[\frac{\pi}{6}(x-1)\right]$

3. $y = 2 + 3\tan\left[\frac{\pi}{4}(x-2)\right]$

4. $y = 2 + 3\cot\left[\frac{\pi}{4}(x-2)\right]$

5. $y = 1 - \csc\left[\frac{\pi}{2}(x+1)\right]$

6. $y = -3\sec\left[\frac{1}{4}(x+\pi)\right]$

7. $y = 2 + \cot\left[\frac{1}{4}(x+\pi)\right]$

8. $y = 1 - \tan\left[\frac{\pi}{8}(x+2)\right]$

9. $y = 4 - 2\sec\left[\frac{\pi}{8}(x+1)\right]$

10. $y = 3 + \csc\left[\frac{1}{3}(x-\pi)\right]$

11. $y = 4 - \frac{1}{2}\tan x$

12. $y = \cot\left[\frac{\pi}{6}(x+2)\right]$

13. $y = -2 + \cot\left[\frac{3\pi}{2}(x-1)\right]$

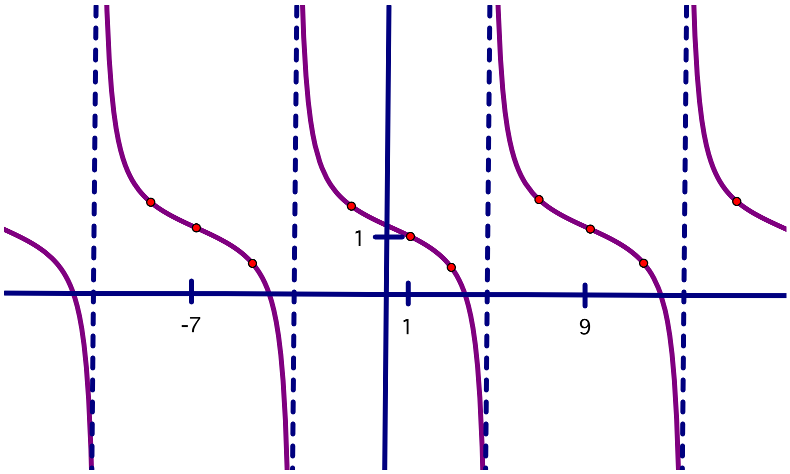
14. $y = -1 + \frac{1}{3}\tan\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right]$

15. $y = 3 + 2\csc\left[\frac{1}{5}(x+3\pi)\right]$

16. $y = -1 + 3\sec\left[\frac{1}{6}(x-\pi)\right]$

Find two cofunction equations for each of these graphs.

17.



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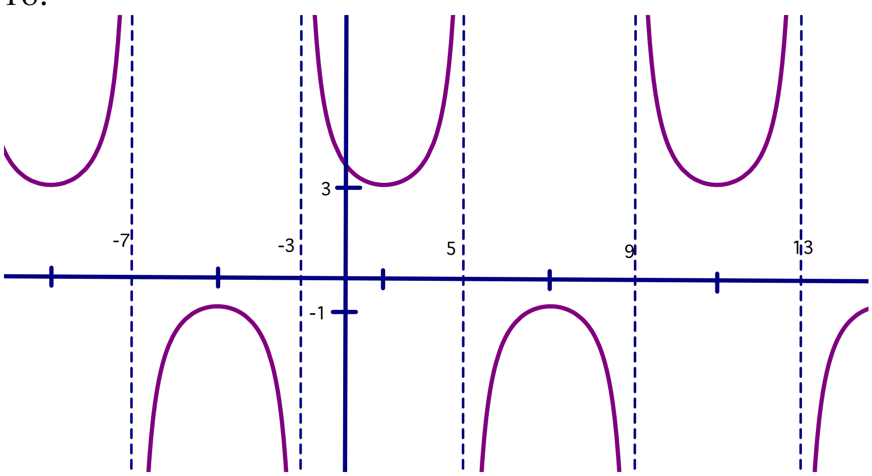

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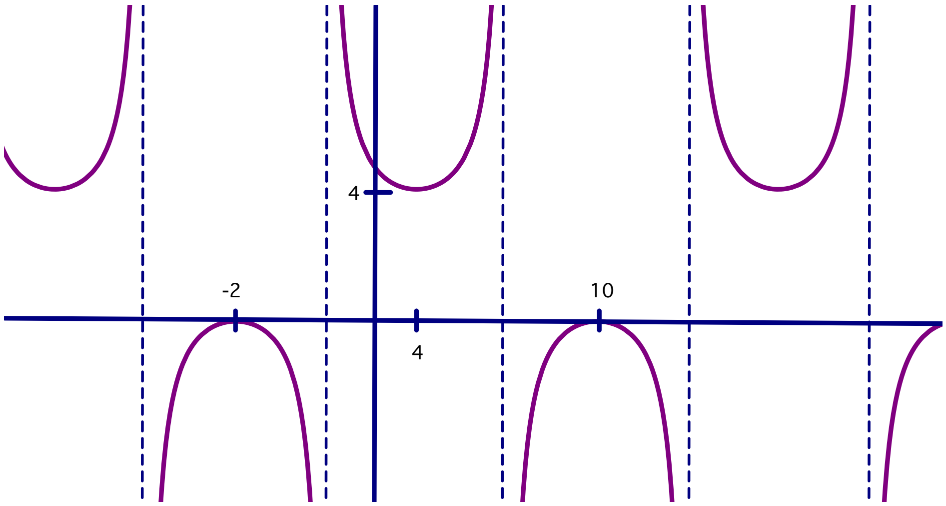
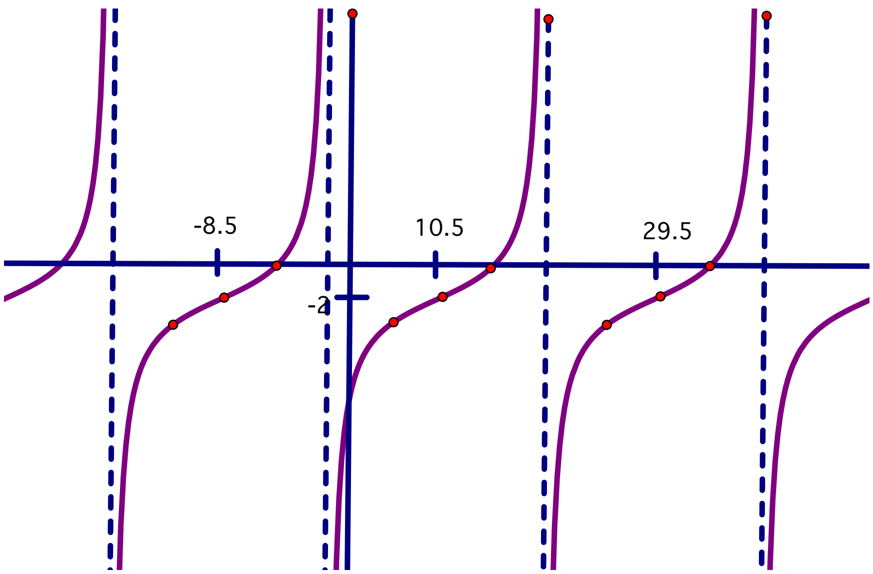


20.



<u>2-4 Multiple Choice Homework</u>

1. If $\tan x = 3$, the numerical value of $\sqrt{\csc x}$ is

a) 0.97 b) 1.03 c) 1.78 d) 3.16 e) 0.32

2. The period of the function $f(x) = k\cos kx$ is $\frac{\pi}{2}$. The amplitude of f is

a) 2 b) $\frac{1}{2}$ c) 1 d) $\frac{1}{4}$ e) 4

---

3. The graph of $f(x)$ can be obtained from the graph of $\sec x$ by applying, in order, a horizontal stretch by a factor of 2, a vertical stretch by a factor of 2, and a vertical shift down 3 units. The equation of $f(x)$ is

a) $f(x) = -3 + 2\sec 2x$

b) $f(x) = -3 + 2\sec\frac{x}{2}$

c) $f(x) = 2 - 3\sec 2x$

d) $f(x) = 2 - 3\sec\frac{x}{2}$

e) None of the above

---

4. Compared to the graph of $y = f(x)$, the graph of $y = f(x+2) - 3$ is

a) shifted 2 units right and 3 units down
b) shifted 2 units left and 3 units down
c) shifted 2 units right and 3 units up
d) shifted 2 units left and 3 units up
e) none of the above

---

5. The horizontal shift of the graph of $y = 3 + 2\cot(4x - \pi)$

a) $\pi/2$ b) π c) 2 d) 3 e) $\pi/4$

---

6. The graph of $f(x)$ can be obtained from the graph of $\sec x$ by applying, in order, a horizontal stretch by a factor of $1/2$, a vertical stretch by a factor of 2, and a vertical shift down 3 units. The equation of $f(x)$ is

a) $f(x) = -3 + 2\sec 2x$

b) $f(x) = -3 + 2\sec\dfrac{x}{2}$

c) $f(x) = 2 - 3\sec 2x$

d) $f(x) = 2 - 3\sec\dfrac{x}{2}$

e) None of the above

---

7. Given $g(x) = 2 + 3\,cot\left[\dfrac{\pi}{8}(x+1)\right]$, which of the following statements is not true?

a) The amplitude of $g(x)$ is 3.

b) The period of $g(x)$ is 8.

c) The phase shift is 1.

d) The vertical shift is 2.

---

8. Which of the following functions has both an amplitude of 2 and a period of 4π?

a) $y = 2\cos\left[\frac{1}{4}(x+0)\right]$ b) $y = 2\sin[4(x+\pi)]$

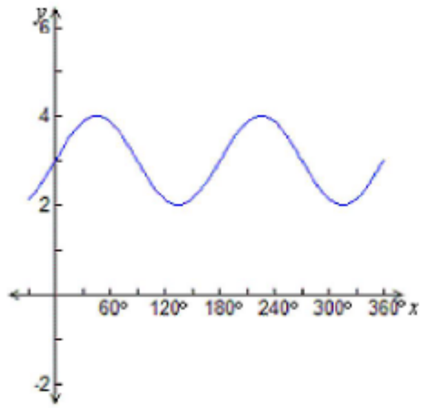
c) $y = -3 - 2\tan\left[\frac{1}{4}(x-\pi)\right]$ d) $y = 2 + sec\left[\frac{1}{2}(x-4\pi)\right]$

Parent Trigonometric Functions Practice Test
CALCULATOR REQUIRED

Round to 3 decimal places. Show all work.

Multiple Choice (3 pts. each)

1. Which equation can be used to model the graph of the function shown?



(a) $y = \sin 2x + 3$
(b) $y = 2\sin x + 3$
(c) $y = \sin 2x - 3$
(d) $y = 2\sin x - 3$
(e) $y = 2\sin 2x + 3$

2. The y-intercept of $y = \left|\sqrt{2}\csc 3\left(x + \frac{\pi}{5}\right)\right|$ is

(a) 0.22
(b) 1.49
(c) 4.58
(d) 0.67
(e) 1.41

3. The interval(s) of decreasing for the sine function in the interval from 0 to 2π is (are)

(a) $x \in (\pi, 2\pi)$
(b) $x \in (0, \pi)$
(c) $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
(d) $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$
(e) $x \in (0, \pi) \cup (\pi, 2\pi)$

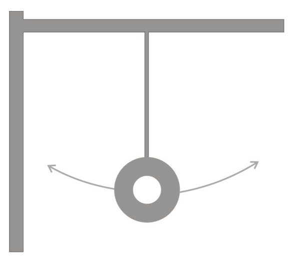
4. During one cycle, a sinusoid has a minimum at $(18,44)$ and a maximum at $(30,68)$. What is the amplitude of this sinusoid?

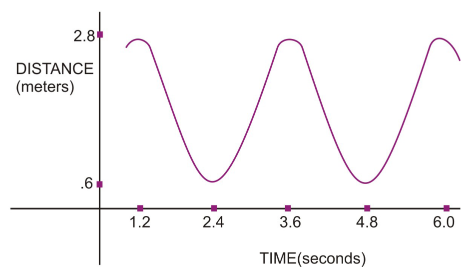
(a) 6
(b) 12
(c) 22
(d) 48
(e) 56

5. In the following diagram, an old car tire is swinging back and forth on a rope. The following graph compares its distance from the vertical wall and the time. Using the diagram answer the following question:

How long does it take for the tire to swing from its closest distance to the wall to its farthest distance from the wall?

(a) 1.2 seconds
(b) 1.4 seconds
(c) 2.4 seconds
(d) 2.8 seconds
(e) 3.2 seconds





Free Response (10 pts. each)
Identify *k*, *A*, *period*, and *h*. Label the 5 coordinates.

1. Sketch the primary cycle of $y = -2 + 3\sin\left[\frac{\pi}{7}(x-1)\right]$.

2. Sketch the primary cycle of $y = 2 - 3\sec\left[\frac{1}{7}(x+\pi)\right]$.

3. Sketch the primary cycle of $y = -2 - 3\cot\left[\frac{\pi}{7}(x-1)\right]$.

4. In front of the Antique Sewing Machine Museum in Arlington, Texas, is the largest sewing machine in the world. The *flywheel*, which turns as the machine sews, is 5 feet in diameter. Write a model for the height h (in feet) of the handle on the flywheel as a function of the time t (in seconds), assuming that the wheel makes a complete revolution every 2 seconds and that the handle starts at its minimum height of 4 feet above the ground.

a) Sketch a model for the height h (in feet) of the handle on the flywheel as a function of time t (in seconds).

b) Find an equation for the graph you sketched in part a).

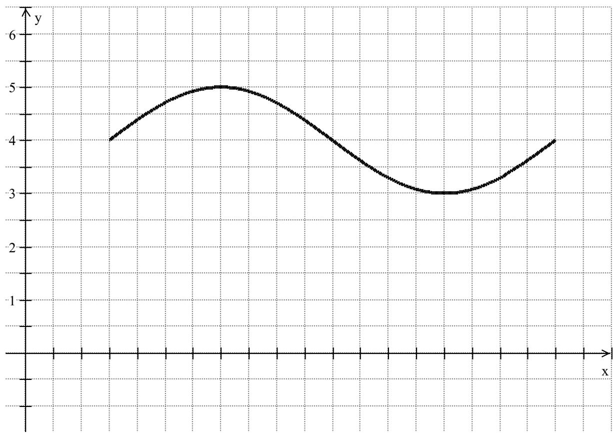
c) What is the height of the handle of the flywheel at 7 seconds?

d) What are the first four times the height of the handle of the flywheel is 6 ft?

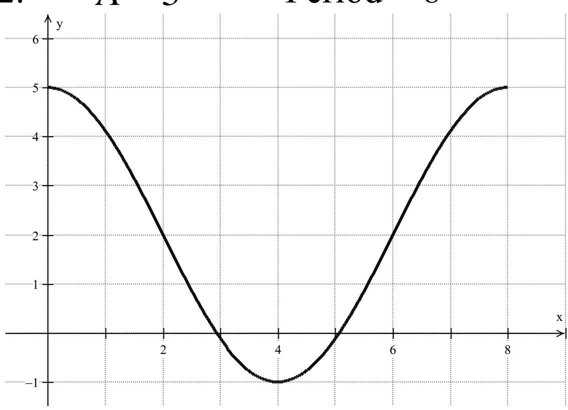
Parent Trigonometric Functions Homework Answer Key

2-1 Free Response Homework

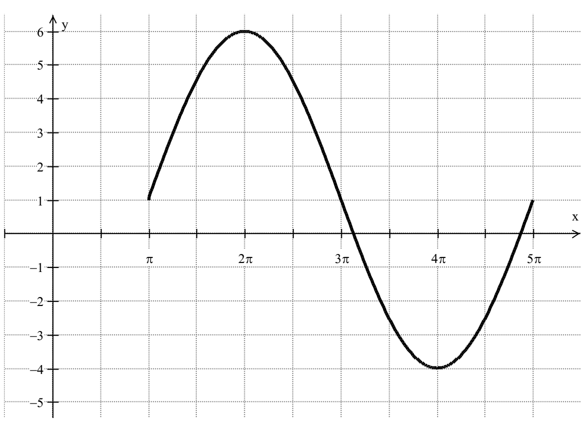
1. $A = 1$ Period = 16 $k = 4$ $h = 3$



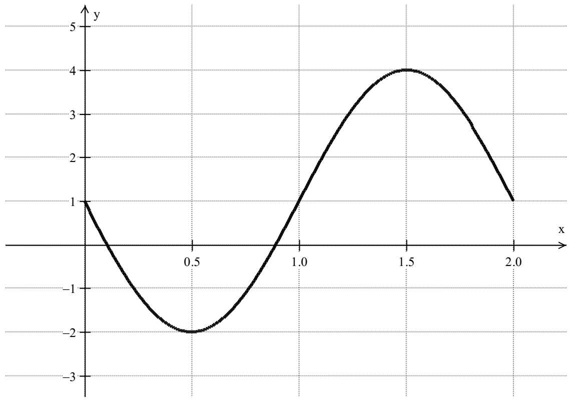
2. $A = 3$ Period = 8 $k = 2$ $h = 0$



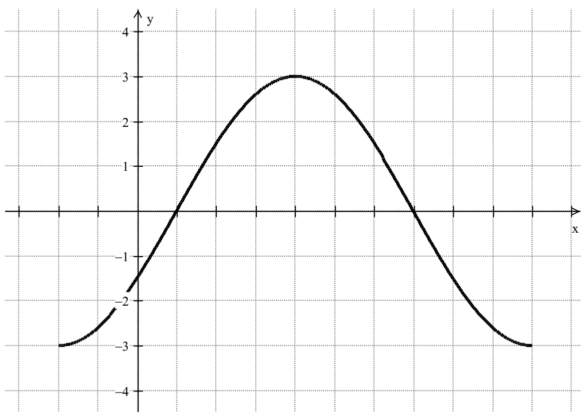
3. $A = 5$ Period = 4π $k = 1$ $h = \pi$



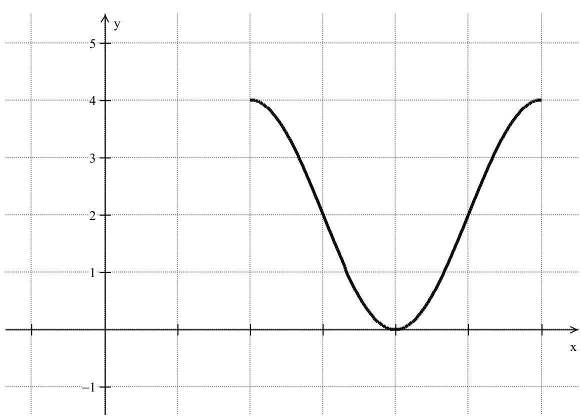
4. $A=3$ Period = 2 $k=1$ $h=0$



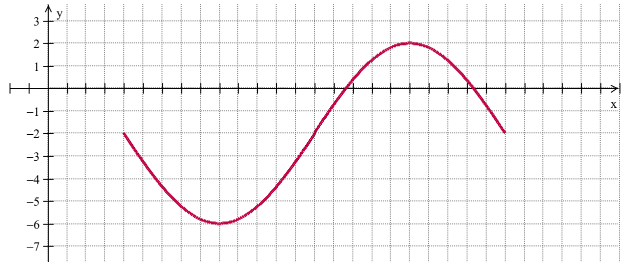
5. $A=3$ Period = 12 $k=0$ $h=-2$



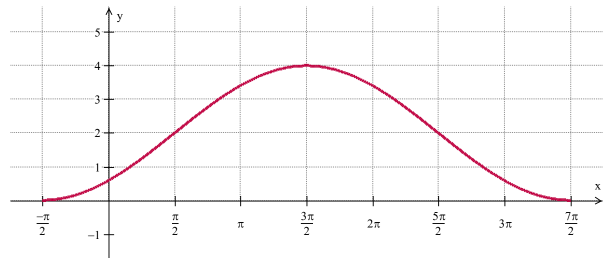
6. $A=1$ Period = 4 $k=2$ $h=2$



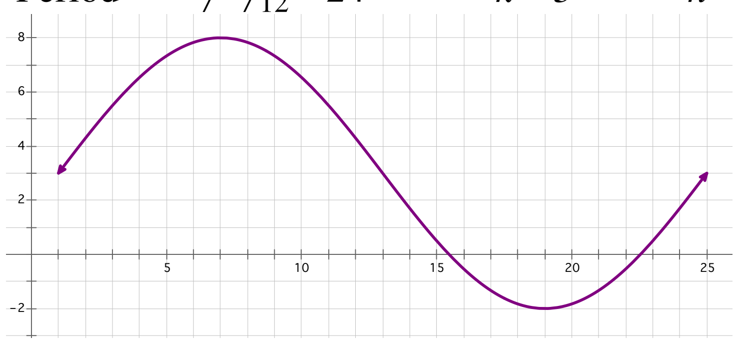
7. $A = 4$ Period = 10 $k = -2$ $h = 2$



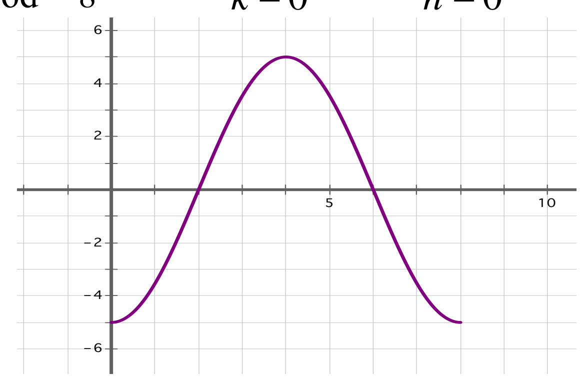
8. $A = -2$ Period = 4π $k = 2$ $h = -\frac{\pi}{2}$



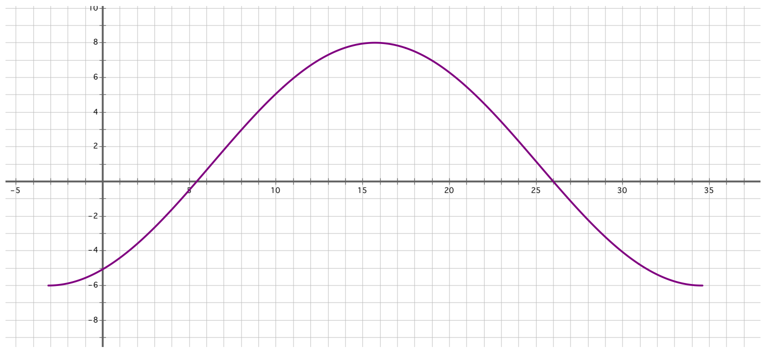
9. $A = 3$ Period $= 2\pi / \pi/_{12} = 24$ $k = 3$ $h = 1$



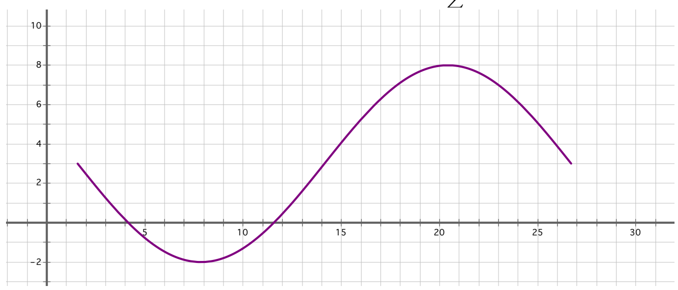
10. $A = 3$ Period = 8 $k = 0$ $h = 0$



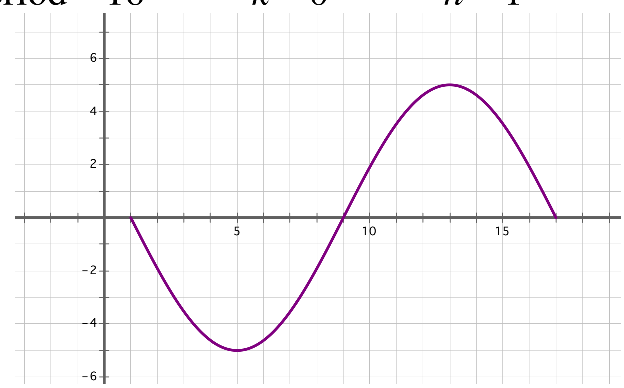
11. $A = 7$ Period $= 12\pi$ $k = 1$ $h = -\pi$



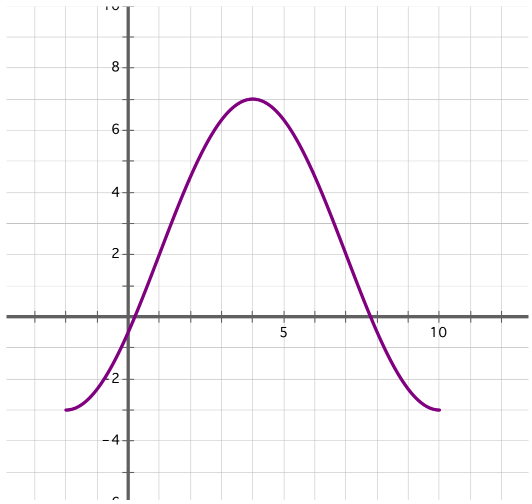
12. $A = 5$ $per = 8\pi$ $k = 3$ $h = \dfrac{\pi}{2}$



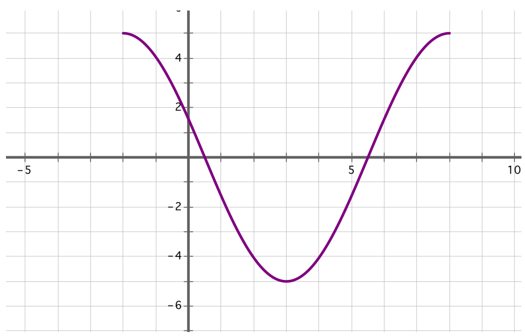
13. $A = 2$ Period $= 16$ $k = 0$ $h = 1$



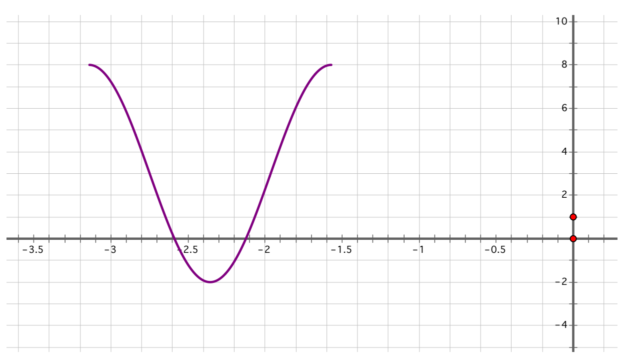
14. $A = 5$ Period = 12 $k = 2$ $h = -2$



15. $A = 5$ Period $= 10$ $k = 0$ $h = -2$



16. $A = 5$ $per = \dfrac{\pi}{2}$ $k = 3$ $h = -\pi$



17a. True b. False c. False d. True

18. $y = 2 + 3\cos\frac{\pi}{4}(x-4)$ $y = 2 + 3\sin\frac{\pi}{4}(x-2)$

19. $y = 2 + \cos\frac{1}{4}(x-3\pi)$ $y = 2 + \sin\frac{1}{4}(x-\pi)$

20. $y = 4\cos\frac{\pi}{2}(x-3)$ $y = 4\sin\frac{\pi}{2}(x-2)$

21. $y = -2 + 4\cos\frac{2}{3}(x-2\pi)$ $y = -2 + 4\sin\frac{2}{3}\left(x - \frac{5\pi}{4}\right)$

22. $y = -2 + 2\cos 2\pi\left(x - \frac{1}{2}\right)$ $y = -2 + 2\sin 2\pi\left(x - \frac{1}{4}\right)$

23. $y = -2 + \cos\frac{\pi}{7}(x-6)$ $y = -2 + \sin\frac{\pi}{7}\left(x - \frac{5}{2}\right)$

24. $y = 45 - 35\cos\frac{\pi}{90}(x+15)$ $y = 45 + 35\sin\frac{\pi}{90}(x-30)$

25. $h = 246 + 197\cos\left(\frac{\pi}{15}(t-10)\right)$ $h = 246 + 197\sin\left(\frac{\pi}{15}(t-3.5)\right)$

Multiple Choice Homework

1.	E	2.	A	3.	B	4.	E
5.	B	6.	B	7.	B	8.	B

2-2 Free Response Homework

1a. 0.433 b. 2.929, 5.071, 10.929

2a. 2.433 b. No solutions

3a. –0.095 b. 2.739, 9.827, 15.305

4a.	4.027	b.	7.612, 14.379, 26.461
5a.	–1.763	b.	–6, –12, –18, –24
6a.	2.951	b.	No solutions
7a.	1.349	b.	–3.025, –14.975, –19.025, – 30.975
8a.	–0.763	b.	–1.108, –1.892, –3.108, –3.892

9. $x \in \{6.863,\ 11.987,\ 19.429,\ 24.553\}$

10. $x \in \{8.357, 17.643\}$

11. $x \in \{-0.327,\ -1.007,\ -2.993,\ -3.673\}$

12. $x \in \{1.426,\ 2.574,\ 7.426,\ 8.574\}$

13. $x \in \{6.325,\ 25.093,\ 31.458,\ 50.225\}$

14. $x \in \{11.706,\ 16.294\}$

15. $x \in \{-29.706,\ -48.836,\ -79.972,\ -99.102\}$

16. $x \in \{-14.219,\ -9.781,\ 5.781,\ 10.219\}$

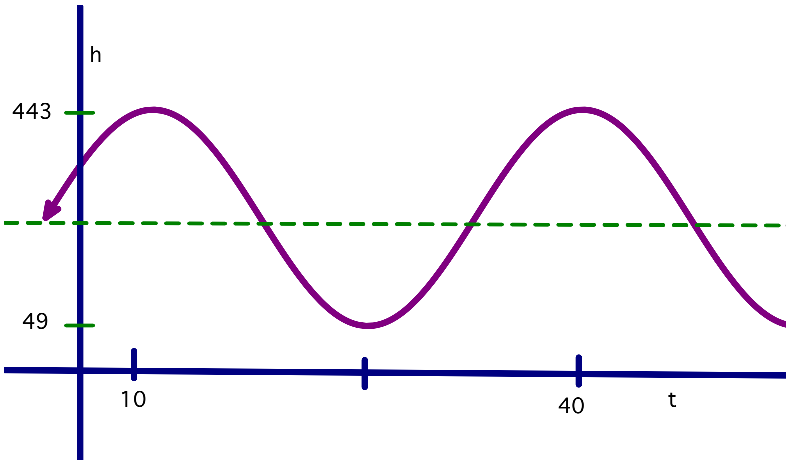
2-2 Multiple Choice Homework

1. C 2. D 3. C 4. D 5. A

2-3 Free Response Homework

1. The London Eye

a.



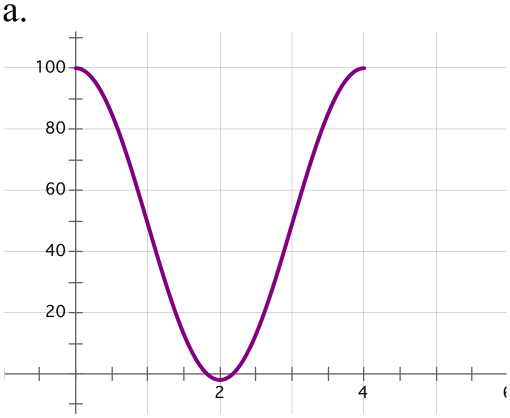
b. $h = 246 + 197\cos\left(\frac{\pi}{15}(t-10)\right)$

c. $h(0) = 147.5 feet$

d. 21.486, 28.514, 51.486 minutes

2. The Mythbusters Bobbing for Apples

a.



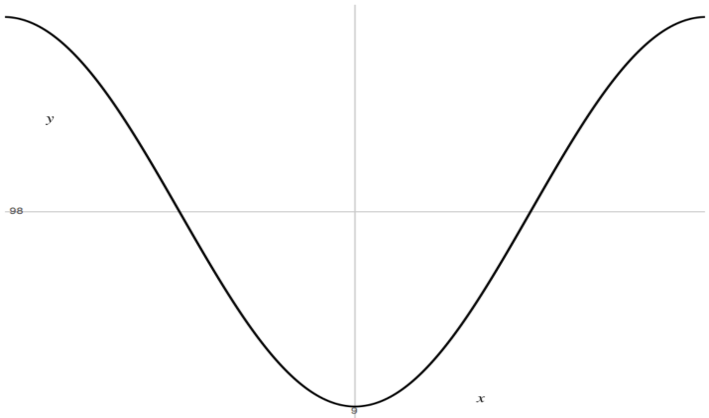
b. $h = 49 + 51\cos\left(\frac{\pi}{2}(t-0)\right)$

c. $h = 19.023\ ft$

d. $t = 2.179\ \text{sec}$

3. The Meditation Problem

a. $y = 98 + 30\cos\left(\frac{\pi}{4}(t-6)\right)$



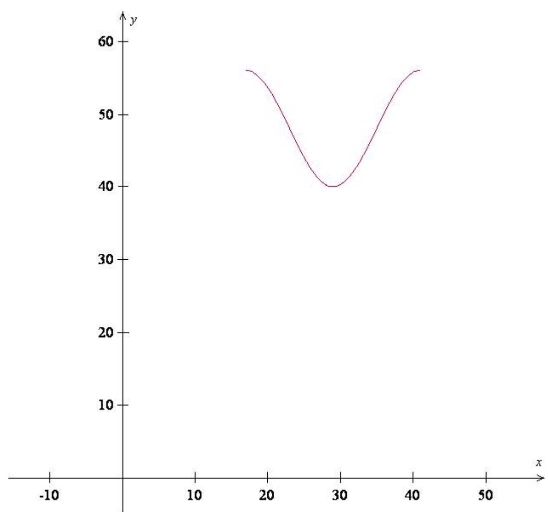
b. $t = 0$ seconds $y = 98\text{ cm}^3$ $t = 3$ seconds $y = 76.787\text{ cm}^3$

$t = 10$ seconds $y = 68\text{ cm}^3$

c. Highest at $t = 6, 14, 22$ seconds

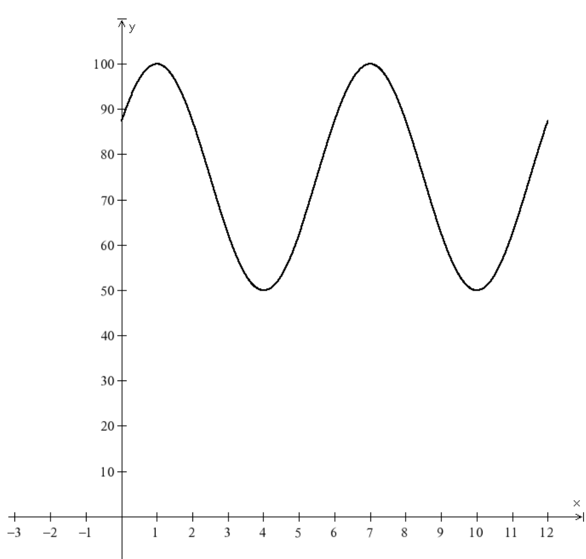
4. The Chemistry Experiment

a.



and the equation is $y = 48 + 8\cos\left[\dfrac{\pi}{12}(t-17)\right]$.

b. $y(0) = 45.929$



c. $t = \{12.468, 21.532, 36.468\}$

5. The Teacher Attention Problem

a.

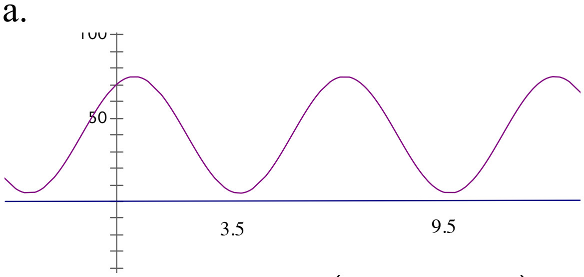
b. $y = 75 + 25\cos\left[\frac{\pi}{3}(t-1)\right]$

c. 87.5 students

d. $t = 2.107$ and 5.893

6. The HWY 280 Problem

a.



b. $s = 40 - 35\cos\left(\frac{\pi}{3}(d-3.5)\right)$

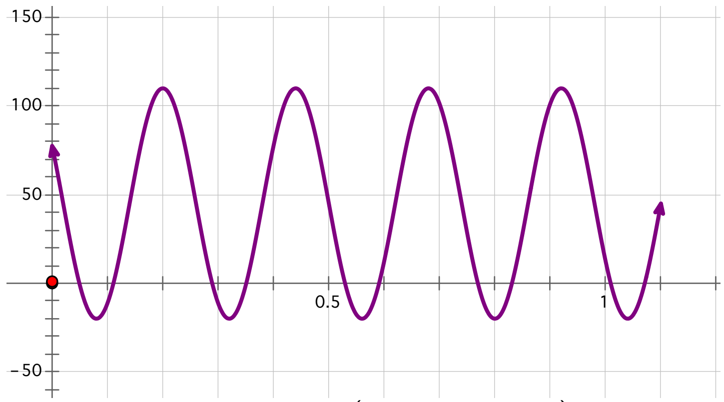
c. $s(7.2) = 66.010\text{mph}$

d. 11.581 and 13.419 miles

7. The Trampoline Problem

If you decide the trampoline surface is zero:

a.

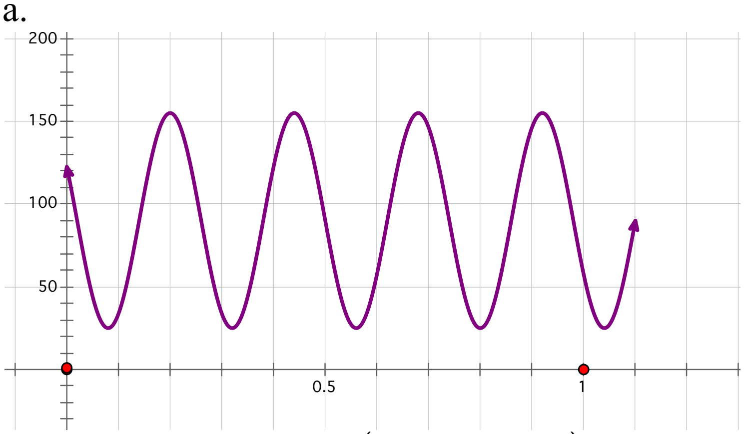


b. $h(t) = 45 + 65\cos\left(\frac{25\pi}{3}(t - 0.2)\right)$

c. $h = 55$ cm $\rightarrow t = 0.019, 0.222, 0.259, 0.462$secs

If you decide the trampoline surface is zero:

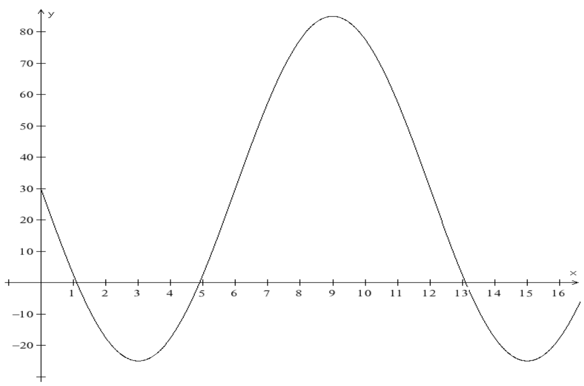
a.



b. $h(t) = 90 + 65\cos\left(\frac{25\pi}{3}(t - 0.2)\right)$

c. $h = 100$ cm $\rightarrow t = 0.019, 0.222, 0.259, 0.462$secs

8. The Tsunmai Problem

a. 

b. $y = -55\sin\left[\frac{\pi}{6}t\right]$ or $y = -55\cos\left[\frac{\pi}{6}(t-3)\right]$

c. 4 minutes 11.9 seconds

9. 143.801 mmHg; 2:21 am, 9:39 am, and 2:21 pm

10. Day 301

11. $t = 114.798$ sec

12. 10.229 minutes

13. Game 2, Game 18, and Game 56

14. $2003 < t < 2015$

15. 4:24am, 7:36am, 4:24pm, and 7:36pm.

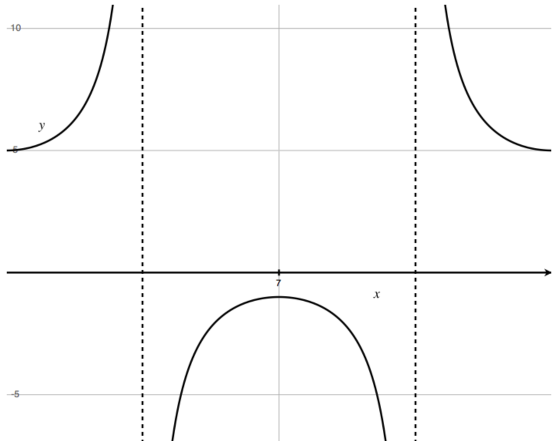
16. $t = 14.044$or 15.956 sec

2-3 Multiple Choice Homework

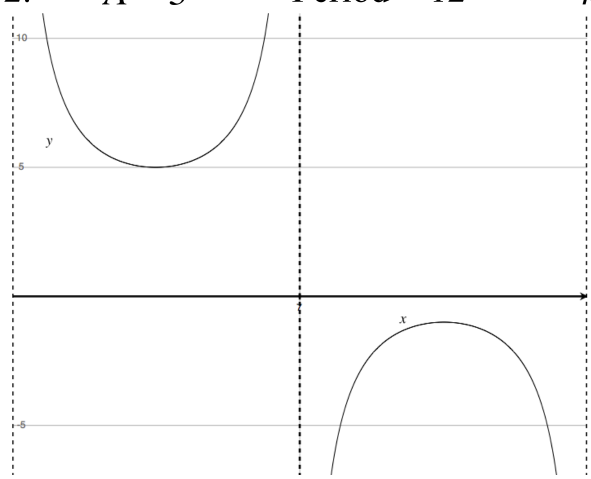
1. D 2. A 3. B 4. B 5. D

2-4 Free Response Homework

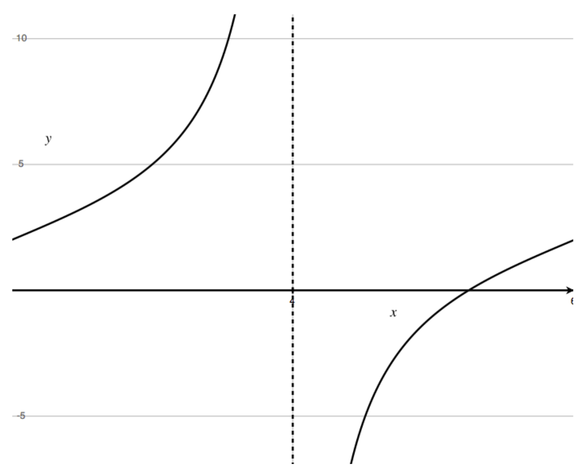
1. $A = 3$ Period = 12 $k = 2$ $h = 1$



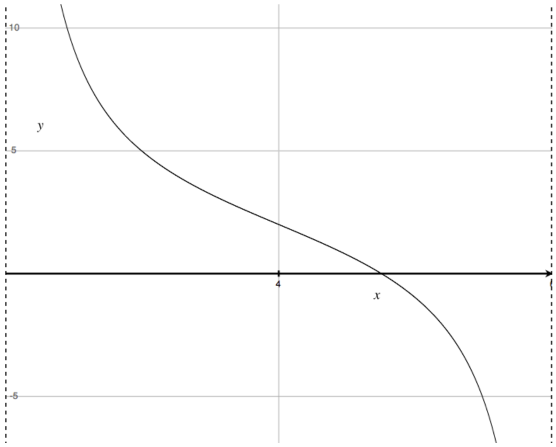
2. $A = 3$ Period = 12 $k = 2$ $h = 1$



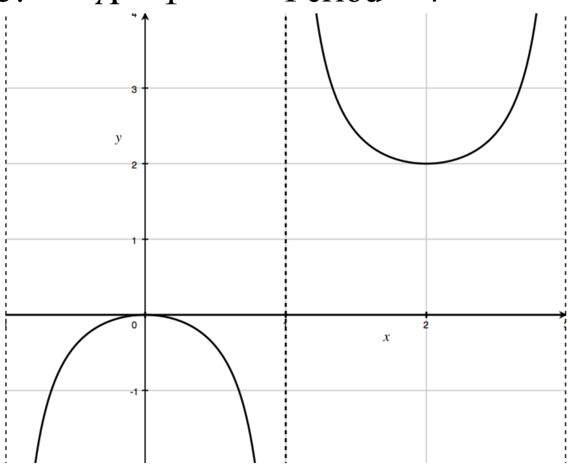
3. $A = 3$ Period = 4 $k = 2$ $h = 2$



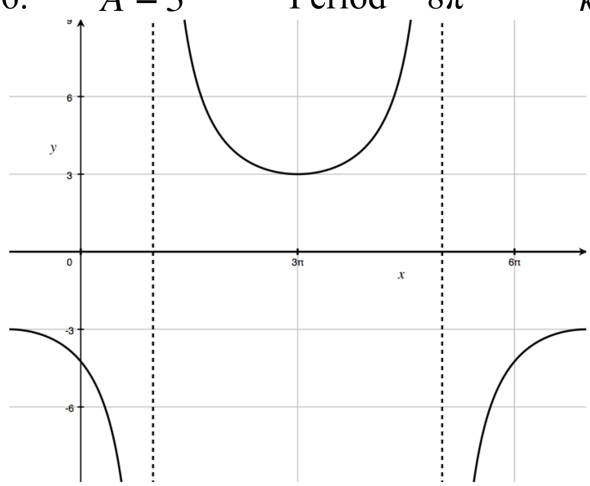
4. $A = 3$ Period = 4 $k = 2$ $h = 2$



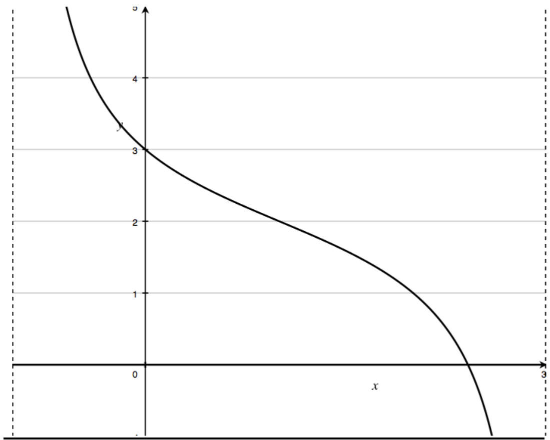
5. $A = 1$ Period = 4 $k = 1$ $h = -1$



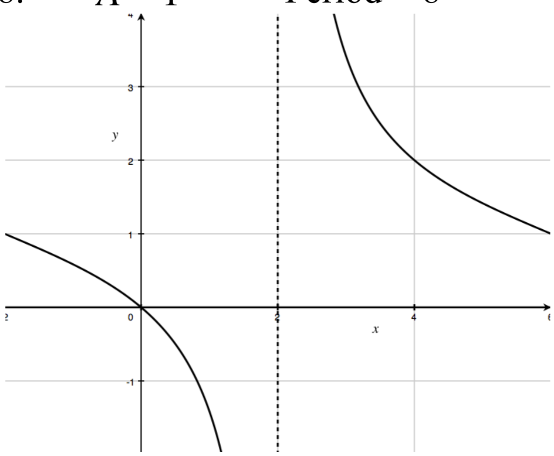
6. $A = 3$ Period = 8π $k = 0$ $h = -\pi$



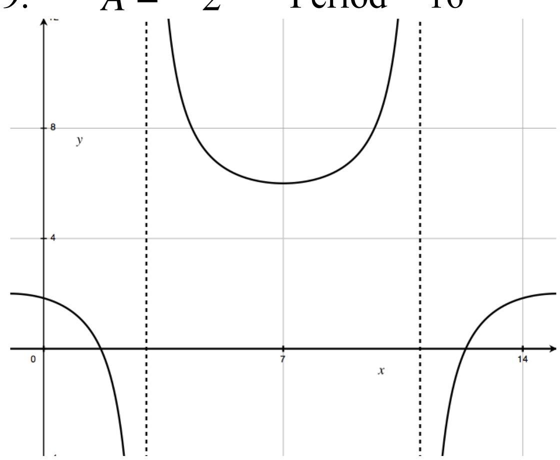
7. $A = 1$ Period = 4π $k = 2$ $h = -\pi$



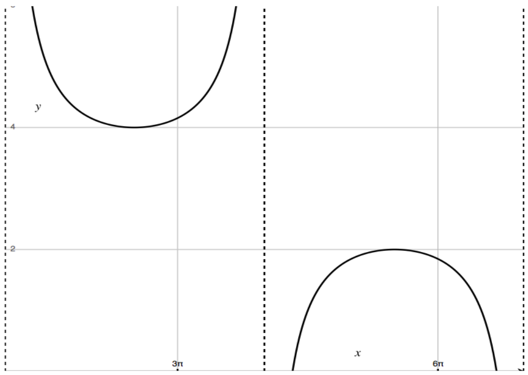
8. $A = 1$ Period = 8 $k = 1$ $h = -2$



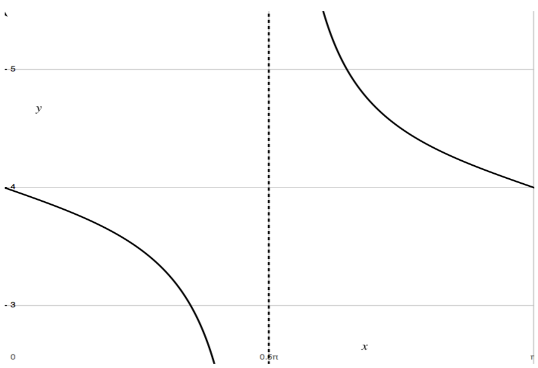
9. $A = -2$ Period = 16 $k = 4$ $h = -1$



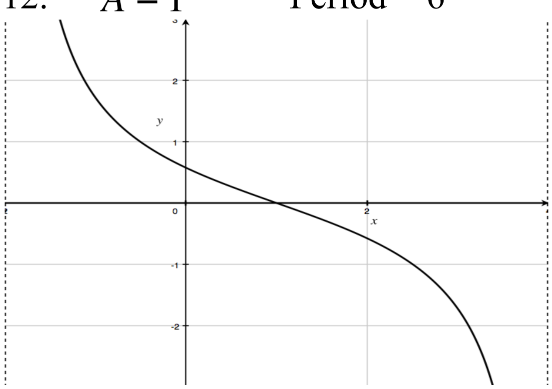
10. $A = 1$ Period = 6π $k = 3$ $h = \pi$



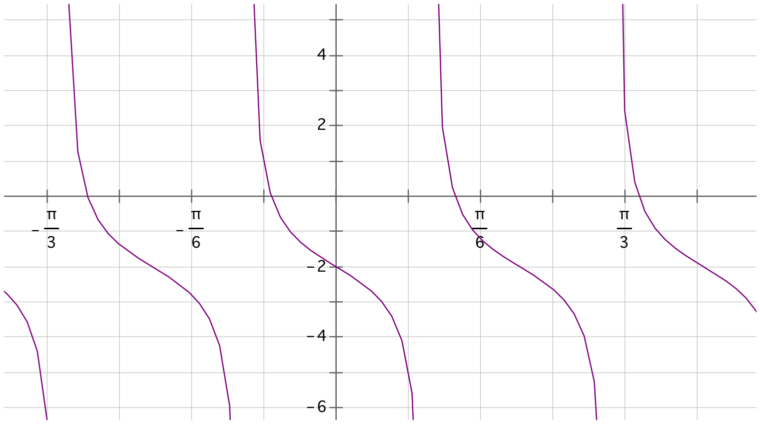
11. $A = -\frac{1}{2}$ Period = π $k = 4$ $h = 0$



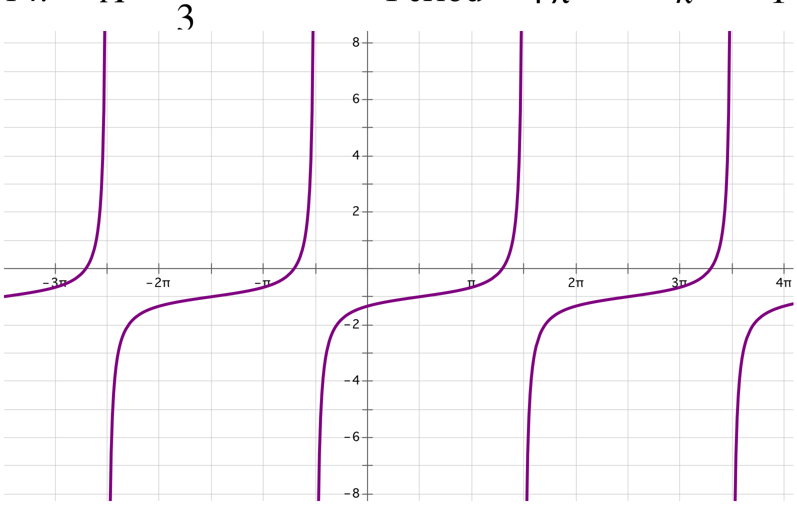
12. $A = 1$ Period = 6 $k = 0$ $h = -2$



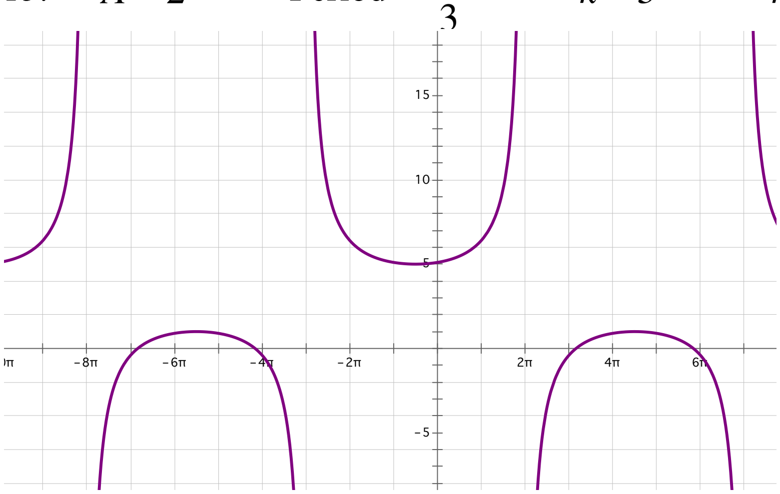
13. $A = 1$ Period = $\frac{4}{3}$ $k = -2$ $h = 1$



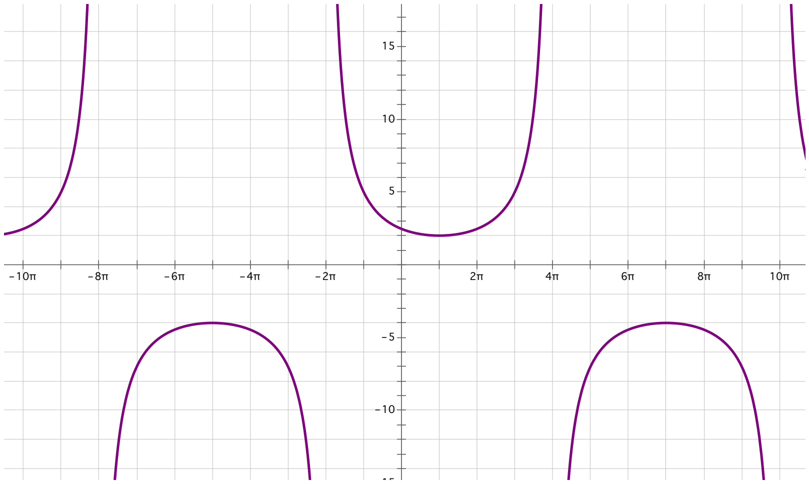
14. $A = \frac{1}{3}$ Period $= 4\pi$ $k = -1$ $h = \frac{\pi}{2}$



15. $A = 2$ Period $= \frac{4}{3}$ $k = 3$ $h = -3\pi$



16. $A = 3$ Period $= 12\pi$ $k = -1$ $h = \pi$



17. $y = 1 - \dfrac{1}{2}\tan\left[\dfrac{\pi}{8}(t-1)\right]$ and $y = 1 + \dfrac{1}{2}\cot\left[\dfrac{\pi}{8}(t+1)\right]$

18. $y = 1 + 2\sec\left[\dfrac{\pi}{8}(t-9)\right]$ and $y = 1 + 2\csc\left[\dfrac{\pi}{8}(t-5)\right]$

19. $y = 2 + 2\sec\left[\dfrac{\pi}{6}(t-1)\right]$ and $y = 2 + 2\csc\left[\dfrac{\pi}{6}(t+2)\right]$

20. $y = -2 + 2\tan\left[\dfrac{2\pi}{19}(t-10.5)\right]$ and $y = -2 - 2\cot\left[\dfrac{2\pi}{19}(t-1)\right]$

2-4 Multiple Choice Homework

1. B 2. E 3. B 4. B 5. E 6. A

7. C 8. C

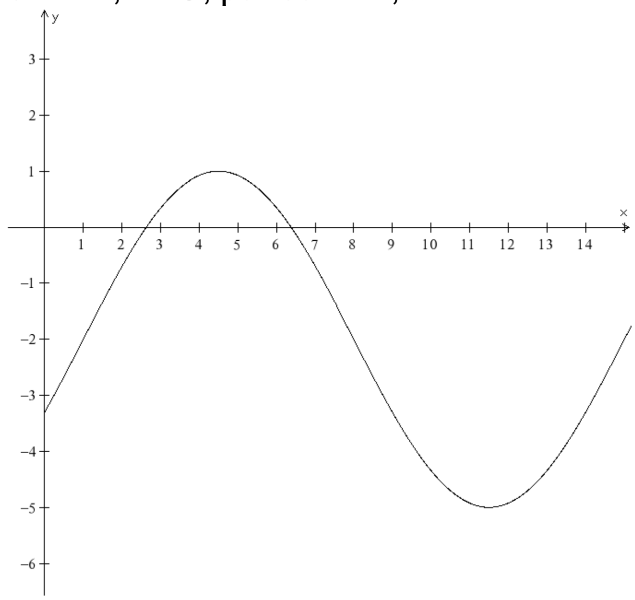
Trigonometric Functions Practice Test Answer Key

Multiple Choice

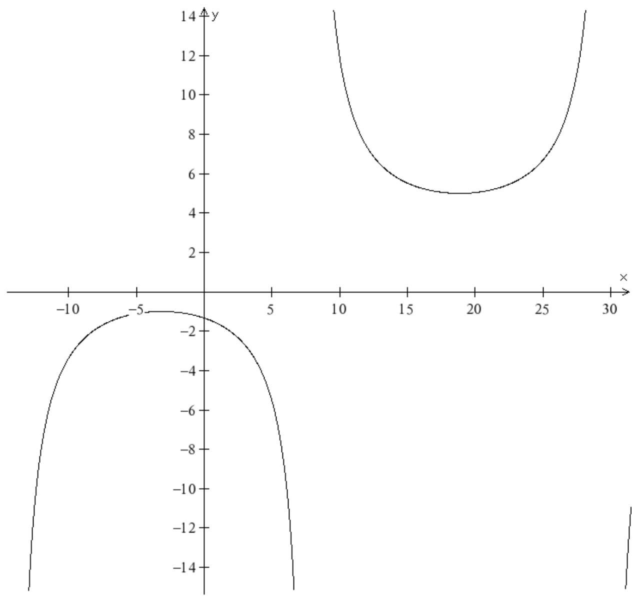
1. A 2. B 3. C 4. B 5. A 6. A

Free Response

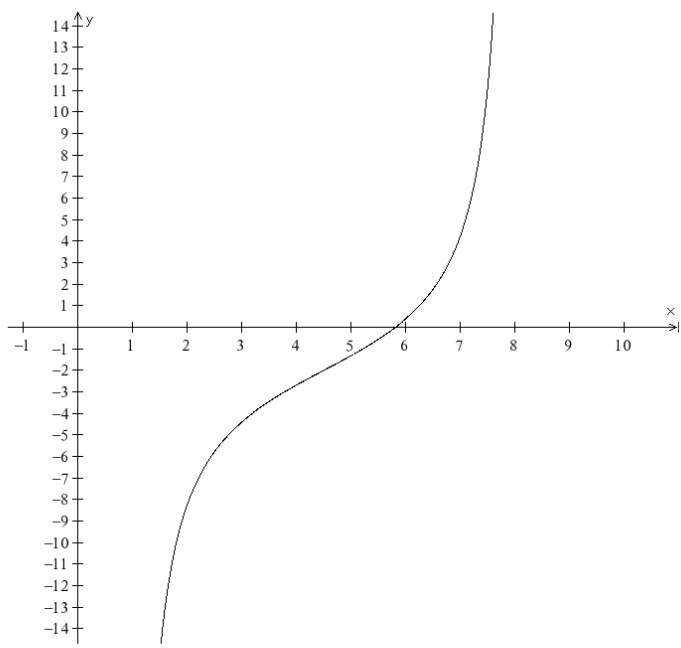
1. $k = -2, A = 3, \text{period} = 14, h = 1$



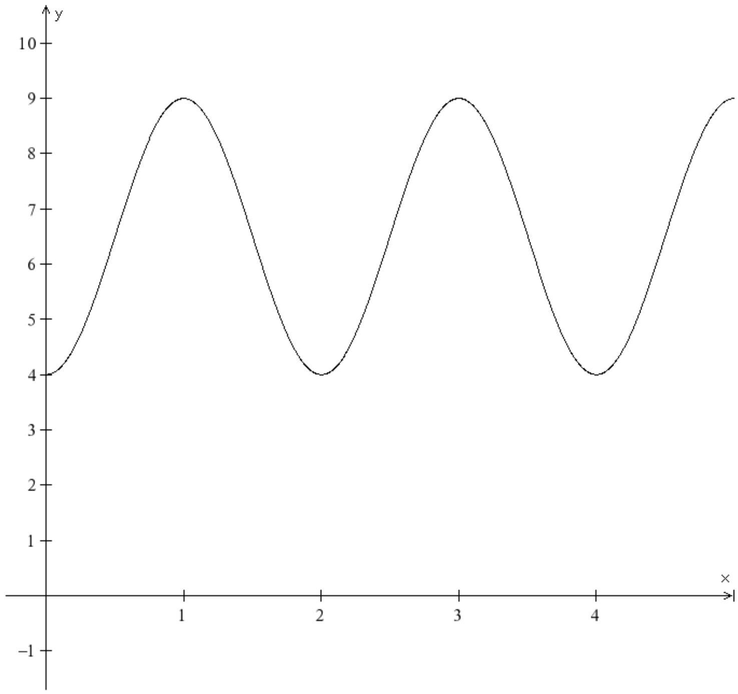
2. $k = 2, A = 3$, period $= 14\pi, h = -\pi$



3. $k = -2, A = 3$, period $= 7, h = 1$



4a.



b. $h = 6.5 - 2.5\cos[\pi(t - 0)]$

c. 9 feet

d. $t = 0.436, 1.537, 2.463,\ 3.537$ sec