Chapter 8:

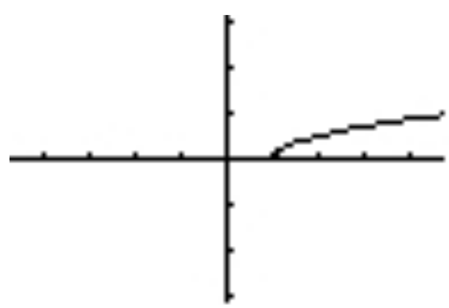
Radical Functions

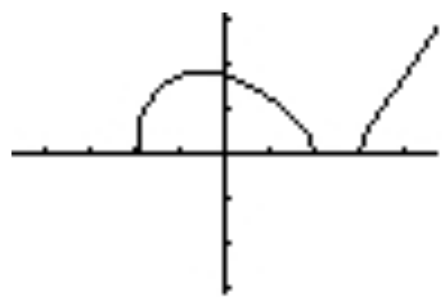
Chapter 8 Overview: Types and Traits of Radical Functions

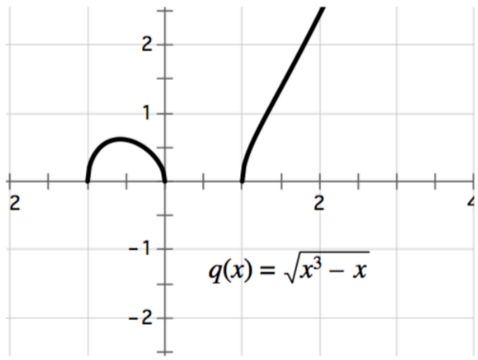
Vocabulary:
1. ***Radical (Irrational) Function*** – an expression whose general equation contains a root of a variable and possibly addition, subtraction, multiplication, and/or division

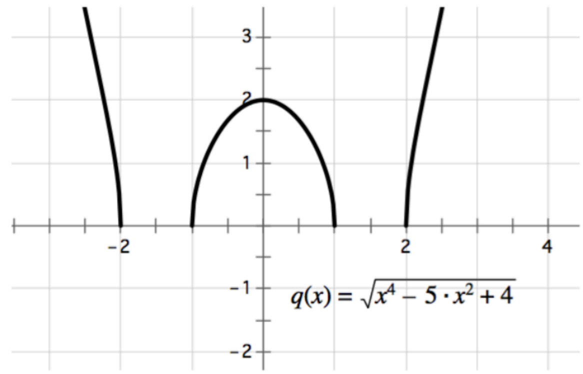
Basically, that means an equation with an *x* in a radical. There are two different kinds of irrational functions—those with an odd index, like $y = \sqrt[3]{a_n x^n + ... + a_0}$:

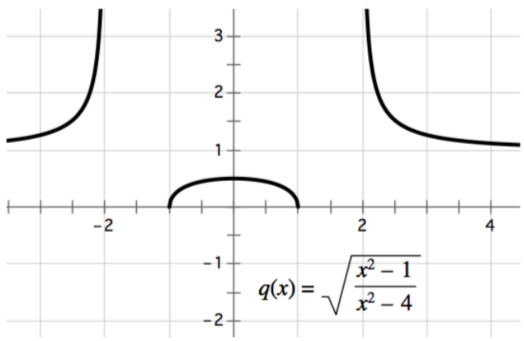
and those with an even index, like $y = \sqrt{a_n x^n + ... + a_0}$:

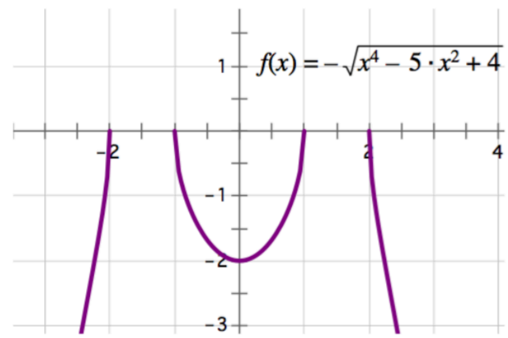












Note that the graphs of the functions with the even index given by a calculator look like they do not touch the x-axis. **Radical functions rarely graph perfectly on a calculator because of the window settings and the numerical values of the traits.** Know what to expect from the graph and realize what is correct rather than just trusting the calculator.

The traits that are common to all radical functions are:

1. Domain
2. Range
3. Zeros
4. *y*-intercepts
5. Extreme Points
6. End Behavior Asymptotes

If the radical has a fraction in it, the traits of a rational function would be included:

7. Vertical Asymptotes
8. Points of Exclusion (POEs)

8-1: Zeros and Domain

> REMEMBER: The only things that can change the domain from all real numbers are:
>
> 1. a zero denominator,
> 2. a negative under an even radical, or
> 3. a non-positive in a log.

This section will concentrate on even radicals. The number that indicates the kind of radical—2 for square root, 3 for cube root, etc.—is known as the **index**.

Domain for Radical Functions:
Odd index – All Reals
Even index – the radicand must be non-negative

> REMEMBER: Uses of Sign Patterns
>
> 1. y sign pattern = above or below x-axis,
> 2. *Velocity* sign pattern = moving right or left, and
> 3. $\frac{dy}{dx}$ sign pattern = increasing or decreasing.

Now we will be adding the sign pattern of the radicand. The radicand being non-negative implies an inequality for the domain. In other words, the domain is found by setting the radicand ≥ 0. And because of the = under the inequality, zeros will be found at the same time as the domain.

LEARNING OUTCOMES

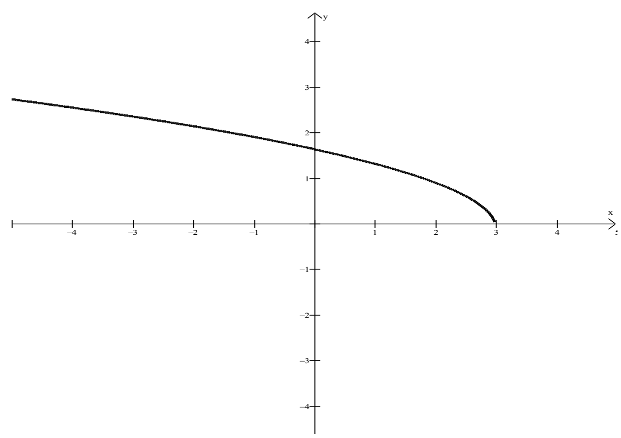
Use sign patterns to determine the domain of radical functions.
Find zeros of radical functions.

EX 1 Find the zeros and domain of $y = \sqrt{3-x}$.

$3 - x \geq 0$
$3 \geq x$ is the domain and
$(3,\ 0)$ is the zero

$3-x$	+	0	−
x		3	

The graph shows a curve only to the left of 3:



$y = \sqrt{3-x}$

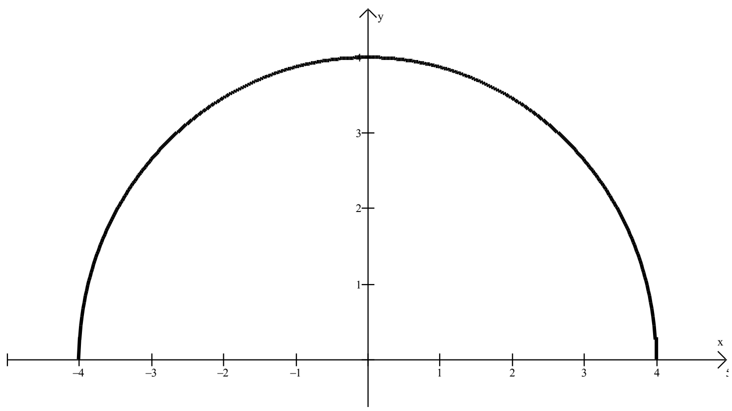
EX 2 Find the zeros and domain of $y = \sqrt{16-x^2}$.

$16 - x^2 \geq 0$
$(4-x)(4+x) \geq 0$

$16-x^2$	−	0	+	0	−
x		-4		4	

So the zeros are $(\pm 4,\ 0)$ and the domain is $x \in [-4,\ 4]$.

The graph shows a curve only between –4 and 4:



$y = \sqrt{16 - x^2}$

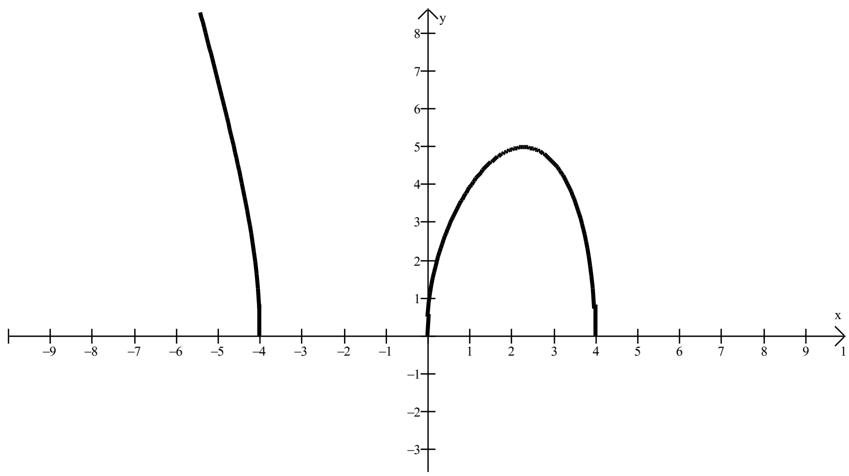
Notice how the curve looks like it does not touch the *x*-axis. But it must because zeros were found. As the technology gets better, this may improve, but for the present, recognize the limitations of the calculator.

EX 3 Find the zeros and domain of $y=\sqrt{16x-x^3}$.

$$16x-x^3 \geq 0$$
$$x(4-x)(4+x) \geq 0$$

$16x-x^3$	+	0	−	0	+	0	−
x		−4		0		4	

So the zeros are $(\pm 4, 0)$ and $(0, 0)$, and the domain is $x \in (-\infty, -4] \cup [0, 4]$.



$y=\sqrt{16x-x^3}$

EX 4 Find the zeros and domain of $y=\sqrt{\dfrac{x-1}{x+2}}$.

$$\frac{x-1}{x+2} \geq 0$$

y^2	+	DNE	−	0	+
x		−2		1	

So the zero is $(1, 0)$, and the domain is $x \in (-\infty, -2) \cup [1, \infty)$.

Note that –2 would be a vertical asymptote, not a zero; therefore, it is not included in the domain.

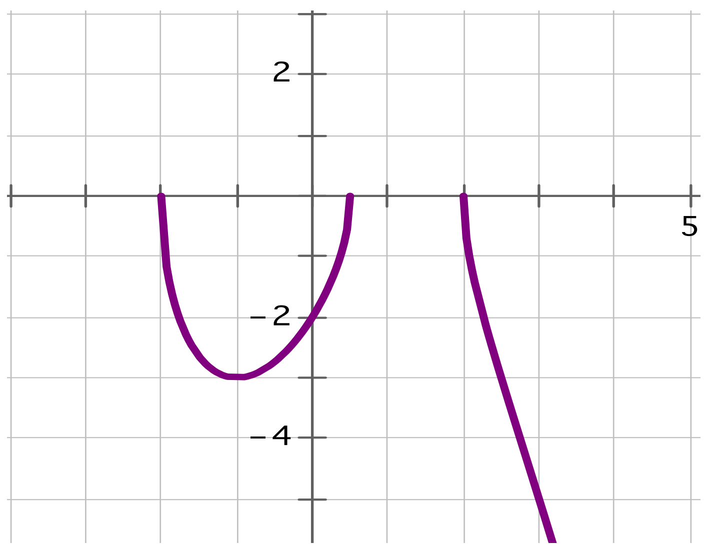
EX 5 Find the zeros and domain of $y=-\sqrt{2x^3-x^2-8x+4}$.

$$2x^3-x^2-8x+4\geq 0$$
$$x^2(2x-1)-4(2x-1)\geq 0$$
$$(x^2-4)(2x-1)\geq 0$$

$2x^3-x^2-8x+4$	−	0	+	0	−	0	+
x		−2		½		2	

Notice that the negative outside the radical does not affect the sign pattern.

So the zeros are $(\pm 2, 0)$ and $\left(\frac{1}{2}, 0\right)$, and the domain is $x\in\left[-2, \frac{1}{2}\right]\cup[2,\infty)$.



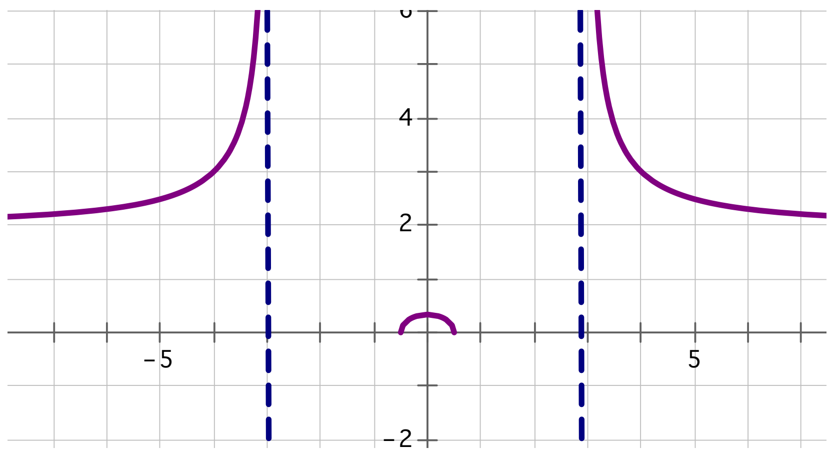
$y=-\sqrt{2x^3-x^2-8x+4}$

EX 6 Find the zeros and domain of $y=\sqrt{\dfrac{4x^2-9}{x^2-9}}$.

$$\frac{4x^2-9}{x^2-9}\geq 0$$

y^2	$+$	VA	$-$	0	$+$	0	$-$	VA	$+$
x		-3		$-\frac{2}{3}$		$\frac{2}{3}$		3	

So the zero is $(1,\ 0)$, and the domain is $x\in(-\infty,\ -3)\cup\left[-\dfrac{2}{3},\dfrac{2}{3}\right]\cup(3,\ \infty)$.



Summary of Domains and zeros of Radical Functions

1. It is all about the RADICAND.

   a. Any sign outside the radical does not affect the sign pattern.

   b. Zeros, VAs, and POEs go on the sign pattern.

2. The domain is where the radicand sign pattern has + or 0.

8-1 Free Response Homework

Find the zeros and domain of each function.

1. $y=\sqrt{9-x^2}$

2. $y=\sqrt{x^2-4x-21}$

3. $y=\sqrt{x^2-2x-5}$

4. $y=\sqrt{x^3+3x^2-18x-40}$

5. $y=\sqrt{x^3-5x^2+7x-3}$

6. $y=\sqrt{x^4+x^3-7x^2-5x+10}$

7. $y=-\sqrt{-x^3-x^2+9x+9}$

8. $y=-\sqrt{-x^2+5x-6}$

9. $y=\sqrt{-x^4+41x^2-400}$

10. $y=\sqrt{2x^3+x^2-32x-16}$

11. $y=-\sqrt{6x^2-5x-21}$

12. $y=-\sqrt{4x^4-37x^2+9}$

Find the zeros, vertical asymptotes, and domain of each function.

13. $y=\sqrt{\dfrac{x}{x^2-4}}$

14. $y=\sqrt{\dfrac{4x}{x^2+4}}$

15. $y=\sqrt{\dfrac{x^2-9}{x^2-4}}$

16. $y=\sqrt{\dfrac{x^2-4}{x^2-9}}$

17. $y=-\sqrt{\dfrac{6x}{x^2+9}}$

18. $y=\sqrt{\dfrac{-4x^3+x^2+16x-4}{4x^3-x^2+36x-4}}$

19. $y=\sqrt{\dfrac{x^2+3x}{x^2+2x-3}}$

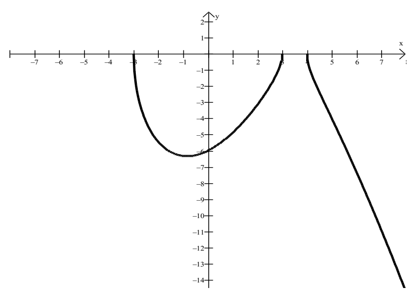
20. $y=\sqrt{\dfrac{x^2+3x}{x^2-2x-3}}$

8-1 Multiple Choice Homework

1. Find the domain of the function $g(u)=\sqrt{u}-\sqrt{9-u}$.

a) $u\in[0,\ \infty)$ b) $u\in(-\infty,\ 0]$ c) $u\in(0,\ 9)$

d) $u\in[0,\ 9]$ e) $u\in(-\infty,\ 9]$



2. Given the graph above, which of the following might be the sign pattern of $f(x)$?

a)
$f(x)$: + 0 – 0 + 0 –
x: ⟵ -3 3 4 ⟶

b)
$f(x)$: – 0 + 0 – 0 +
x: ⟵ -3 3 4 ⟶

c)
$f(x)$: + 0 0 + 0
x: ⟵ -3 3 4 ⟶

d)
$f(x)$: – 0 0 – 0
x: ⟵ -3 3 4 ⟶

e)
$f(x)$: 0 – 0 0 –
x: ⟵ -3 3 4 ⟶

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3. The derivative of $\sqrt{x}-\dfrac{1}{x\sqrt[3]{x}}$ is

a) $\frac{1}{2}x^{-1/2}-x^{-4/3}$ b) $\frac{1}{2}x^{-1/2}+\frac{4}{3}x^{-7/3}$ c) $\frac{1}{2}x^{-1/2}-\frac{4}{3}x^{-1/3}$

d) $-\frac{1}{2}x^{-1/2}+\frac{4}{3}x^{-7/3}$ e) $-\frac{1}{2}x^{-1/2}-\frac{4}{3}x^{-1/3}$

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4. The function $f(x)=\sqrt{\dfrac{x}{x-1}}$ has as its domain all values of x such that

a) $x>1$ b) $x\le 0$ c) $x<1$ d) $0\le x<1$ e) $x>1$ or $x\le 0$

---

5. Given this sign pattern

$$\begin{array}{c|ccccccc} y^2 & - & 0 & + & 0 & - & 0 & + \\ \hline x & & -4 & & 1 & & 2 & \end{array}$$

(on a number line $\longleftrightarrow$), the domain of $y=f(x)$ is

a) $x\in(-\infty,-4)\cup(1,2)$ b) $x\in(-\infty,-4]\cup[1,2]$

c) $x\in(-4,1)\cup(2,\infty)$ d) $x\in[-4,1]\cup[2,\infty]$

e) $x\in[-4,1]\cup[2,\infty)$

8-2: The Chain Rule

Just as rational functions needed a separate rule because they could not be written in the form needed for the Power Rule, radical functions often need another rule. Some radicals do not need a new rule—namely those that can be written as $y = x^n$. Remember that radicals are actually fractional exponents, where the index is the denominator and the power is the numerator. So $\frac{d}{dx}\left[\sqrt{x^5}\right] = \frac{d}{dx}\left[x^{5/2}\right] = \frac{5}{2}x^{3/2}$.

But most radical functions are not a single variable to a power but are polynomials or rationals to a power. In other words, they are **composite functions.**

Vocabulary:
1. ***Composite Function*** – a function made of two other functions, one inside the other. For example, $y = \sqrt{16x - x^3}$, $y = \sin x^3$, $y = \cos^3 x$, or $y = (x^2 + 2x - 5)^3$. The general symbol is $f(g(x))$.

EX 1 If $f(x) = 2x + 1$ and $g(x) = x^2$, determine the value of:

a) $f(g(3))$ b) $g(f(3))$ c) $f(g(x))$

a) To find $f(g(3))$, first find $g(3)$:

$$g(x) = x^2, \text{ so } g(3) = 3^2 = 9$$
$$f(g(3)) = f(9)$$
$$f(x) = 2x + 1$$
$$f(9) = 2(9) + 1$$
$$= 19$$

So, $f(g(3)) = 19$.

b) For $g(f(3))$, $f(x)=2x+1 \to f(3)=2(3)+1=7$

$$g(f(3))=g(7)=49$$

c) $f(g(x))=2(g(x))+1=2x^2+1$

EX 2 If $f(x)=\sqrt{x}$ and $g(x)=2x+5$, find $f(g(x))$.

$$f(g(x))=\sqrt{g(x)}=\sqrt{2x+5}$$

A question of how to take the derivative of a composite function arises in Calculus. There are now two (or more) functions that must be differentiated, but, since one is inside the other, the derivatives cannot be taken at the same time. Just as a radical cannot be distributed over addition, a derivative cannot be distributed over a composite function. While this section concerns radical functions, the Chain Rule applies to all composite functions and will come up again and again. It is one of the cornerstones of Calculus.

The Chain Rule: $\frac{d}{dx}\left[f(g(x))\right]=f'(g(x))\cdot g'(x)$

LEARNING OUTCOME

Find the derivative of radical functions.

EX 3 $y=\sqrt{16x-x^3}$; find $\dfrac{dy}{dx}$

$$y=\sqrt{16x-x^3}=\left(16x-x^3\right)^{1/2}$$
$$\frac{dy}{dx}=\frac{1}{2}\left(16x-x^3\right)^{-1/2}\left(16-3x^2\right)$$
$$=\frac{\left(16-3x^2\right)}{2\left(16x-x^3\right)^{1/2}}$$

In this case, the $\sqrt{\ }$ is the *f* function and the polynomial $16x-x^3$ is the *g* function. Each derivative is found using the Power Rule, but, as $16x-x^3$ is inside the $\sqrt{\ }$, it is inside the derivative of the $\sqrt{\ }$.

EX 4 $D_x\left[\sqrt[3]{x^4+5x^2+4}\right]$

$$D_x\left[\sqrt[3]{x^4+5x^2+4}\right]=D_x\left[\left(x^4+5x^2+4\right)^{1/3}\right]$$
$$=\frac{1}{3}\left(x^4+5x^2+4\right)^{-2/3}\left(4x^3+10x\right)$$
$$=\frac{4x^3+10x}{3\left(x^4+5x^2+4\right)^{2/3}}$$
$$=\frac{2x\left(2x^2+5\right)}{3\left(x^4+5x^2+4\right)^{2/3}}$$

Note that the last step of this example included factoring to determine whether the fraction could be further simplified. Always express answers in simplest form.

The Chain Rule applies to any composite function, not just radicals.

EX 5 $\frac{d}{dx}\left[\left(4x^2-2x-1\right)^{10}\right]$

$$\frac{d}{dx}\left[\left(4x^2-2x-1\right)^{10}\right]=10\left(4x^2-2x-1\right)^9\left(8x-2\right)$$
$$=20\left(4x^2-2x-1\right)^9\left(4x-1\right)$$

EX 6 $\frac{d}{dx}\left[\sqrt{\left(x^2+1\right)^5+7}\right]$

$$\frac{d}{dx}\left[\sqrt{\left(x^2+1\right)^5+7}\right]=\frac{d}{dx}\left[\left(\left(x^2+1\right)^5+7\right)^{1/2}\right]$$
$$=\frac{1}{2}\left(\left(x^2+1\right)^5+7\right)^{-1/2}5\left(x^2+1\right)^4\left(2x\right)$$
$$=\frac{5x\left(x^2+1\right)^4}{\left(\left(x^2+1\right)^5+7\right)^{1/2}}$$

EX 7 $G(x)=\sqrt{\dfrac{54x}{x^2+9}}$; find $G'(x)$.

$$G(x)=\sqrt{\frac{54x}{x^2+9}}=\left(\frac{54x}{x^2+9}\right)^{1/2}$$

$$G'(x)=\frac{1}{2}\left(\frac{54x}{x^2+9}\right)^{-1/2}\bullet\left[\frac{(x^2+9)(54)-(54x)(2x)}{(x^2+9)^2}\right]$$

$$=\frac{1}{2}\bullet\frac{(x^2+9)^{1/2}}{(54x)^{1/2}}\bullet\frac{(54x^2+486-108x^2)}{(4-x)^2}$$

$$=\frac{(x^2+9)^{1/2}(-54x^2+486)}{2(54x)^{1/2}(x^2+9)^2}$$

$$=\frac{243-27x^2}{(54x)^{1/2}(x^2+9)^{3/2}}=\frac{27(9-x^2)}{(54x)^{1/2}(x^2+9)^{3/2}}$$

EX 8 $F(x)=\sqrt{\dfrac{3x+2}{4-x}}$; find $F'(x)$.

$$F(x)=\sqrt{\frac{3x+2}{4-x}}=\left(\frac{3x+2}{4-x}\right)^{1/2}$$

$$F'(x)=\frac{1}{2}\left(\frac{3x+2}{4-x}\right)^{-1/2}\bullet\frac{(4-x)(3)-(3x+2)(-1)}{(4-x)^2}$$

$$=\frac{1}{2}\bullet\frac{(4-x)^{1/2}}{(3x+2)^{1/2}}\bullet\frac{12-3x+3x+2}{(4-x)^2}$$

$$=\frac{(4-x)^{1/2}\bullet 14}{2(3x+2)^{1/2}(4-x)^2}$$

$$=\frac{7}{(3x+2)^{1/2}(4-x)^{3/2}}$$

Ex 9 Given this table of values, find $\frac{d}{dx}\left[f(g(x))\right]$ and $\frac{d}{dx}\left[g(f(x))\right]$ at $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

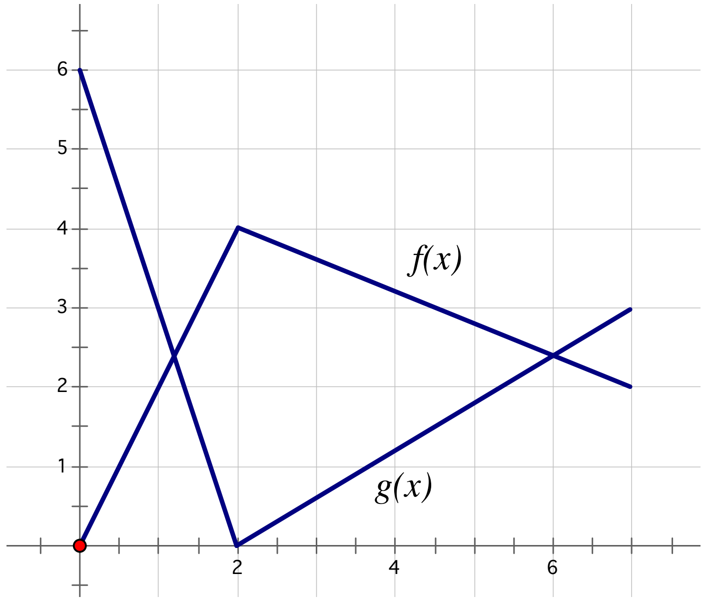
These are two different, but similar problems, so let us consider them individually:

$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$ at $x = 1$,

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(1))g'(1)$$
$$= f'(2)(6)$$
$$= (5)(6)$$
$$= 30$$

$\frac{d}{dx}\left[g(f(x))\right] = g'(f(x))f'(x)$ at $x = 1$,

$$\frac{d}{dx}\left[g(f(x))\right] = g'(f(1))f'(1)$$
$$= g'(3)(4)$$
$$= (9)(4)$$
$$= 36$$



Ex 10 Given the graph above, find

(a) $u'(1)$ if $u = f(g(x))$
(b) $v'(1)$ if $v = g(f(x))$
(c) $w'(1)$ if $w = f(x)g(x)$

(a) $u'(1)$ if $u = f(g(x))$

$$\Rightarrow u'(x) = f'(g(x)) \cdot g'(x)$$

$$\Rightarrow u'(1) = f'(g(1)) \cdot g'(1)$$

$$\Rightarrow u'(1) = f'(3) \cdot (-3)$$

$$\Rightarrow u'(1) = \left(-2/5\right) \cdot (-3) = 6/5$$

(b) $v'(1)$ if $v = g(f(x))$

$$\Rightarrow v'(x) = g'(f(x)) \cdot f'(x)$$

$$\Rightarrow v'(1) = g'(f(1)) \cdot f'(1)$$

$$\Rightarrow v'(1) = g'(2) \cdot 2$$

$$\Rightarrow \text{dne}$$

Note that $g'(2)$ does not exist. The slope cannot be determined at $x = 1$ because the slopes to the left and right of $x = 1$are different. The is called a corner point or a cusp point and will be explored further in a later chapter.

(c) $w'(1)$ if $w = f(x)g(x)$

$$\Rightarrow w'(x) = f'(x)(g(x)) + g'(x)f(x)$$

$$\Rightarrow w'(1) = f'(1)(g(1)) + g'(1)f(1)$$

$$\Rightarrow w'(1) = (2)(3) + \left(-\frac{1}{3}\right)(2)$$

$$\Rightarrow w'(1) = 6 + \left(-2/3\right) = 16/3$$

8-2 Free Response Homework

Find the following derivatives.

1. $\frac{d}{dx}\left[\sqrt{2x+25}\right]$

2. $D_x\left[\sqrt[3]{3x+2}\right]$

3. $y=\sqrt{x^2+x+13}$; find y'

4. $\frac{d}{dx}\left[\sqrt{3x^2-4x+9}\right]$

5. $D_x\left[\sqrt{1-4x^2}\right]$

6. $\frac{d}{dx}\left[(5-3x)^{3/2}\right]$

7. $y=\sqrt{-x^4+41x^2-400}$

8. $y=\sqrt{2x^3+x^2-32x-16}$

9. $y=\sqrt{-x^3-x^2+9x+9}$

10. $y=\sqrt{-x^2+5x-6}$

11. $y=\sqrt{x^2-5x-21}$

12. $y=\sqrt{4x^4-37x^2+9}$

13. $y=\sqrt{\frac{x}{x^2-4}}$

14. $y=\sqrt{\frac{4x}{x^2+4}}$

15. $y=\sqrt{\frac{x^2-9}{x^2-4}}$

16. $y=\sqrt{\frac{x^2-4}{x^2-9}}$

17. $y=-\sqrt{\frac{6x}{x^2+9}}$

18. $y=\sqrt{\frac{-4x^3+x^2+16x-4}{4x^3-x^2+36x-9}}$

19. $y=\sqrt{\frac{x^2+3x}{x^2+2x-3}}$

20. $y=\sqrt{\frac{x^2+3x}{x^2-2x-3}}$

21. $y=\left(4x^2-5x+1\right)^5$; find y'

22. $\dfrac{d}{dx}\left[\sqrt[6]{\left(x^2-4x+4\right)^5}\right]$

23. $y=\sqrt[4]{1-2x}$; find $\dfrac{dy}{dx}$

24. $y=\sqrt[7]{x^3-2x}$; find $\dfrac{dy}{dx}$

25. $y=\sqrt[10]{16-x^2}$; find $\dfrac{dy}{dx}$

26. $\dfrac{d}{dx}\left[\left(\dfrac{4x-1}{3-x}\right)^{10}\right]$

27. $y=\dfrac{1}{\sqrt{25-x^2}}$; find y'

28. $f(x)=\sqrt[5]{\dfrac{2x-1}{x^2-4}}$; find $f'(x)$

29. $\dfrac{d}{dx}\left[\sqrt{\dfrac{5x+6}{5x-4}}\right]$

30. $D_x\left[\sqrt[4]{\dfrac{x^4+4x^2-5}{x-1}}\right]$

31. $D_x\left[\sqrt{\dfrac{2x^3-5x^2+2x}{x^2-4}}\right]$

32. $\dfrac{d}{dx}\left[\sqrt[3]{\left(\dfrac{x^4-13x^2+36}{2x^3-3x^2-8x+12}\right)^7}\right]$

33. $\dfrac{d}{dx}\left[\sqrt[3]{\dfrac{2x+1}{1-x}}\right]$

34. $f(x)=\sqrt{4-\dfrac{4}{9}x^2}$; find $f'\left(\sqrt{5}\right)$

8-2 Multiple Choice Homework

1. Find *a,* such that the function $f(x)=4x+\sqrt{a-x^2}$ has the domain $[-5,\ 5]$.

a) $a=5$ b) $a=25$ c) $a=-25$ d) $a=\sqrt{5}$ e) $a=-\sqrt{5}$

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2. If $h(x)=(x^2-4)^{3/4}+1$, then the value of $h'(2)$ is

a) 3 b) 2 c) 1 d) 0 e) DNE

---

3. Find the point on the graph of $y=\sqrt{x}$ between $(1,\ 1)$ and $(9,\ 3)$ at which the tangent to the graph has the same slope as the line through $(1,\ 1)$ and $(9,\ 3)$.

a) $(1,\ 1)$ b) $(2,\ \sqrt{2})$ c) $(3,\ \sqrt{3})$ d) $(4,\ 2)$

e) none of these

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4. What is $\lim\limits_{x\to\infty}\dfrac{\sqrt{9x^2+2}}{4x+3}$?

a) $\dfrac{3}{2}$ b) $\dfrac{3}{4}$ c) $\dfrac{\sqrt{2}}{3}$ d) 1 e) The limit does not exist.

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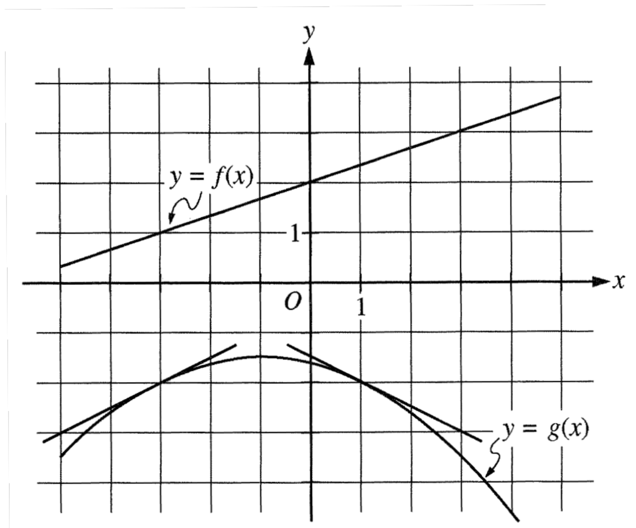
5. Let h be a differentiable function, and let f be function defined by $f(x) = h(x^2 - 3)$. Which of the following is equal to $f'(2)$?

a) $h'(1)$ b) $4h'(1)$ c) $4h'(2)$ d) $h'(4)$ e) $4h'(4)$

---

6. The figure below shows the graph of the functions f and g. The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = g(f(x))$, what is $B'(-3)$?



a) $-\frac{1}{2}$

b) $-\frac{1}{6}$

c) $\frac{1}{6}$

d) $\frac{1}{3}$

e) $\frac{1}{2}$

---

7. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	8	8	4	3
8	6	-12	2	4

Given that $h(x) = g(f(x))$, $h'(4) =$

(a) 32 (b) 10 (c) -6 (d) 24 (e) 16

The correct answer is A

---

8. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

Given that $h(x) = f(g(x))$, $h'(8) =$

a) -12 b) -2 c) -1 d) -8 e) 10

The correct answer is D

---

9. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

Given that $h(x) = g(g(x))$, $h'(8) =$

a) 1 b) 2 c) 3 d) 4 e) 8

The correct answer is D

8-3: Extreme Points of Radical Functions

Previously, it was found that the critical values of the extreme points were where the derivative equaled zero, where the derivative was undefined, and endpoints of an arbitrarily stated domain. Now this concept will be explored in the context of radical functions.

REMEMBER:

Steps to Finding the Extreme Points of a Rational Function

1. Check for POEs before differentiating.

   a. If there is a POE, reduce the function.

2. Find $\frac{dy}{dx}$ using the Quotient Rule.

3. Find the Critical Values:

   i. $\frac{dy}{dx} = 0 \rightarrow$ numerator $= 0 \rightarrow$ solve for x.

   ii. $\frac{dy}{dx}$ *dne* $\rightarrow$ denominator $= 0 \rightarrow$ solve for x.
   [*Note that the numbers that solve this equation will the vertical asymptotes of the y-equation, therefore they will not lead to extreme points.*]

   iii. Identify the x-coordinates of any given domain.

4. Find the y-coordinates for each of the critical values (from the y-equation).

   a. Note that this can be done as each critical value is found.

5. List answers as points.

Because a radical function, which usually starts with no denominator, might have a derivative with a denominator, there are numbers that are not excluded from the domain of y, which can make $\frac{dy}{dx}$ not exist.

LEARNING OUTCOME Find the critical values and extreme points of radical functions.

EX 1 Find the critical values of $y = \sqrt{16x - x^3}$.

$$y = \sqrt{16x - x^3} = \left(16x - x^3\right)^{1/2}$$
$$\frac{dy}{dx} = \frac{1}{2}\left(16x - x^3\right)^{-1/2}\left(16 - 3x^2\right)$$
$$= \frac{\left(16 - 3x^2\right)}{2\left(16x - x^3\right)^{1/2}}$$

i) $$\frac{\left(16 - 3x^2\right)}{2\left(16x - x^3\right)^{1/2}} = 0$$
$$16 - 3x^2 = 0$$
$$x = \pm\frac{4}{\sqrt{3}}$$

Recall from an example in a previous section the domain is $x \in \left(-\infty,\ -4\right] \cup \left[\,0,\ 4\right]$. Therefore, $x = -\frac{4}{\sqrt{3}}$ is not actually a critical value because it is not in the domain. This leaves $x = \frac{4}{\sqrt{3}}$ as a critical value.

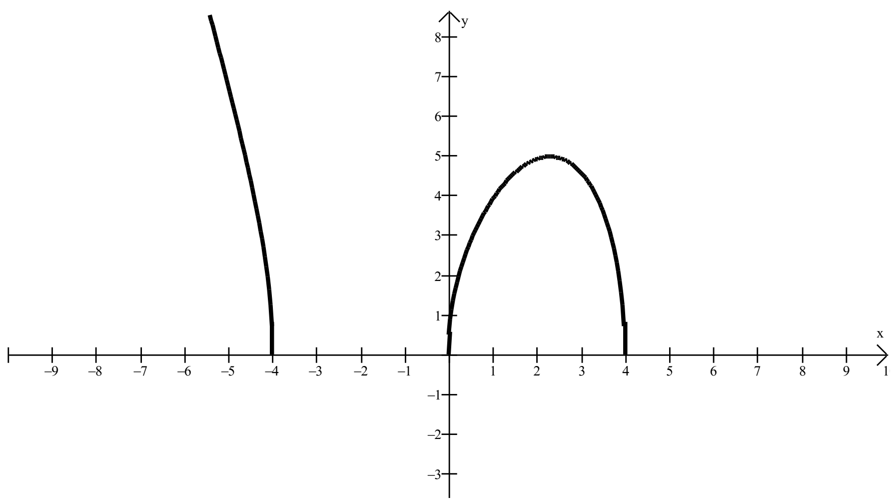
ii) $\frac{\left(16 - 3x^2\right)}{2\left(16x - x^3\right)^{1/2}}$ does not exist

$$16x - x^3 = 0$$

$$x = 0,\ 4,\ -4$$

iii) There is no arbitrary domain stated.

Therefore, the critical values are $x = \frac{4}{\sqrt{3}}$, 0, 4, and -4. The graph shows that $x = \frac{4}{\sqrt{3}}$ is at the maximum point, while the others are at the minimum points.



$y = \sqrt{16x - x^3}$

Note that there are three sign patterns involved in this problem.

For the Domain:

$16x - x^3$	+	0	−	0	+	0	−
x		-4		0		4	

For the function:

y	+	0		0	+	0	
x		-4		0		4	

And for the derivative:

$\frac{dy}{dx}$	$-$	dne		dne	$+$	0	$-$	dne
x		-4		0		$\frac{4}{\sqrt{3}}$		4

Each one means something different in the context of the problem. This is why labeling the sign pattern is so important.

<u>Steps to Finding the Extreme Points of Composite Functions</u>

1. FIND THE DOMAIN FIRST!!!

2. Find $\frac{dy}{dx}$ using the Chain Rule.

3. Find the Critical Values:

   i. $\frac{dy}{dx} = 0 \rightarrow$ numerator $= 0 \rightarrow$ solve for x.

   ii. $\frac{dy}{dx}$ $dne \rightarrow$ denominator $= 0 \rightarrow$ solve for x.

   iii. Identify the x-coordinates of any domain restriction.

4. Reject critical values that do not fall within the domain.

5. Find the y-coordinates for each of the critical values (from the y-equation).

6. List answers as points.

EX 2 Find the extreme points of $y = -\sqrt{x^4 - 6x^2 + 8}$.

The domain of the function is $x \in \left(-\infty,\ -2\right] \cup \left[-\sqrt{2},\ \sqrt{2}\right] \cup \left[2,\ \infty\right)$.

$$\frac{dy}{dx} = -\frac{1}{2}\left(x^4 - 6x^2 + 8\right)^{-1/2}\left(4x^3 - 12x\right)$$
$$= \frac{6x - 2x^3}{\left(x^4 - 6x^2 + 8\right)^{1/2}}$$

i) $$\frac{dy}{dx} = \frac{6x - 2x^3}{\left(x^4 - 6x^2 + 8\right)^{1/2}} = 0$$
$$6x - 2x^3 = 0$$
$$2x\left(3 - x^2\right) = 0$$

$x = 0,\ \pm\sqrt{3}$, but $x = \pm\sqrt{3}$ are not in the domain. This leaves $x = 0$ as a critical value. So $\left(0,\ -\sqrt{8}\right)$ or $\left(0,\ -2\sqrt{2}\right)$ is an extreme point.

ii) $$\frac{dy}{dx} = \frac{6x - 2x^3}{\left(x^4 - 6x^2 + 8\right)^{1/2}} \quad \text{DNE}$$

$$\left(x^4 - 6x^2 + 8\right)^{1/2} = 0$$
$$x = \pm 2,\ \pm\sqrt{2}$$

iii) As with EX 1, there is no arbitrary domain.

The critical values are $x = 0,\ \pm\sqrt{2}$, and ± 2.

The extreme points are $\left(0,\ -2\sqrt{2}\right)$, $\left(\pm 2,\ 0\right)$, and $\left(\pm\sqrt{2},\ 0\right)$.

EX 3 Find the extreme points of $f(x)=\sqrt{\dfrac{x^2-4}{x^2-9}}$ on $x\in[-4,\ 4]$.

Domain: $x\in[-4,\ -3)\cup[-2,\ 2]\cup(3,\ 4]$

$$f'(x)=\frac{1}{2}\left(\frac{x^2-4}{x^2-9}\right)^{-1/2}\bullet\frac{(x^2-9)(2x)-(x^2-4)(2x)}{(x^2-9)^2}$$

$$=\frac{1}{2}\frac{(x^2-9)^{1/2}}{(x^2-4)^{1/2}}\bullet\frac{-10x}{(x^2-9)^2}$$

$$=\frac{-5x}{(x^2-4)^{1/2}(x^2-9)^{3/2}}$$

i) $\dfrac{dy}{dx}=0$ means $-5x=0$, so $x=0$

ii) $\dfrac{dy}{dx}$ does not exist means $x^2-4=0$ or $x^2-9=0$, so $x=\pm 2$ or $x=\pm 3$.
But $x=\pm 3$ are not in the domain, leaving $x=\pm 2$ as critical values.

iii) Endpoints of this arbitrary domain are 4 and –4.

The critical values are $x=0,\ \pm 2,\ \pm 4$.

The extreme points are $\left(0,\ \frac{2}{3}\right)$, $(2,\ 0)$, $(-2,\ 0)$, $\left(4,\ \frac{2\sqrt{3}}{\sqrt{7}}\right)$, $\left(-4,\ \frac{2\sqrt{3}}{\sqrt{7}}\right)$.

EX 4 Find the intervals on which $y = -\sqrt{x^4 - 6x^2 + 8}$ is increasing.

As seen in EX 2 the domain is $x \in \left(-\infty,\ -2\right] \cup \left[-\sqrt{2}\,,\ \sqrt{2}\,\right] \cup \left[2,\ \infty\right)$.

Also seen in EX 2, $\dfrac{dy}{dx} = \dfrac{6x - 2x^3}{\left(x^4 - 6x^2 + 8\right)^{1/2}}$ and the critical values are $x = 0,\ \pm 2,\ \pm\sqrt{2}$. The sign pattern for the DERIVATIVE would be

$\dfrac{dy}{dx}$	+	DNE		DNE	−	0	+	DNE		DNE −
x		-2		$-\sqrt{2}$		0		$\sqrt{2}$		2

Note that, because of the domain of the radical, there are some intervals on the sign pattern that have no sign. That is because the function does not have real values there.

The intervals of increasing are where $\dfrac{dy}{dx} > 0$. They would be

$$x \in \left(-\infty,\ -2\right) \cup \left(0,\ \sqrt{2}\right)$$

EX 5 Find the extreme points of $G(x)=\sqrt{\dfrac{54x}{x^2+9}}$.

The domain here is easy. Since the denominator is always positive, the domain is $x \le 0$

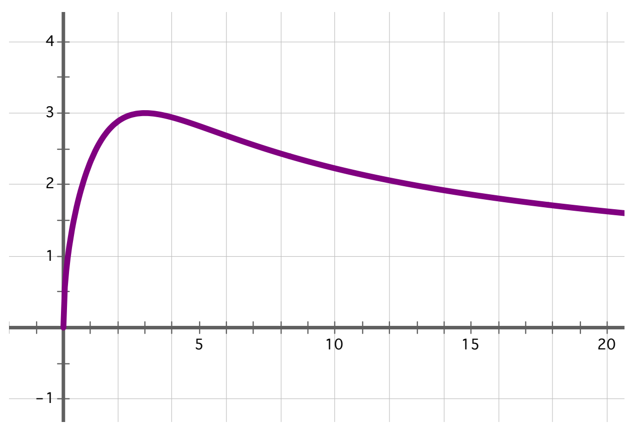
$$G'(x)=\frac{27\left(9-x^2\right)}{\left(54x\right)^{1/2}\left(x^2+9\right)^{3/2}}$$

i) $\dfrac{dy}{dx}=0$ means $9-x^2=0 \rightarrow x=\pm 3$, so $x=0$. But $x=-3$ are not in the domain, leaving $x=3$ as a critical value.

ii) $\dfrac{dy}{dx}$ does not exist means $54x=0$ or $x^2+9=0$, so $x=0$.

iii) Endpoints: None given.

So the extreme points are $(0, 0)$ and $(3, 3)$



8-3 Free Response Homework

1. Use the equation of the tangent line to $f(x)=\sqrt{6-x}$ at $x=2$ to approximate $f(1.9)$.

2. Use the equation of the tangent line to $f(x)=\sqrt{x+5}$ at $x=-1$ to approximate $f(-0.9)$.

State the domain and critical values.

3. $y=(5-3x)^{3/2}$

4. $y=\sqrt{x^2-4x-5}$

5. $y=\sqrt{9x^3-4x^2-27x+12}$

6. $y=\sqrt{-x^4+13x^2-36}$

7. $y=\sqrt{9-x}$

8. $y=\sqrt{\dfrac{x^2-1}{x^2-4}}$

9. $y=\sqrt{\dfrac{x^2-4}{x^2-1}}$

10. $y=\sqrt{\dfrac{-10x}{x^2+25}}$

Find the extreme points.

11. $y=\sqrt{-x^4+41x^2-400}$

12. $y=\sqrt{2x^3+x^2-32x-16}$

13. $y=\sqrt{-x^3-x^2+9x+9}$

14. $y=\sqrt{-x^2+5x-6}$

15. $y=\sqrt{x^2-4x-21}$

16. $y=\sqrt{4x^4-37x^2+9}$

17. $y=\sqrt{\dfrac{-8x}{x^2+4}}$

18. $y=-\sqrt{\dfrac{8x}{x^2-4}}$

19. $y=-\sqrt{\dfrac{x^2-9}{x^2-4}}$

20. $y=\sqrt{\dfrac{x^2-5x+6}{16-x^2}}$

21. $y=\sqrt{\dfrac{x^2-4x}{x^2-4}}$

22. $y=-\sqrt{\dfrac{x}{9-x^2}}$

23. $y=\sqrt{\dfrac{x^2+3x}{x^2+2x-3}}$

24. $y=\sqrt{\dfrac{x^2+3x}{x^2-2x-3}}$

25. $y=\sqrt[7]{x^3-2x}$

26. $g(x)=\sqrt[6]{\left(x^2-4x+4\right)^5}$

27. $f(x)=\sqrt{\dfrac{2x-1}{x^2-4}}$

28. $F(x)=\sqrt{4-\dfrac{4}{9}x^2}$

State the domain, find the critical values, and determine if the critical values occur at a maximum point, a minimum point, or neither.

29. $y=\sqrt{9-4x^2}$

30. $y=\sqrt{x^4-8x^2+12}$

31. $y=\sqrt{\dfrac{3x}{9+x^2}}$

32. $y=\sqrt{\dfrac{3x}{9-x^2}}$

33. The position of a particle moving along the x-axis at time $t\geq 0$ is described by $x(t)=\sqrt{5+t^2}$. Find the velocity at $t=2$.

34. A particle's position $\left(x(t),\, y(t)\right)$ at time $0\leq t\leq 10$ is described by the parametric equations $x(t)=\sqrt{t^2+4}$ and $y(t)=\sqrt{10t-t^2}$.

a) What is the particle's velocity at $t=2$?

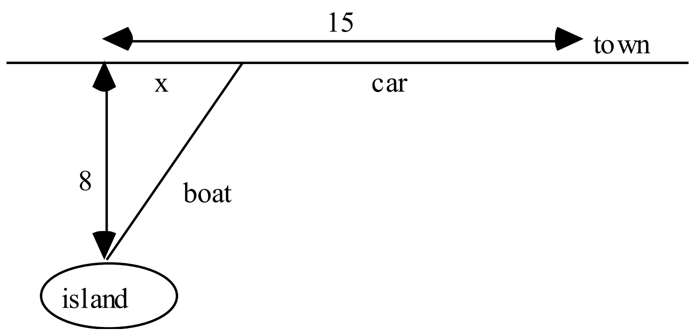
b) What is the particle's acceleration at $t=2$?

35. A particle's position $\left(x(t),\, y(t)\right)$ at time $0\leq t\leq 10$ is described by the parametric equations $x(t)=\sqrt{t^2+4}$ and $y(t)=\sqrt{10t-t^2}$.

a) When is the particle moving right and down?

b) When is the particle at rest?

36. A person is on an island 8 miles from a straight shore. She needs to go to a town 15 miles down the coast. A boat costs \$16 per mile and a car costs \$11 per mile. How far down the shore should she land the boat in order to minimize the cost? What is that minimum cost?



8-3 Multiple Choice Homework

1. Given this sign pattern

$\frac{dy}{dx}$	$+\ 0$	$0\ -$	$0\ +$	0	$0\ -$
x	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2

, at what value of x does f have a local minimum?

a) -2 b) $-\sqrt{2}$ c) 0 d) $\sqrt{2}$ e) 2

---

2. The table below gives the values of at selected values of f, f', g, and g'. If $h(x) = f(g(x))$, then $h'(1) =$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–1	6	5	3	–2
1	3	–3	–1	2
3	1	–2	2	3

a) 5 b) 6 c) 9 d) 10 e) 12

3. Find the absolute maximum value of $y=\sqrt{36-x^2}$ on the interval $x\in[-2,\ 2]$.

a) 5 b) 6 c) 7 d) 0 e) 1

4. The graph of the function $f(x)=2x^{5/3}-5x^{2/3}$ is increasing on which of the following intervals

I. $1<x$ II. $0<x<1$ III. $x<0$

a) I only b) II only c) III only

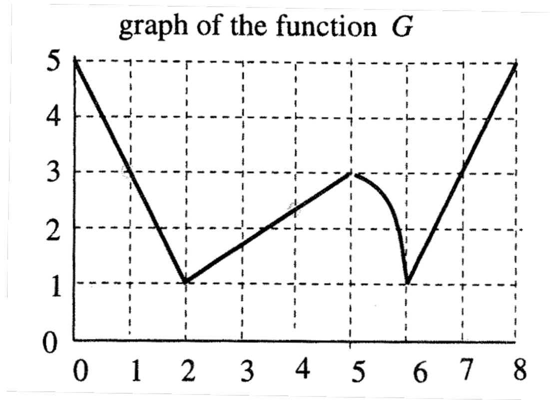
d) I and II only e) I and III only

5. The table below gives values of the differentiable functions *f* and *g* and of their derivatives f' and g', at selected values of *x*. If $h(x)=f(g(x))$, what is the slope of the graph of *h* at $x=2$?

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
–1	–5	1	3	0
0	–2	0	1	1
1	0	–3	0	0.5
2	5	–1	5	2

a) –10 b) –5 c) 5 d) 6 e) 10

6. The function F is defined by $F(x) = G(x + G(x))$, where the graph of the function G is shown in the figure. The approximate value of $F'(1)$ is



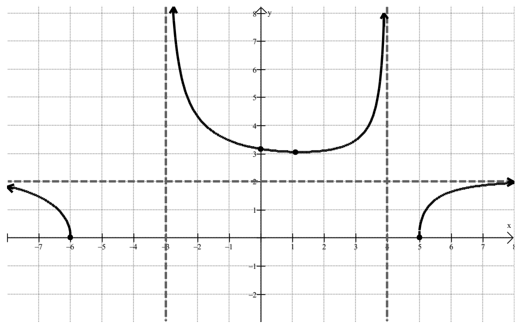
a) $\frac{7}{3}$

b) $\frac{2}{3}$

c) –2

d) –1

e) $-\frac{2}{3}$



7. Which of the following sign patterns apply to the graph above?

I.

y	+	0	−	DNE	+	DNE	−	0	+
x		−6		−3		4		5	

II.

y^2	+	0	−	DNE	+	DNE	−	0	+
x		−6		−3		4		5	

III.

$\frac{dy}{dx}$	−	DNE		DNE	−	0	+	DNE		DNE	+
x		−6		−3		1		4		5	

a) I only   b) II only   c) I and II only

d) II and III only   e) I, II, and III

---

8. The absolute maximum of $y=-\sqrt{25-x^2}$ on $x\in[-2, 4]$ is

a) -2 b) 0 c) -5 d) $-\sqrt{21}$ e) -3

8-4: General Radical Curve Sketching

REMEMBER: Radical Traits

1. Domain
2. Zeros
3. y-intercept
4. Extreme Points
5. Range
6. End Behavior (EB)

And, if there is a rational function within the radical:

7. POE
8. Vertical Asymptotes

LEARNING OUTCOME

Find all the traits and sketch a fairly accurate radical curve algebraically.

EX 1 Given the traits below, sketch a graph of the function.

Domain: $x \in (-\infty, -6] \cup (-3, 4) \cup [5, \infty)$

Zeros: $(-6, 0)$, $(5, 0)$

VA: $x = -3$, $x = 4$

POE: None

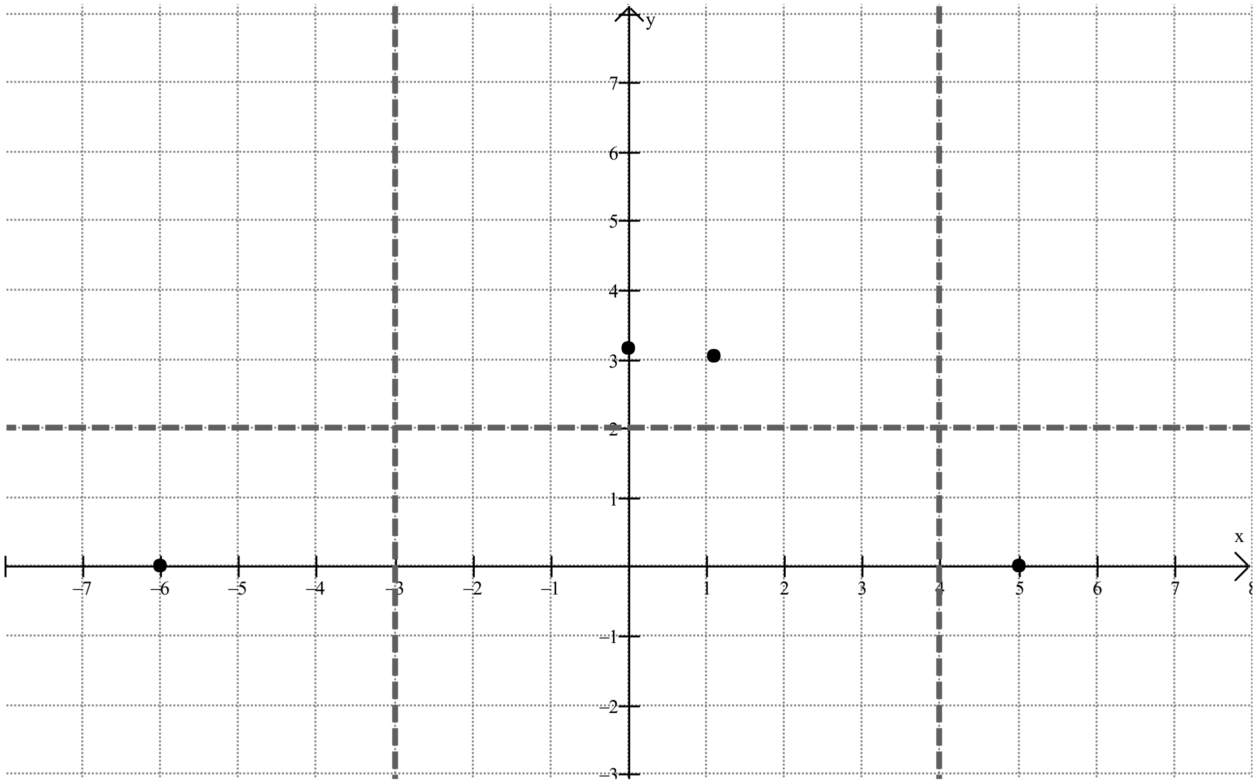
y-int: $(0, 3.15)$

Extreme Points: $(-6, 0)$, $(5, 0)$, $(1.1, 3.05)$

End Behavior: $y = 2$

Range: $y \in [0, 2) \cup [3.05, \infty)$

Graph all the asymptotes and points. Note that the only point that is not an extreme point is the y-intercept (in this case). Note also, that because of the domain there will be large sections with no graph.



For $x \in (-\infty, -6]$**:**

$(-6, 0)$ is both a zero and an extreme point. Because of the range this means the zero is a minimum point. There are no other extreme points, so the curve must flatten out towards $y = 2$.

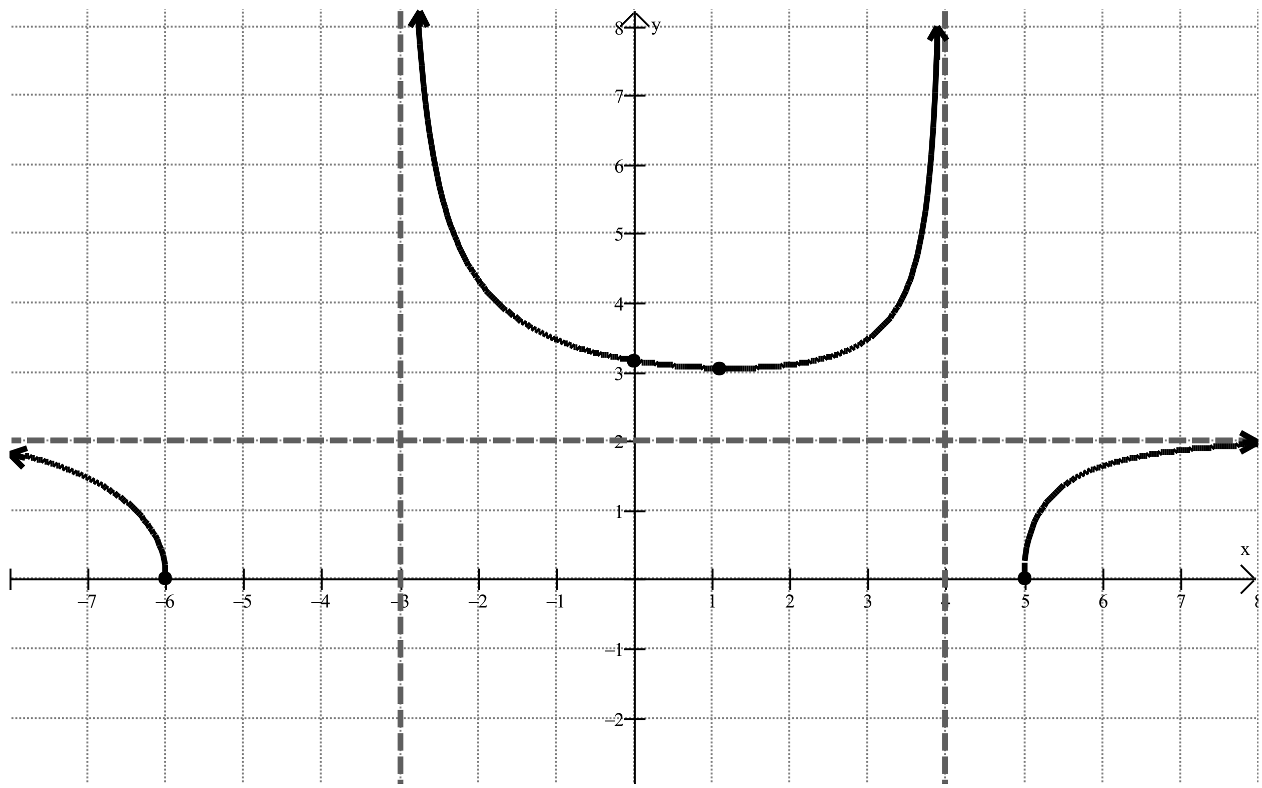
For $x \in (-3, 4)$**:**

Notice that the extreme point is a little lower than the *y*-intercept, so it is a minimum point. That means that the curve has to head up to the vertical asymptotes.

For $x \in [5, \infty)$**:**

$(5, 0)$ is both a zero and an extreme point. Because of the range the zero is a minimum point. There are no other extreme points in this region of the graph, so the curve must head to the end behavior asymptote $(y = 2)$.

Note that because of the domain, there is no curve graphed outside the domain.



EX 2 Find the traits and sketch $y=\sqrt{6+5x-x^2}$.

1. Domain: $6+5x-x^2 \geq 0$
   $(6-x)(1+x) \geq 0$

   $$\begin{array}{c|ccccc} y^2 & - & 0 & + & 0 & - \\ x & & -1 & & 6 & \end{array}$$, so the domain is $x \in [-1,\ 6]$

2. Zeros: $(-1,\ 0)$ and $(6,\ 0)$

3. y-int: $\left(0,\ \sqrt{6}\right)$

4. Extreme Points: $\frac{dy}{dx}=\frac{1}{2}\left(6+5x-x^2\right)^{-1/2}\left(5-2x\right)$

$$\frac{dy}{dx}=0\rightarrow\left(5-2x\right)=0$$

$$x=\frac{5}{2};\; y=\frac{7}{2}$$

An extreme point is $\left(\frac{5}{2},\ \frac{7}{2}\right)$.

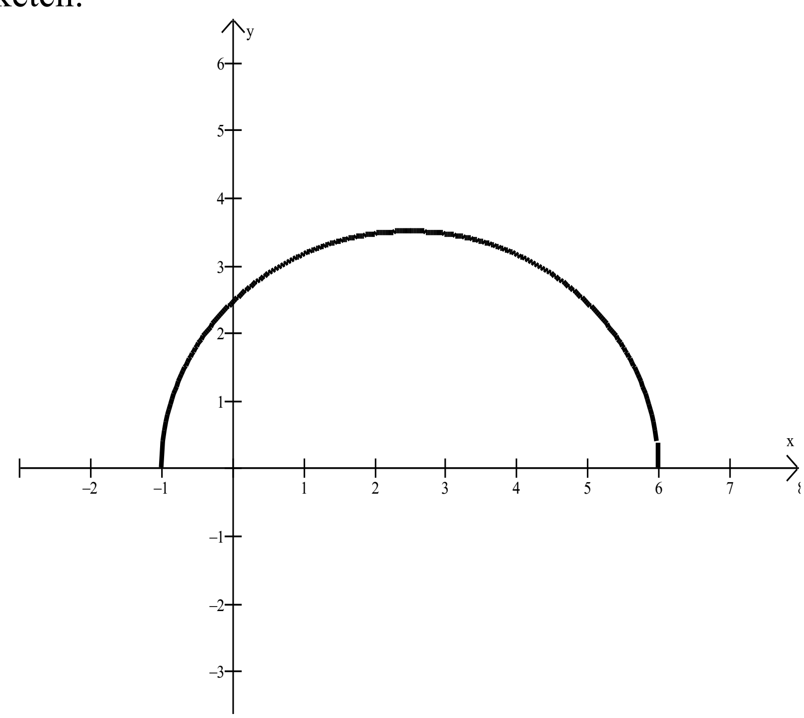
$\frac{dy}{dx}$ does not exist $\rightarrow 6+5x-x^2=0$

$\left(-1,\ 0\right)$ and $\left(6,\ 0\right)$

5. Range: $y\in\left[0,\ 3.5\right]$

6. End Behavior: Recall that end behavior describes the behavior of the function at $x=\pm\infty$. The domain of this function is $x\in\left[-1\, ,6\right]$. Since neither $\pm\infty$ are in the domain, there is no end behavior to consider.

7. Sketch:



$$y=\sqrt{6+5x-x^2}$$

EX 3 Find the traits and sketch $y=-\sqrt{x^3-4x^2-9x+36}$.

1. Domain: $y^2=x^3-4x^2-9x+36\geq 0$
$(x-4)(x+3)(x-3)\geq 0$

$$\begin{array}{c|ccccccc} y^2 & - & 0 & + & 0 & - & 0 & + \\ x & & -3 & & 3 & & 4 & \end{array}$$
, so the domain is $x\in[-3,\ 3]\cup[4,\ \infty)$

2. Zeros: $(\pm 3,\ 0)$ and $(4,\ 0)$

3. y-int: $(0,\ -6)$

4. Extreme Points: $\frac{dy}{dx}=-\frac{1}{2}\left(x^3-4x^2-9x+36\right)^{-1/2}\left(3x^2-8x-9\right)$

$\frac{dy}{dx}=0\rightarrow\left(3x^2-8x-9\right)=0$

$$x=\frac{8\pm\sqrt{8^2-4(3)(-9)}}{2(3)}=\begin{cases}-0.852\\ 3.519\end{cases}$$

But 3.519 is not in the domain.

$\frac{dy}{dx}$ does not exist $\rightarrow x^3-4x^2-9x+36=0$

$x=-3,\ 3$ or 4

The extreme points are $(-0.852,\ -6.336)$, $(-3,\ 0)$, $(3,\ 0)$, and $(4,\ 0)$.

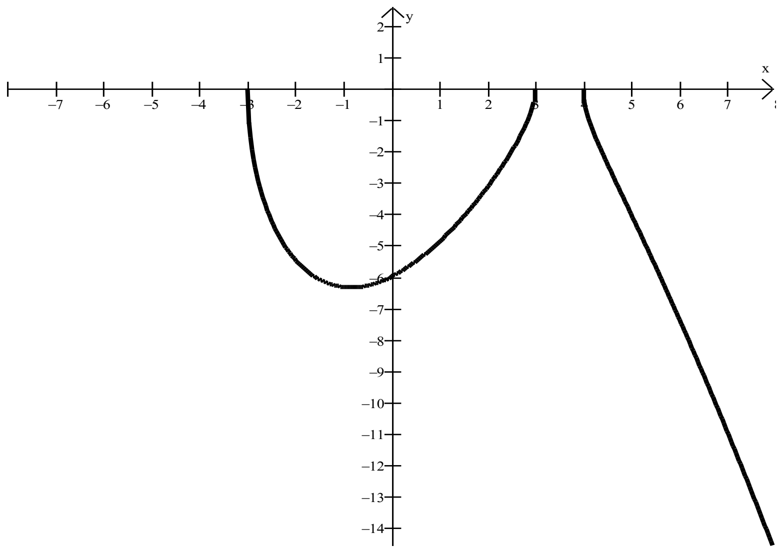
5. Range: $y\in(-\infty,\ 0]$

6. End Behavior: The domain of this function is $x \in [-3,\ 3] \cup [4,\ \infty)$. Since only $+\infty$ is in the domain, consider only the limit as x goes to $+\infty$.

$$\lim_{x \to +\infty} -\sqrt{x^3 - 4x^2 - 9x + 36} = -\infty$$

Thus, the right end of the graph heads down. There is no EB on the left.

7. Sketch:



$$y = -\sqrt{x^3 - 4x^2 - 9x + 36}$$

With more complicated radical functions, it might be a good idea to look at just the sketching part before putting a whole problem together.

EX 4 Find the traits and sketch $y=-\sqrt{\dfrac{x}{x^2-4}}$.

1. Domain: $\dfrac{x}{x^2-4}\ge 0$

$$\frac{x}{(x-2)(x+2)}\ge 0$$

y^2	$-$	DNE	$+$	0	$-$	DNE	$+$
x		-2		0		2	

The domain is $x\in(-2,\ 0]\cup(2,\ \infty)$.

2. Zero: $(0,\ 0)$

3. y-int: $(0,\ 0)$

4. Extreme Points: $\dfrac{dy}{dx}=-\dfrac{1}{2}\left(\dfrac{x}{x^2-4}\right)^{-1/2}\left(\dfrac{(x^2-4)(1)-x(2x)}{(x^2-4)^2}\right)$

$$=\frac{-(x^2-4)^{1/2}}{2x^{1/2}}\left(\frac{-x^2-4}{(x^2-4)^2}\right)$$

$$=\frac{x^2+4}{2x^{1/2}(x^2-4)^{3/2}}$$

i) $\dfrac{dy}{dx}=0\rightarrow x^2+4=0\rightarrow x=$ no solution;

ii) $\dfrac{dy}{dx}=$ does not exist $\rightarrow x=0,\ 2,\ -2$

$y(0)=0$

$y(\pm 2)\rightarrow$ DNE

So $(0,\ 0)$ is the only extreme point.

5. Range: $y \in (-\infty, 0]$

6. The domain of this function is $x \in (-2,\ 0] \cup (2,\ \infty)$.

EB on the left: None

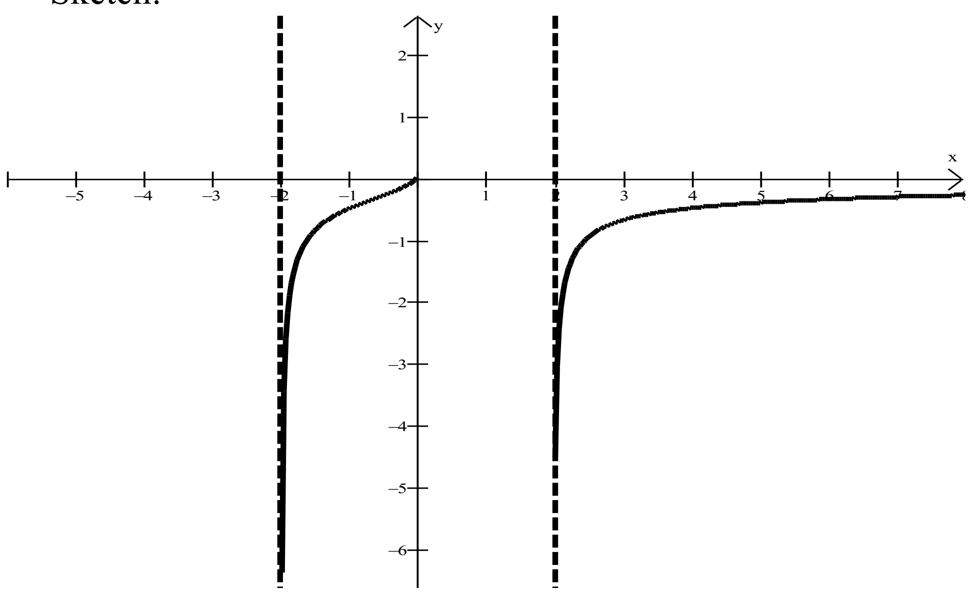
EB on the right: $\lim_{x \to +\infty} -\sqrt{\frac{x}{x^2 - 4}} = 0$

Therefore, there is a horizontal asymptote at $y = 0$ on the right.

7. VA: $x = \pm 2$

8. POE: None

9. Sketch:



$$y = -\sqrt{\frac{x}{x^2 - 4}}$$

EX 5 Find the traits and sketch $y = -\sqrt{x^3 - 4x^2 - 9x + 36}$ on $x \in [-2, 7]$.

As we saw in EX 3, here are the traits and sketch without the arbitrary domain:

Domain: $x \in [-3, 3] \cup [4, \infty)$

Zeros: $(\pm 3, 0)$ and $(4, 0)$

y-int: $(0, -6)$

Extreme Points: $(-0.852, -6.336)$, $(-3, 0)$, $(3, 0)$, and $(4, 0)$

Range: $y \in (-\infty, 0]$

End Behavior: Left end none, right end down

With the domain restriction, the traits become

Domain: $x \in [-2, 3] \cup [4, 7]$

Note that the restrictionn $x \in [-2, 7]$ overlays the domain $x \in [-3, 3] \cup [4, \infty)$ which is inherent to the function.

Zeros: $(3, 0)$ and $(4, 0)$

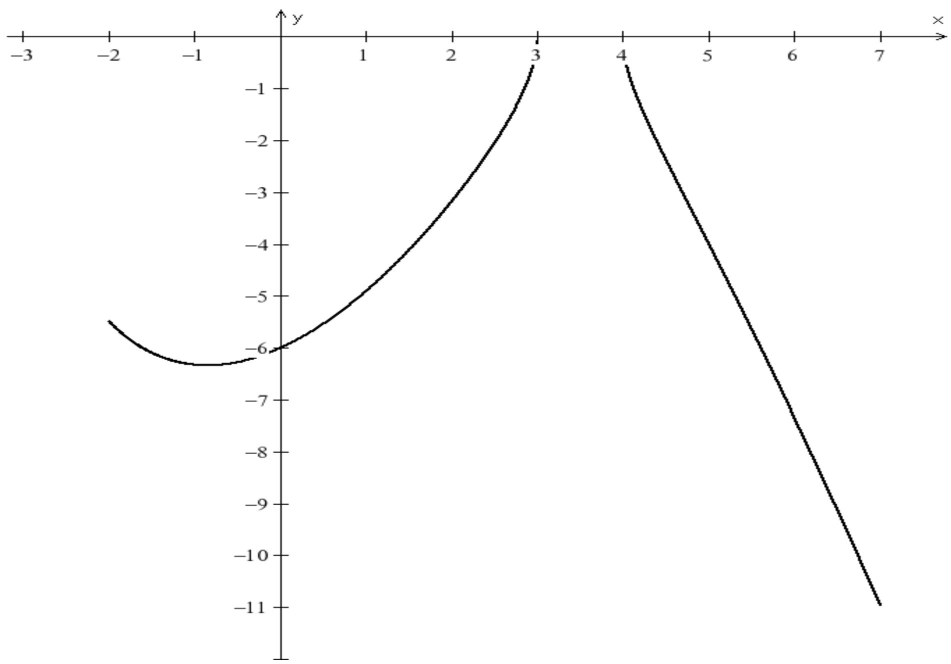
y-int: $(0, -6)$

Extreme Points: $(-0.852, -6.336)$, $(-2, -5.477)$, $(3, 0)$, $(4, 0)$, and $(7, -10.954)$

Range: $y \in [-10.954, 0]$

End Behavior: None

And the sketch becomes



8-4 Free Response Homework

1. Given the traits below, sketch a graph of the function.

Domain: $x \in [-1.2, 2] \cup (3, 5) \cup (5, \infty)$
Zeros: $(-1.2, 0)$, $(2, 0)$
VA: $x = 3$
POE: $(5, 6)$
y-int: $(0, 2)$
Extreme Points: $(-1.2, 0)$, $(1, 2.5)$, $(2, 0)$, $(4, 5.8)$
End Behavior: Left: None Right: $y = \frac{1}{2}x + 3$
Range: $y \in [0, 2.5] \cup [5.8, \infty)$

2. Given the traits below, sketch a graph of the function.

Domain: $x \in [-5, -3] \cup (-1, 1) \cup [3, 5]$
Zeros: $(-5, 0)$, $(-3, 0)$, $(-5, 0)$, $(-3, 0)$
VA: $x = \pm 1$
POE: None
y-int: $(0, 3.5)$
Extreme Points: $(-5, 0)$, $(-4, 2)$, $(-3, 0)$, $(0, 3.5)$, $(3, 0)$, $(4, 2)$, $(5, 0)$
End Behavior: Left: None Right: None
Range: $y \in [0, 2] \cup [3.5, \infty)$

Find the traits and sketch.

3. $y = \sqrt{x^2 - 4x - 5}$

4. $y = -\sqrt{9 - 4x^2}$

5. $y = \sqrt{5 - 4x - x^2}$

6. $y=\sqrt{5-x}$ on $x\in[0,\ 5]$

7. $y=\sqrt{x^3+3x^2-6x-8}$ on $x\in[-4,\ 4]$

8. $y=-\sqrt{6x-x^2-x^3}$

9. $y=-\sqrt{\dfrac{x^2-1}{x^2-4}}$

10. $y=\sqrt{\dfrac{2x^2+7x-4}{x^3+3x^2-6x-8}}$

11. $y=-\sqrt{x^4-3x^2-10}$ on $x\in[-5,\ 3]$

12. $y=\sqrt{-x^4+13x^2-36}$

13. $F(x)=\sqrt{\dfrac{3x}{x^2+9}}$ on $x\in[0,\ 4]$

14. $y=\sqrt{x^4-29x^2+100}$ on $x\in[-6,\ 7]$

15. $y=\sqrt{\dfrac{2x^3+x^2-32x-16}{2x^2-5x-3}}$ on $x\in[-4,\ 4]$

16. $y=\sqrt{-x^3-x^2+9x+9}$ on $x\in[-5,\ 3]$

17. $y=\sqrt{-x^2+5x-6}$

18. $y=\sqrt{x^2-4x-21}$ on $x\in[-5,\ 10]$

19. $y=\sqrt{4x^4-37x^2+9}$ on $x\in[-4,\ 4]$

8-4 Multiple Choice Homework

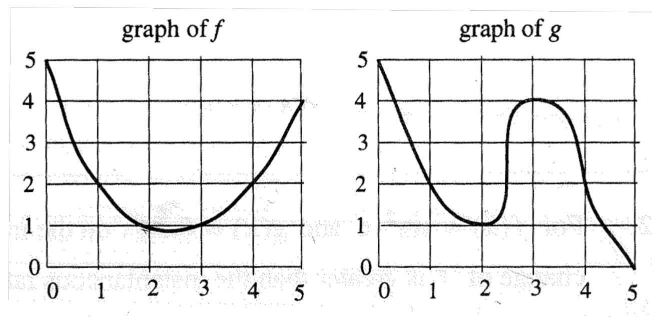
1. Find the absolute maximum value of $y=\sqrt{81-x^2}$ on the interval $[-3,\ 3]$.

a) 10 b) 0 c) 9 d) 8 e) 1

---

2. The graphs of the functions f and g are shown in the figures. If $h(x)=g(f(x))$, which of the following statements are true about the function h?

I. $h(0)=4$
II. h is increasing at $x=2$
III. The graph of h has a horizontal tangent at $x=4$.



a) I only b) II only c) I and II only

d) II and III only e) I, II, and III

---

3. The end behavior of $y=\sqrt{12x-4x^2-x^3}$ is

a) Up on both ends

b) Up on the left and down on the right

c) Up on the left and none on the right

d) Down on the left and none on the right

e) Up on the right and none on the left

---

4. Which of the following statements is true about $y=-\sqrt{\dfrac{x^2-1}{x^2-4}}$.

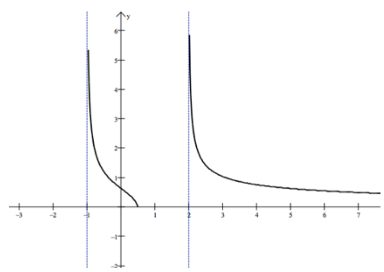
I. The end behavior is $y = 1$

II. The range is $y \le 0$

III. The y-intercept is $\left(0, \frac{1}{2}\right)$

a) I only b) II only c) I and II only

d) II and III only e) I, II, and III



5. Which of the following sign patterns apply to the graph above?

I.

y	DNE	+	0		DNE	+
x	-1		$\frac{1}{2}$		2	

II.

y^2	DNE	+	0		DNE	+
x	-1		$\frac{1}{2}$		2	

III.

$\frac{dy}{dx}$	DNE	$-$	DNE		DNE	$-$
x	-1		$\frac{1}{2}$		2	

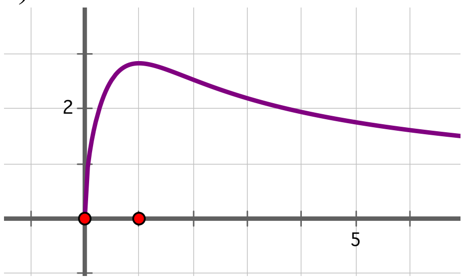
a) I only b) II only c) I and II only

d) I and III only e) I, II, and III

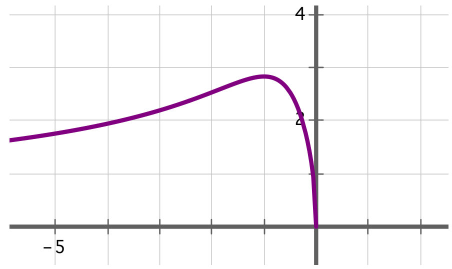
---

6. Which of the following graphs match the equation $y = \sqrt{\frac{x^2}{x^2 - 4}}$?

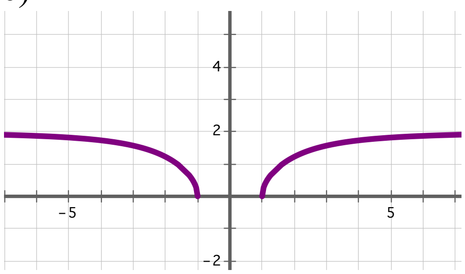
a)



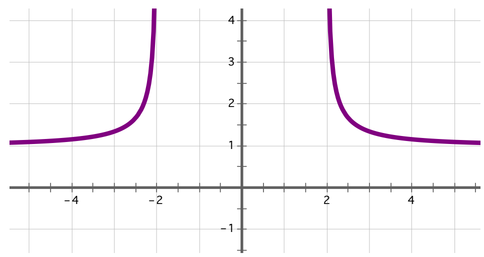
b)



c)



d)



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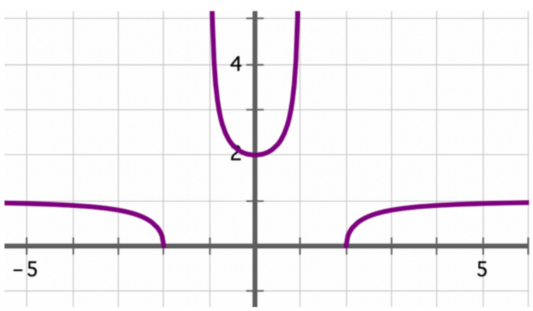
7. Which of the following equations matches the graph below?



a) $y=\sqrt{\dfrac{x^2-4}{x^2-1}}$ b) $y=\sqrt{\dfrac{x^2-1}{x^2-4}}$ c) $y=\sqrt{\dfrac{4-x^2}{x^2-1}}$

d) $y=\sqrt{\dfrac{1-x^2}{x^2-4}}$ e) $y=\sqrt{\dfrac{x^2-4}{x^2}}$

8-5: Implicit Differentiation

In the Chain Rule section, composite functions were denoted as $f\left(g\left(x\right)\right)$. But if y is a function of x, expressions like y^3 or $4y^3+8y-5$ are also composite functions, because $y=g\left(x\right)$. For example, if $y=x+\sqrt{x^2+1}$, then $y^3=\left(x+\sqrt{x^2+1}\right)^3$.

> LEARNING OUTCOMES
>
> Take derivatives of relations implicitly.
> Use implicit differentiation to find higher order derivatives.

EX 1 Find $\frac{d}{dx}\left[y^5\right]$.

Note that in $\frac{d}{dx}\left[y^5\right]$, the derivative in terms of x. If it were in terms of y, or $\frac{d}{dy}\left[y^5\right]$, the derivative would be simple:

$$\frac{d}{dy}\left[y^5\right]=5y^4$$

But for $\frac{d}{dx}\left[y^5\right]$, it is implied that y is a function of x. By the chain rule, $\frac{d}{dx}\left[y^5\right]$should equal $5y^4$ times the derivative of y. The actual function that y equals is unknown, but its derivative is $\frac{dy}{dx}$. So,

$$\frac{d}{dx}\left[y^5\right]=5y^4\frac{dy}{dx}$$

In terms of functions, this may not be very interesting or important. But consider a non-function, like this circle.

EX 2 Find $\frac{dy}{dx}$ if $x^2+y^2=25$.

$$\frac{d}{dx}\left[x^2+y^2=25\right]\to\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}\left(y^2\right)=\frac{d}{dx}\left(25\right)$$
$$2x+2y\frac{dy}{dx}=0$$

Now isolate $\frac{dy}{dx}$

$$2y\frac{dy}{dx}=-2x$$
$$\frac{dy}{dx}=-\frac{x}{y}$$

The Conics share a common standard equation, namely,

$$Ax^2+Cy^2+Dx+Ey+F=0.$$

For now, we will ignore the *Bxy* term because it complicates everything. Assuming the conic is non-degenerate, we could identify the shape of the conic by the A and C in the equation.

Circle	A = C
Parabola	A or C is 0
Ellipse	$A\neq C$, but both are positive
Hyperbola	A or C is negative

EX 3 Find $\frac{dy}{dx}$ for the hyperbola $x^2 - 3y^2 + 4x - 12y - 2 = 0$.

$$\frac{d}{dx}\left[x^2 - 3y^2 + 4x - 12y - 2 = 0\right]$$
$$2x - 6y\frac{dy}{dx} + 4 - 12\frac{dy}{dx} = 0$$
$$2x + 4 = 6y\frac{dy}{dx} + 12\frac{dy}{dx}$$
$$2x + 4 = (6y + 12)\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{2x+4}{6y+12}$$
$$= \frac{x+2}{3y+6}$$

EX 4: Find $\frac{dy}{dx}$ for $\frac{y^2+1}{x^2-1} = 3y$.

$$\frac{d}{dx}\left[\frac{y^2+1}{x^2-1} = 3y\right]$$
$$\frac{(x^2-1)\left(2y\frac{dy}{dx}\right) - (y^2+1)(2x)}{(x^2-1)^2} = 3\frac{dy}{dx}$$
$$(x^2-1)\left(2y\frac{dy}{dx}\right) - (y^2+1)(2x) = 3\frac{dy}{dx}$$
$$(x^2-1)\left(2y\frac{dy}{dx}\right) - 3\frac{dy}{dx} = (y^2+1)(2x)$$
$$(2x^2y - 2y - 3)\frac{dy}{dx} = (y^2+1)(2x)$$
$$\frac{dy}{dx} = \frac{2x(y^2+1)}{2x^2y - 2y - 3}$$

8-5 Free Response Homework

Find $\frac{dy}{dx}$ for each of these relations.

1. $2x^2+2y^2-10x+2y-5=0$

2. $y^2+4x+16y+4=0$

3. $9x^2+4y^2+36x-8y+4=0$

4. $12x^2-4y^2+72x+16y+44=0$

5. $y^2=\frac{x-1}{x+1}$

6. $x^2=\frac{x-y}{x+y}$

7. $x^2+y^2=\frac{x}{y}$

8. $y^2=\frac{x-y}{x+y}$

9. $y^2=\frac{x^2-1}{x+2}$

10.

Find the line tangent to each of these conics at the given point.

11. $x^2+y^2+6x-14y+58=0$ at $(0,\ 8)$

12. $x^2+10x-20y+25=0$ at $(-5,\ 0)$

13. $x^2+25y^2+6x-100y+9=0$ at $(3,\ 3.6)$

14. $x^2-y^2-6y-3=0$ at $(\sqrt{3},\ 0)$

15. $9x^2+4y^2+36x-8y+4=0$ at $(0,\ -2)$

16. $12x^2-4y^2+72x+16y+44=0$ at $(-1,\ -3)$

<u>8-5 Multiple Choice Homework</u>

1. If $y^2-3x=7$, then $\frac{dy}{dx}=$

a) $-\frac{6}{7y^3}$ b) $-\frac{3}{y^3}$ c) 3 d) $\frac{3}{2y}$ e) $-\frac{9}{4y^3}$

---

2. For what values of x does the curve $y^2-x^3-15x^2=8$ have horizontal tangent lines?

a) $x=-10$ only b) $x=0$ only c) $x=10$ only

d) $x=0$ and $x=-10$ e) $x=-10$, $x=0$, and $x=10$

---

3. An equation of the tangent line to the curve $x^2+y^2=169$ at the point $(5,\ -12)$ is

a) $5y-12x=-120$ b) $5x-12y=119$ c) $5x-12y=169$

d) $12x+5y=0$ e) $12x+5y=169$

---

4. If $x^2 + y^3 = 10$, then when $x = 2$, $\frac{dy}{dx} =$

a) $-\frac{2}{3}$ b) $-\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{3}$ e) $\frac{3}{2}$

---

5. Use Implicit Differentiation to find the points on $x^3 - y^2 + x^2 - 1 = 0$ which have horizontal tangent lines.

a) $(0, \pm 1)$

b) $(-1.5, 0)$ only

c) $\left(-\frac{2}{3}, \pm 1.072\right)$ only

d) $(0, \pm 1)$ and $\left(-\frac{2}{3}, \pm 1.072\right)$

e) The tangent line is never horizontal

---

6. At what point on the curve $x^2 - y^2 + x = 2$ is the tangent line vertical?

a) $(1, 0)$ only

b) $(-2, 0)$only

c) $\left(1, \sqrt{2}\right)$ only

d) $(1, 0)$ and $(-2, 0)$

e) The tangent line is never vertical.

8-6: Related Rates

Derivatives have primarily been interpreted as the slope of the tangent line. But, as with rectilinear motion, there are other contexts for the derivative. One overarching concept is that a derivative is a **Rate of Change**. The tendency is to think of rates as distance per time unit, like miles/hour or feet/second, but even slope is a rate of change—it is just that rise and run are both measured as distances.

LEARNING OUTCOME
Solve related rates problems.

The idea behind related rates is two-fold. First, change is occurring in two or more measurements that are related to each other by the geometry (or algebra) of the situation. Second, an implicit chain rule situation exists in that the x and y-values are functions of time, which may or may not be a variable in the problem. Therefore, when taking the derivative of an x or y, an **Implicit Rate Term** $\left(\frac{dx}{dt} \text{ or } \frac{dy}{dt}\right)$ occurs. A classic problem is the falling ladder.

Common Formulas for Related Rates Problems

Pythagorean Theorem:

$x^2 + y^2 = r^2$

Area Formulas:

Circle $A = \pi r^2$

Rectangle $A = lw$

Trapezoid $A = \frac{1}{2}h\left(b_1 + b_2\right)$

Volume Formulas:

Sphere $V = \frac{4}{3}\pi r^3$

Right Prism $V = Bh$

Cylinder $V = \pi r^2 h$

Cone $V = \frac{1}{3}\pi r^2 h$

Right Pyramid $V = \frac{1}{3}Bh$

Surface Area Formulas:

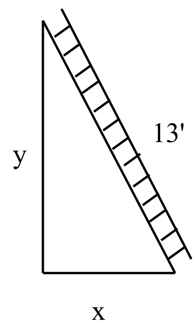
Sphere $S = 4\pi r^2$

Cylinder $S = 2\pi r^2 + 2\pi rl$

Cone $S = \pi r^2 + \pi rl$

Right Prism $S = 2B + Ph$

EX 1 A 13-foot-tall ladder is leaning against a wall. The bottom of the ladder slides away from the wall at 4 ft/sec. How fast is the top of the ladder moving down the wall when the ladder is 5 feet from the wall?



As can be seen in the picture, the height of the top of the ladder and the distance the bottom of the ladder is from the wall are related by the Pythagorean theorem. Both are variables, because the ladder is moving. Therefore,

$$x^2 + y^2 = 13^2$$

4 ft/sec is the rate at which the *x*-value is changing—i.e. $\frac{dx}{dt}$. To find $\frac{dy}{dt}$, differentiate $x^2 + y^2 = 13^2$ to get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

This is essentially an equation in four variables. But *x* and $\frac{dx}{dt}$ are known, and $y = 12$ (by the Pythagorean theorem).
So,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2(5)(4) + 2(12)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{5}{3} \text{ ft / sec}$$

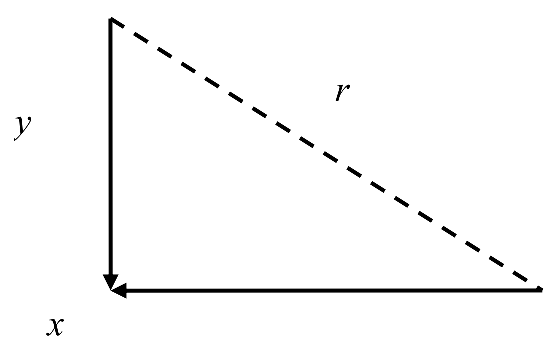
It should make sense that $\frac{dy}{dt}$ is negative since the top of the ladder is falling.

Another common related rate problem is where a tank of a particular shape is filling or draining.

Process for Related Rates Problems:

1. Determine what is being asked.
   a. Look at the units to determine what is given and what is asked.
2. Determine the equation that relates the variable to each other. (NB. This will be the one to be differentiated)
3. Determine what is given.
   a. Look at the units to determine what is given and what is asked.
4. If there is a product of two variables, eliminate the product by either multiplying or substituting a secondary equation.
   a. Note: This is only because we have not learned the Product Rule yet.
5. Differentiate in terms of time.
   a. Do not forget the implicit fractions.
6. Substitute and solve for the missing variable.

Ex 2 Two cars approach an intersection, one traveling south at 20 mph and the other traveling west at 30 mph. How fast is the direct distance between them decreasing when the westbound car is .6 miles and the southbound car is .8 miles from the intersection?



As we can see in the picture, the distance between the two cars are related by the Pythagorean Theorem.

$$x^2 + y^2 = r^2$$

We know several pieces of information. The southbound car is moving at 20 mph; i.e. $\frac{dy}{dt} = -20$. By similar logic we can deduce each of the following:

$$\frac{dy}{dt} = -20 \qquad \frac{dx}{dt} = -30$$
$$y = 0.8 \qquad x = 0.6$$

And, by the Pythagorean Theorem, $r = 1.0$

Now we take the derivative of the Pythagorean Theorem and get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$

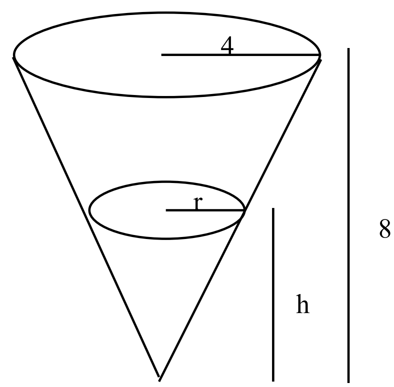
This is essentially an equation in six variables. But we know five of those six variables, so just substitute and solve.

$$2(0.8)(-30) + 2(0.6)(-20) = 2(1.0)\frac{dr}{dt}$$

$$\frac{dr}{dt} = -36 \ \ miles/hour$$

It should make sense that $\frac{dr}{dt}$ is negative since the two cars are approaching one another. We know the units based on the fraction, . Since *r* was in miles and *t* was in hours, our final units must be miles/hour.

EX 3 A tank shaped like an inverted cone 8 feet in height and with a base diameter of 8 feet is filling at a rate of 10 ft^3/minute. How fast is the height changing when the water is 6 feet deep?



The units on the 10 tell us that it is the change in volume, or $\frac{dV}{dt}$. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. But this equation has too many variables for us to differentiate it as it stands. Since the rate of change of the height—i.e., $\frac{dh}{dt}$—was what the question is, eliminate the *r* from the equation. By similar triangles, $\frac{r}{h} = \frac{4}{8}$ and $r = \frac{1}{2}h$. Substitution gives a volume equation in terms of height only:

$$\begin{aligned} V &= \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h \\ &= \frac{\pi}{12}h^3 \end{aligned}$$

Differentiate and plug into to solve for $\frac{dh}{dt}$.

$$\begin{aligned} V &= \frac{\pi}{12}h^3 \\ \frac{dV}{dt} &= \frac{\pi}{4}h^2\frac{dh}{dt} \\ 10 &= \frac{\pi}{4}(6)^2\frac{dh}{dt} \end{aligned}$$

$$\frac{dh}{dt}=\frac{10}{9\pi}\text{ ft/min}$$

Ex 4 If two resistors are connected in parallel, then the total resistance is given by the formula $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$, where all values for *R* are in Ohms (Ω). If R_1 and R_2 are increasing at rates of $0.3\ \Omega/\text{sec}$ and $0.2\ \Omega/\text{sec}$, respectively, find how fast R is changing when $R_1=80\ \Omega$ and $R_2=100\ \Omega$.

$$\frac{d}{dt}\left[\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}\right]\rightarrow\frac{d}{dt}\left[R^{-1}=\left(R_1\right)^{-1}+\left(R_2\right)^{-1}\right]$$

$$-R^{-2}\left(\frac{dR}{dt}\right)=-R_1^{-2}\left(\frac{dR_1}{dt}\right)-R_2^{-2}\left(\frac{dR_2}{dt}\right)$$

$$-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-2}\left(\frac{dR}{dt}\right)=-R_1^{-2}\left(\frac{dR_1}{dt}\right)-R_2^{-2}\left(\frac{dR_2}{dt}\right)$$

$$-\left(\frac{1}{80}+\frac{1}{100}\right)^{-2}\left(\frac{dR}{dt}\right)=-\left(80\right)^{-2}\left(0.3\right)-\left(100\right)^{-2}\left(0.2\right)$$

$$\frac{dR}{dt}=\frac{107}{810}\text{ or }0.132\ \Omega/\text{sec}$$

8-6 Free Response Homework

1. Two boats leave an island at the same time, one heading north and one heading east. The northbound boat is moving at 12 mph and the eastbound boat is traveling at 5 mph. At $t = 0.2$ hours, the northbound boat is 1.4 miles away from the island and the eastbound boat is 1 mile from the island.

a) Draw a picture of the situation at any time *t*.

b) What variables are given in the problem?

c) What and known quantities are given? And what is the unknown for which to solve?

d) What equation(s) relates the quantities? And which one will be differentiated?

e) How fast is the distance between the two ships increasing at $t = 0.2$ hours?

2. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the intersection and watches an eastbound train traveling 60m/sec.

a) Draw a picture of the situation at any time *t*.

b) What variables are given in the problem?

c) What and known quantities are given? And what is the unknown for which to solve?

d) What equation(s) relates the quantities? And which one will be differentiated?

e) At how many m/sec is the train moving away from the observer 4 second after it passes the intersection?

3. A circular ink stain is spreading (i.e. the radius is changing) at half an inch per minute.

a) Draw a picture of the situation at any time *t*.

b) What variables are given in the problem?

c) What and known quantities are given? And what is the unknown for which to solve?

d) What equation(s) relates the quantities? And which one will be differentiated?

e) How fast is the area changing when the stain has a 1-inch diameter?

4. A screensaver has a rectangular logo that expands and contracts as it moves around the screen. The ratio of the sides stays constant, with the long side being 1.5 times the short side. At a particular moment, the long side is 3 cm, and perimeter is changing by .25 cm/sec.

a) Draw a picture of the situation at any time *t*.
b) What variables are given in the problem?
c) What and known quantities are given? And what is the unknown for which to solve?
d) What equation(s) relates the quantities? And which one will be differentiated?
e) How fast is the area of the rectangle changing at that moment?

5. Sand is dumped onto a pile at 30π ft^3/min. The pile forms a cone with the height always equal to the base diameter.
a) Draw a picture of the situation at any time *t*.
b) What variables are given in the problem?
c) What and known quantities are given? And what is the unknown for which to solve?
d) What equation(s) relates the quantities? And which one will be differentiated?
e) How fast is the height changing when the pile is 5 feet high?

6. A cylindrical oil tank of height 30' and radius 10' is leaking at a rate of 300 ft^3/min.

a) What variables are given in the problem?
b) What and known quantities are given? And what is the unknown for which to solve?
c) What equation(s) relates the quantities? And which one will be differentiated?
d) How fast is the oil level dropping?

7. Water is leaking out of an inverted conical tank at a rate of 5000 cm^3/min. If the tank is 8 m tall and has a diameter of 4 m.
a) Draw a picture of the situation at any time *t*?
b) What variables are given in the problem?
c) What and known quantities are given? And what is the unknown for which to solve?
d) What equation(s) relates the quantities? And which one will be differentiated?
e) Find the rate at which the height is decreasing when the water level is at 3 m.
f) Then find the rate of change of the radius at that same instant.

8. A spherical balloon is being inflated so that its volume is increasing at a rate of 6 ft^3/min. How fast is the radius changing when $r = 10$ ft?

9. Sand is dumped onto a pile at 30π ft^3/min. The pile forms a cone with the height always equal to the base diameter.

a) How fast is the radius of the base of the pile changing when the pile is 5 feet high?

b) How fast is the base area changing when the pile is 10 feet high?

10. The side of a cube is expanding at a constant rate of 6 inches per second. What is the rate of change of the volume, in in^3 per second, when the total surface area of the cube is 54 in^2?

11. A 25-foot tall ladder is leaning against a wall. The bottom of the ladder is pushed toward the wall at 5 ft/sec. How fast is the top of the ladder moving up the wall when it is 7 feet up?

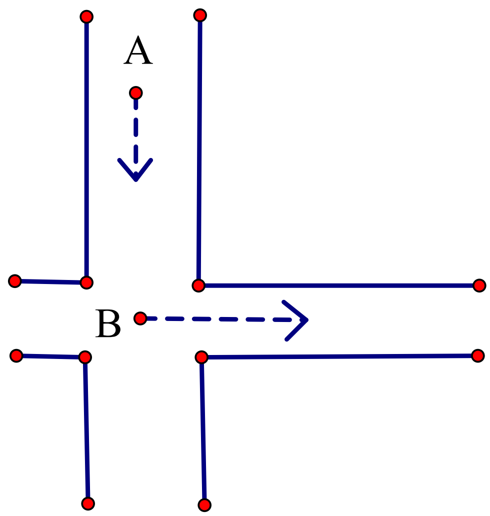
12. The altitude of a triangle is increasing at a rate of 2 cm/sec at the same time that the area of the triangle is increasing at a rate of 5 cm^2/sec. At what rate is the base increasing when the altitude is 12 cm and the area is 144 cm^2?

13. Two cars start moving away from the same point. One travels south at 60 mph, and the other travels west at 25 mph. At what rate is the distance between the cars increasing two hours later?

14. According to Boyle's Law, gas pressure varies directly with temperature and inversely with volume $\left(\text{or } P = \frac{kT}{V}\right)$. Suppose that the temperature is held constant while the pressure increases at 20 kPa/min. What is the rate of change of the volume when the volume is 600 in^3 and the pressure is 150 kPa?

---

15. Person A is 220 feet north of an intersection and walking toward it at $10\ {}^{ft}/_{sec}$. Person B starts at the intersection and walks east at $5\ {}^{ft}/_{sec}$.



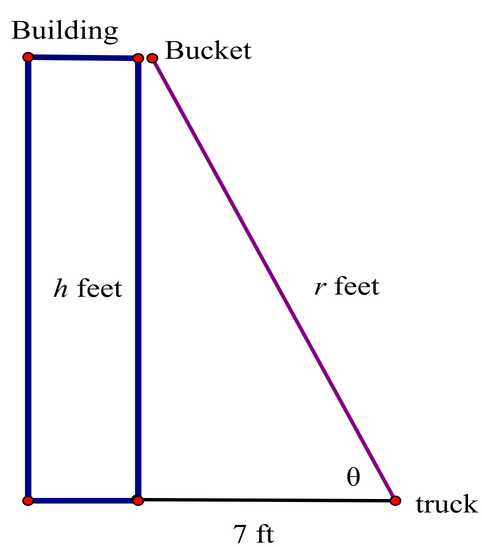
a) At $t = 10$ seconds, how far is each person from the intersection?

b) At $t = 10$ seconds, how far apart are the two people?

c) How fast is the distance between the two people changing at $t = 10$ seconds?

---

16. A fire truck is parked 7 feet away from the base of a building and its ladder is extended to the top of the building. The ladder retracts at a rate of 0.5 feet per second, while the angle of the ladder changes such that the bucket at the end of the ladder comes down vertically.



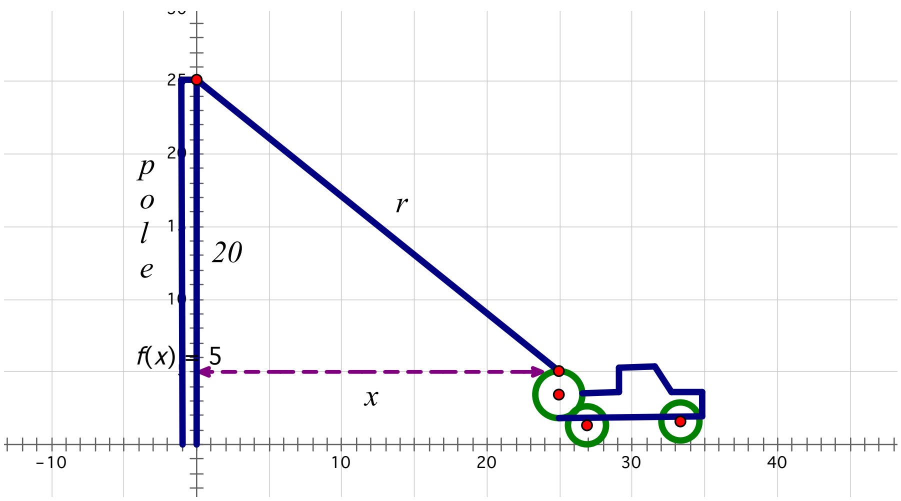
a) How far is the ladder extended when the bucket is 10 feet above the ground?

b) Find the rate at which the bucket is dropping vertically when the bucket is 10 feet above the ground.

c) What is the relationship between the angle θ and the height of the bucket? Find θ, in radians, when the bucket is 10 feet above the ground.

---

17. A telephone crew is replacing a phone line from one telephone pole to the next. The line is on a spool on the back of a truck, and one end is attached to the top of a 25' pole. The vertical distance from the top of the pole to the level of the spool is 20'.



The truck moves down the street at $20\ {}^{ft}\!/_{sec}$.

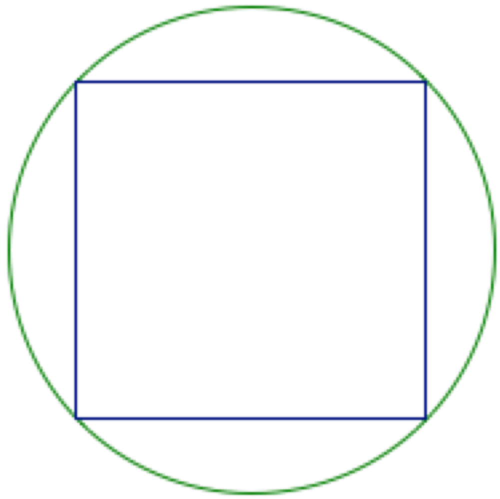
a) Find the length of line that has been rolled out when $t = 15$ sec.

b) Find the rate at which the telephone line is coming off the spool when the truck is 50 feet from the pole.

c) What is the relationship between the angle θ and the truck's distance from the pole? EC. Find θ, in radians, when the truck is 40 feet from the pole.

---

18. A square is inscribed in a circle as shown. As the square expands, the circle expands to maintain the four contact points. The perimeter of the square is increasing at a rate of 4 inches per second.

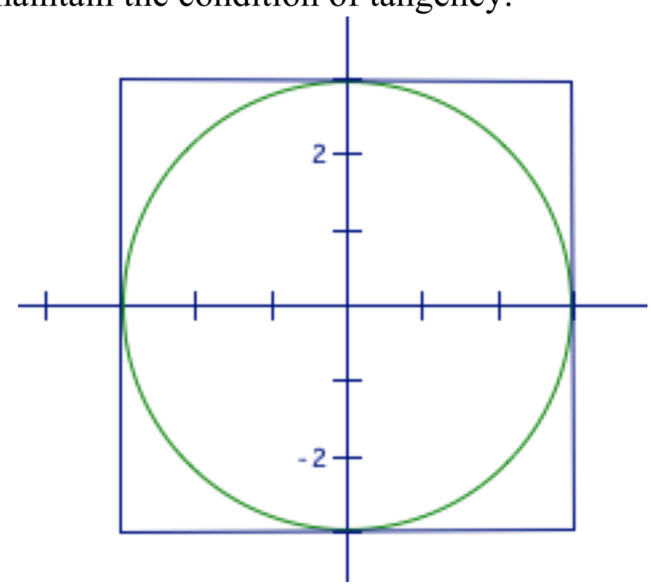


a. Find the rate of change of the circumference of the circle.

b. At the instant when the area of the square is 16 square inches, find the rate at which the area between the square and the circle is increasing.

---

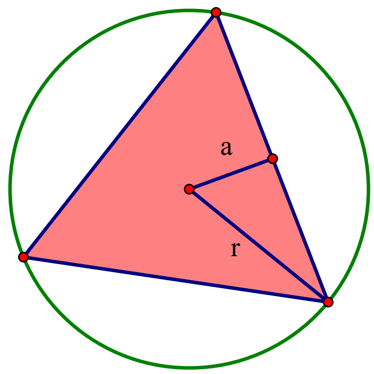
19. A circle is inscribed in a square as shown. The circumference of the circle is increasing at a constant rate of 4 inches per second. As the circle expands, the square expands to maintain the condition of tangency.



a) Find the rate of change of the perimeter of the square.
b) At the instant when the area of the circle is 16π square inches, find the rate at which the area between the square and the circle is increasing.

---

20. An equilateral triangle is inscribed in a circle. The circle's circumference is expanding at 6π in/sec and the triangle maintains the contact of its corners with the circle.



Given that the area of an equilateral triangle is equal to half the apothem a times the perimeter p, find out how fast the area inside the circle but outside the triangle is expanding when the area of the circle is $64\pi\ in$. [Hint: Find p and a in terms of r.]

<u>8-6 Multiple Choice Homework</u>

1. The side of a cube is expanding at a constant rate of 6 inches per second. What is the rate of change of the volume, in in^3 per second, when the total surface area of the cube is 54 in^2?

a) 324 b) 108 c) 18 d) 162 e) 54

---

2. Gravel is being dumped from a conveyor belt at a rate of 35 ft^3/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 15ft high?

a) 0.27 ft/min b) 1.24 ft/min c) 0.14 ft/min

d) 0.2 ft/min e) 0.6 ft/min

---

3. Two cars start moving from the same point. One travels south at 28 mi/h and the other travels west at 70 mi/h. At what rate is the distance between the cards increasing 5 hours later?

a) 75.42 mi/h b) 75.49 mi/h c) 76.4 mi/h

d) 75.39 mi/h e) 75.38 mi/h

---

4. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 cm, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area *S* of a sphere with radius *r* is $S = 4\pi r^2$.)

a) -108π b) -72π c) -48π d) -24π e) -16π

---

5. Water is flowing into a spherical tank with 6-foot radius at the constant rate of 30π cu ft per hour. When the water is *h* feet deep, the volume of the water in the tank is given by $V = \frac{\pi h^2}{3}(18 - h)$. What is the rate at which the depth of the water in the tank is increasing the moment when the water is 2 feet deep?

a) 0.5 ft/hour b) 1.0 ft/hour c) 1.5 ft/hour

d) 2.0 ft/hour e) 2.5 ft/hour

---

6. The radius of a circle is in increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

a) 0.04π m^2/sec b) 0.4π m^2/sec c) 4π m^2/sec

d) 20π m^2/sec e) 100π m^2/sec

---

7. If the rate of change of a number *x* with respect to time *t*, is *x*, what is the rate of change of the reciprocal of the number when $x = -\frac{1}{4}$?

a) –16 b) –4 c) $-\frac{1}{48}$ d) $\frac{1}{48}$ e) 4

---

8. A Golden Rectangle is one where the ratio (called ϕ)of the length to the short side *w* to the long side *l* is equal to the ratio of the long side to the sum of the two sides. In other words, $l = 1.236w$. If a Golden Rectangle changes such that *w* is growing at 2 in/min, how fast is the area changing when *w* is 5 inches?

a) 1.236 in^2/min b) 12.36 in^2/min c) 2.472 in^2/min

d) 24.72 in^2/min e) 30.9 in^2/min

Radical Functions Practice Test
Part 1: CALCULATOR REQUIRED

Round to 3 decimal places. Show all work.

Multiple Choice (3 points each)

1. Use the linear approximation of the function $f(x)=\sqrt{9-x}$ at $a=0$ to approximate the number $\sqrt{9.09}$.

(a) 3.02
(b) 0.15
(c) 7.44
(d) 7.40
(e) 2.25

2. Find the tangent to the semicircle $y=\sqrt{4-x^2}$ at the point $\left(1,\ \sqrt{3}\right)$.

(a) $y=-0.58x+3.31$
(b) $y=-0.58x+2.31$
(c) $y=0.42x+2.31$
(d) $y=-1.58x+1.31$
(e) None of these

Free Response (10 pts. each)

1. Find the zeros and domain of $y=-\sqrt{2x^3+5x^2+x-2}$.

2. Find the zeros and domain of $y=\sqrt{\dfrac{3x^2-4x+1}{3x^3-13x^2+13x-3}}$.

3. Make a sign pattern, find the intervals of increasing, and find the extreme points for $y=\sqrt{\dfrac{3x^2-4x+1}{3x^3-13x^2+13x-3}}$.

4. List all traits and **sketch** $y=-\sqrt{2x^3+5x^2+x-2}$.

Domain:

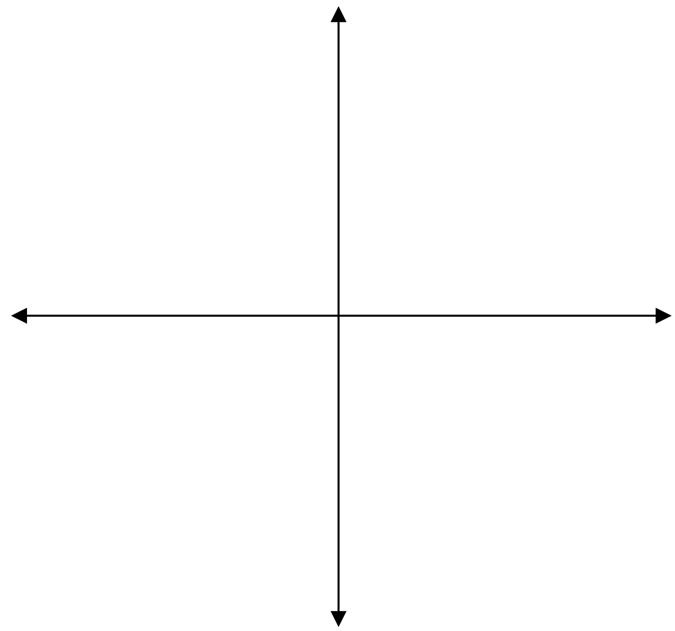
Zeros:

y-int:

VAs:

EB:

POEs:



Extreme Points:

Range:

Radical Functions Practice Test
Part 2: NO CALCULATOR ALLOWED

Round to 3 decimal places. Show all work.

Multiple Choice (3 pts. each)

3. $\lim_{x\to\infty}\frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}$

(a) 4
(b) 1
(c) $\frac{1}{4}$
(d) 0
(e) –1

4. If $\frac{d}{dx}\left[f(x)\right]=g(x)$ and $\left[g(x)\right]=f(3x)$, then $\frac{d}{dx}\left[f(x^2)\right]$ is

(a) $4x^2 f\left(3x^2\right)+2g\left(x^2\right)$
(b) $f\left(3x^2\right)$
(c) $f\left(x^4\right)$
(d) $2x\,f\left(3x^2\right)+2g\left(x^2\right)$
(e) $2x\,f\left(3x^2\right)$

Free Response (10 pts. each)

5. List all traits and **sketch** $y=\sqrt{\dfrac{3x^2-4x+1}{3x^3-13x^2+13x-3}}$.

Domain:

Zeros:

y-int:

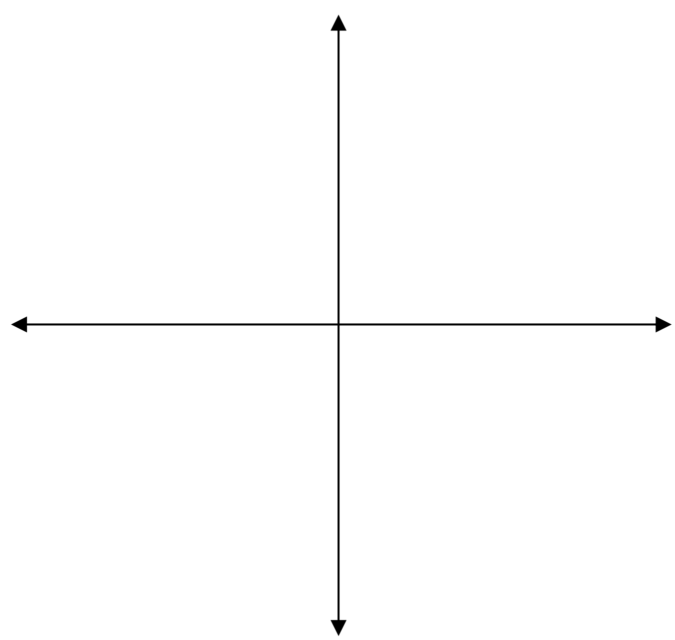
VAs:

POEs:

EB:

Extreme Points:

Range:



Chapter 8 Homework Answer Key

8-1 Free Response Homework

1. Zeros: $(-3,0),(3,0)$; Domain: $x\in[-3,3]$

2. Zeros: $(-3,0),(7,0)$; Domain: $x\in(-\infty,-3]\cup[7,\infty)$

3. Zeros: $(1\pm\sqrt{6},0)$; Domain: $x\in(-\infty,1-\sqrt{6}]\cup[1+\sqrt{6},\infty)$

4. Zeros: $(-5,0),(-2,0),(4,0)$; Domain: $x\in[-5,-2]\cup[4,\infty)$

5. Zeros: $(1,0),(3,0)$; Domain: $x\in\{1\}\cup[3,\infty)$

6. Zeros: $(1,0),(\pm\sqrt{5},0),(2,0)$; Domain: $x\in(-\infty,-\sqrt{5}]\cup$ $[-2,\ 1]\cup[\sqrt{5},\infty)$

7. Zeros: $(-1,0),(\pm 3,0)$; Domain: $x\in(-\infty,-3]\cup[-1,3]$

8. Zeros: $(2,0),(3,0)$; Domain: $x\in[2,3]$

9. Zeros: $(\pm 4,0),(\pm 5,0)$; Domain: $x\in[-5,-4]\cup[4,5]$

10. Zeros: $\left(-\frac{1}{2},0\right),(\pm 4,0)$; Domain: $x\in\left[-4,-1/2\right]\cup(4,\infty)$

11. Zeros: $\left(\frac{7}{3},0\right),\left(-\frac{3}{2},0\right)$; Domain: $x\in(-\infty,-\frac{3}{2}]\cup[\frac{7}{3},\infty)$

12. Zeros: $\left(\pm\frac{1}{2},0\right),(\pm 3,0)$; Domain: $\in(-\infty,-3]\cup\left[-1/2,1/2\right]\cup[3,\infty)$

13. Zeros: $(0,0)$; Domain: $x\in(-2,0]\cup(2,\infty)$

14. Zeros: $(0,0)$; VAs: none; Domain: $x\in[0,\ \infty)$

15. Zeros: $(\pm 3,0)$; VAs: $x=\pm 2$; Domain: $x\in(-\infty,-3]\cup(-2,\ 2)\cup[3,\ \infty)$

16. Zeros: $(\pm 2,0)$; VAs: $x = \pm 3$; Domain: $x \in (-\infty, -3) \cup [-2,\ 2] \cup (3,\ \infty)$

17. Zeros: $(0,0)$; VAs: none; Domain: $x \in [0,\ \infty)$

18. Zeros: $(\pm 2,0)$; VAs: none; POE at $x = \frac{1}{4}$; Domain:
$x \in [-2,\ -\frac{1}{4}) \cup (-\frac{1}{4},\ 2]$

19. Zeros: $(0,0)$; VAs: $x = -1$; POE at $x = -3$; Domain:
$x \in (-\infty, -3] \cup (-1,\ 0] \cup (3,\ \infty)$

20. Zeros: $(-3,0)$, $(0,0)$; VAs: $x = 3,\ -1$; Domain:
$x \in (-\infty, -3) \cup (-3,\ 0] \cup (1,\ \infty)$

8-1 Multiple Choice Homework

1. D 2. E 3. B 4. E 5. E

8-2 Free Response Homework

1. $\dfrac{1}{(2x+25)^{1/2}}$ 2. $\dfrac{1}{(3x+2)^{2/3}}$ 3. $\dfrac{2x+1}{2(x^2+x+13)^{1/2}}$

4. $\dfrac{3x-2}{(3x^2-4x+9)^{1/2}}$ 5. $\dfrac{-4x}{(1-4x^2)^{1/2}}$ 6. $-\dfrac{9}{2}(5-3x)^{1/2}$

7. $\dfrac{dy}{dx} = \dfrac{-2x^3+41x}{\sqrt{-x^4+41x^2-400}}$ 8. $\dfrac{dy}{dx} = \dfrac{3x^2+x-16}{\sqrt{2x^3+x^2-32x-16}}$

9. $\dfrac{dy}{dx} = \dfrac{-3x^2 - 2x + 9}{2\sqrt{-x^3 - x^2 + 9x + 9}}$

10. $\dfrac{dy}{dx} = \dfrac{-2x + 5}{2\sqrt{-x^2 + 5x - 6}}$

11. $\dfrac{dy}{dx} = \dfrac{2x - 5}{2\sqrt{x^2 - 5x - 21}}$

12. $\dfrac{dy}{dx} = \dfrac{8x^3 - 37x}{\sqrt{4x^4 - 37x^2 + 9}}$

13. $\dfrac{dy}{dx} = \dfrac{-x^2 - 4}{2x^{1/2}\left(x^2 - 4\right)^{3/2}}$

14. $\dfrac{dy}{dx} = \dfrac{-x^2 + 4}{(4x)^{1/2}\left(x^2 + 4\right)^{3/2}}$

15. $\dfrac{dy}{dx} = \dfrac{5x}{\left(x^2 - 9\right)^{1/2}\left(x^2 - 4\right)^{3/2}}$

16. $\dfrac{dy}{dx} = \dfrac{-5x}{\left(x^2 - 4\right)^{1/2}\left(x^2 - 9\right)^{3/2}}$

17. $\dfrac{-3\left(x^2 - 9\right)}{(6x)^{1/2}\left(x^2 + 9\right)^{3/2}}$

18. $\dfrac{dy}{dx} = \dfrac{-13x}{\left(4 - x^2\right)^{1/2}\left(x^2 + 9\right)^{3/2}}$

19. $\dfrac{dy}{dx} = \dfrac{-1}{2(x)^{1/2}(x - 1)^{3/2}}$

20. $\dfrac{dy}{dx} = \dfrac{-5x^2 - 6x - 9}{2\left(x^2 + 3x\right)^{1/2}\left(x^2 - 2x - 3\right)^{3/2}}$

21. $5\left(4x^2 - 5x + 1\right)^4(8x - 5)$

22. $\dfrac{5}{3}(x - 2)^{2/3}$

23. $\dfrac{-1}{2(1 - 2x)^{3/4}}$

24. $\dfrac{3x^2 - 2}{7\left(x^3 - 2x\right)^{6/7}}$

25. $\dfrac{-x}{5\left(16 - x^2\right)^{9/10}}$

26. $\dfrac{110(4x - 1)^9}{(3 - x)^{11}}$

27. $\dfrac{x}{\left(25 - x^2\right)^{3/2}}$

28. $\dfrac{-2x^2 + 2x - 8}{5(2x - 1)^{4/5}\left(x^2 - 4\right)^{6/5}}$

29. $\dfrac{-25}{(5x+6)^{1/2}(5x-4)^{3/2}}$

30. $\dfrac{3x^2+2x+5}{4\left(x^3+x^2+5x+5\right)^{3/4}}$

31. $\dfrac{x^2+4x-1}{\left(2x^2-x\right)^{1/2}(x+2)^{3/2}}$

32. $\dfrac{14\left(x^2-3x+9\right)\left(x^2-9\right)^{4/3}}{3(2x-3)^{10/3}}$

33. $\dfrac{1}{(2x+1)^{2/3}(1-x)^{4/3}}$

34. $f'\left(\sqrt{5}\right)=\dfrac{-\sqrt{5}}{3}$

8-2 Multiple Choice Homework

1.	B	2.	E	3.	D	4.	B
5.	B	6.	B	7.	A	8.	D
9.	D						

8-3 Free Response Homework

1. 2.025

3. Domain: $x\in\left(-\infty,\dfrac{5}{3}\right]$ c.v.: $x=\dfrac{5}{3}$

4. Domain: $x\in(-\infty,-1]\cup[5,\infty)$ c.v.s: $x=-1,5$

5. Domain: $x\in\left[-\sqrt{3},\dfrac{4}{9}\right]\cup[\sqrt{3},\infty)$ c.v.s: $x=\dfrac{4}{9},\pm\sqrt{3},-0.863$

6. Domain: $x\in[-3,-2]\cup[2,3]$ c.v.s: $x=\pm\sqrt{\dfrac{13}{2}},\pm 3,\pm 2$

7. Domain: $x \in (-\infty, 9]$ c.v.: $x = 9$

8. c.v.s: $x = 0, -1, 1$ Domain: $x \in (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$

9. c.v.s: $x = 0, \pm 2$ Domain: $x \in (-\infty, -2] \cup (-1, 1) \cup [2, \infty)$

10. c.v.s: $x = 0, -5$ Domain: $x \in [-\infty, 0]$

11. $(\pm 2, 0), (\pm 5, 0), (\pm\sqrt{20.5}, 4.5)$

12. $\left(-\frac{1}{2}, 0\right), (\pm 4, 0), (-2.482, 6.245)$

13. $(\pm 3, 0), (-1, 0), (1.431, 4.111)$

14. $\left(\frac{5}{2}, \frac{1}{2}\right), (2, 0), (3, 0)$

15. $(7, 0), (-3, 0)$

16. $(\pm 3, 0), \left(\pm\frac{1}{2}, 0\right), (0, 3)$

17. $(0, 0), (-2, \sqrt{2})$

18. $(0, 0)$

19. $\left(0, \frac{3}{2}\right), (\pm 3, 0)$

20. $(2, 0), (3, 0)$

21. $(0, 0), \left(3\sqrt{3}, -\frac{3}{\sqrt{2}}\right)$

22. $(0, 0)$

23. No extreme points

24. $(0, 0)$ and $(-3, 0)$

25. $(-0.816, 1.012), (0.816, -1.012)$

26. $(2, 0)$

27. No extreme points

28. $(\pm 3, 0)$ and $(0, 2)$

29. $x = 0$ is at a maximum, and $x = -\frac{3}{2}$ and $x = \frac{3}{2}$ are both at minimums.

30. $(\pm\sqrt{2}, 0), (\pm\sqrt{6}, 0)$ are minimums and $(0, 2\sqrt{3})$ is a maximum.

31. $(0,0)$ is a minimum and $\left(3,\frac{1}{\sqrt{2}}\right)$ is a maximum.

32. $(0,0)$ is a minimum

33. $v(2)=\frac{2}{\sqrt{5+(2)^2}}=\frac{2}{3}$

34a. $\left(\frac{1}{\sqrt{2}},\frac{3}{4}\right)$ b. $(0.177,-0.391)$

35a. $t\in(5,10)$ b. Never at rest

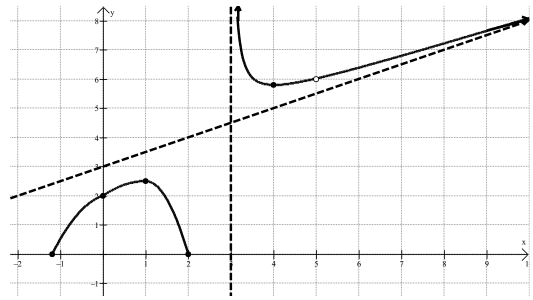
36. $x=7.574$ miles Cost = $257.95

8-3 Multiple Choice Homework

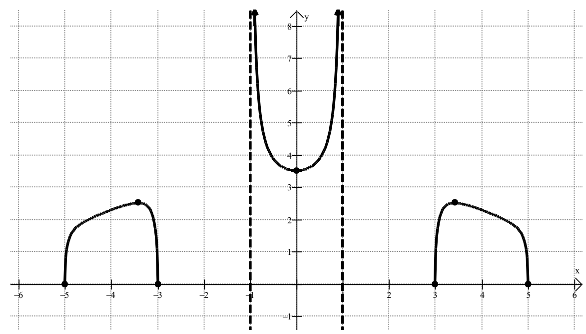
1.	C	2.	D	3.	B	4.	E
5.	D	6.	E	7.	D	8.	E

8-4 Free Response Homework

1.



2.



3. Domain: $x \in (-\infty, -1] \cup [5, \infty)$

Range: $y \in [0, \infty)$

Zeros: $(-1,0),(5,0)$

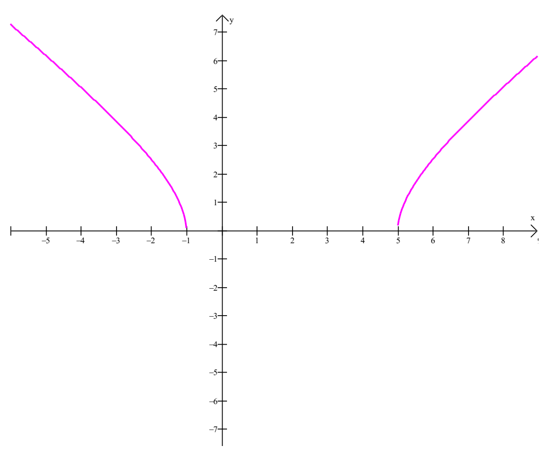
y-int: None

VA: None

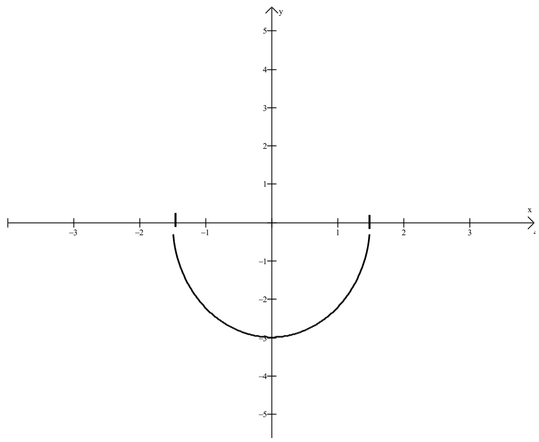
POE: None

EB: Both ends up

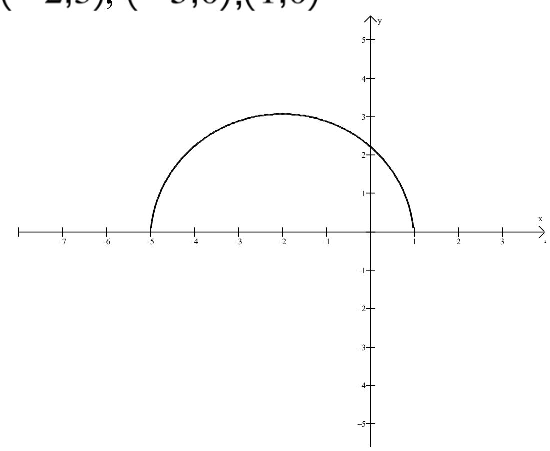
Extreme Points: $(-1,0),(5,0)$



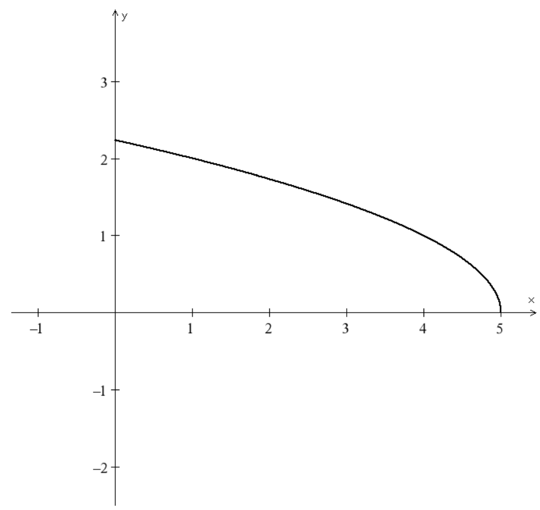
4. Domain: $x \in [-1.5, 1.5]$ Range: $y \in [-3, 0]$
Zeros: $(-1.5, 0), (1.5, 0)$ y-int: $(0, -3)$
VA: None POE: None
EB: None Extreme Points: $(0, -3)$, $(-1.5, 0), (1.5, 0)$



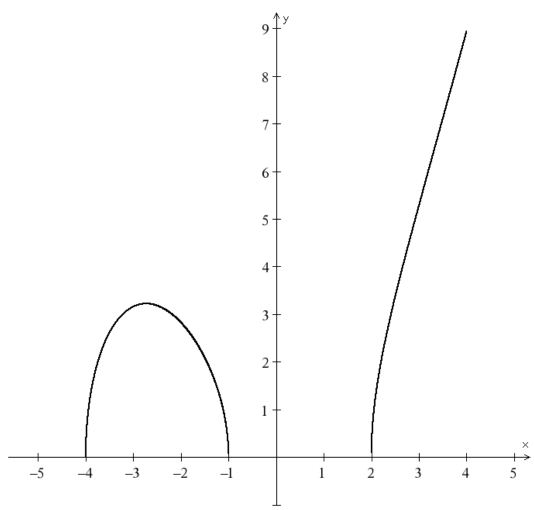
5. Domain: $x \in [-5, 1]$ Range: $y \in [0, 3]$
Zeros: $(-5, 0), (1, 0)$ y-int: $(0, \sqrt{5})$
EB: None VA: None POE: None
Extreme Points: $(-2, 3)$, $(-5, 0), (1, 0)$



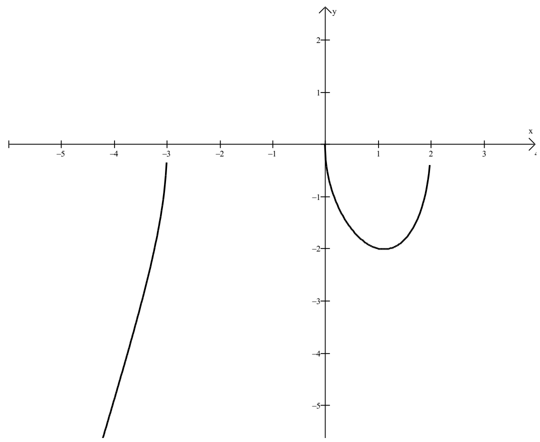
6. Domain: $x \in [0,5]$ Range: $y \in [0,\sqrt{5}]$
Zero: $(5,0)$ y-int: $(0,\sqrt{5})$ EB: None
VA: None POE: None Extreme Points: $(5,0)$, $(0,\sqrt{5})$



7. Domain: $x \in [-4,-1] \cup [2,4]$ Range: $y \in [0,8.994]$
Zeros: $(-4,0),(-1,0),(2,0)$ y-int: None
EB: None VA: None POE: None
Extreme Points: $(-4,0)$, $(-2.732,3.224)$, $(-1,0)$, $(2,0)$, $(4,8.994)$



8. Domain: $x \in (-\infty, -3] \cup [0,2]$ Range: $y \in (-\infty, 0]$
Zeros: $(-3,0),(0,0),(2,0)$ y-int: $(0,0)$
EB: Left end down, end none
VA: None POE: None
Extreme Points: $(-3,0),(0,0),(1.120,-2.015),(2,0)$

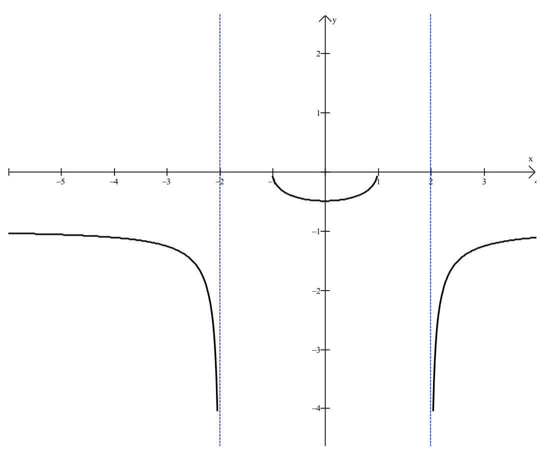


9. Domain: $x \in (-\infty, -2) \cup [-1,1] \cup (2,\infty)$ Range:
$y \in (-\infty, -1) \cup \left[-\frac{1}{2}, 0\right]$

Zeros: $(1,0),(-1,0)$ y-int: $\left(0, -\frac{1}{2}\right)$

VA: $x = -2, x = 2$ POE: None EB: HA $y = -1$

Extreme Points: $\left(0, -\frac{1}{2}\right)$, $(1,0),(-1,0)$



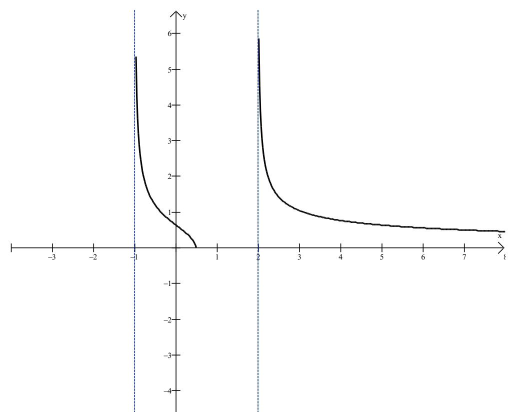
10. Domain: $x \in (-1, \frac{1}{2}] \cup (2, \infty)$ Range: $y \in [0, \infty)$

Zeros: $\left(\frac{1}{2}, 0\right)$ y-int: $\left(0, \frac{1}{\sqrt{2}}\right)$

VA: $x = -1, x = 2$ POE: None

EB: HA $y = 0$ on right, left end none

Extreme Points: $\left(\frac{1}{2}, 0\right)$

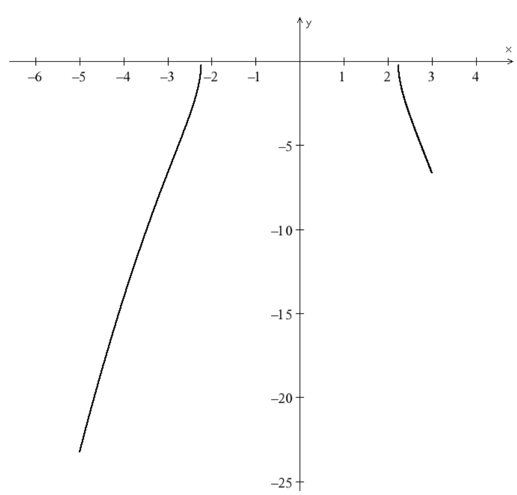


11. Domain: $x \in [-5, -\sqrt{5}] \cup [\sqrt{5}, 3]$ Range: $y \in [-23.238, 0]$

Zeros: $(\pm\sqrt{5}, 0)$ y-int: None

VA: None POE: None EB: None

Extreme Points: $(\pm\sqrt{5}, 0)$,

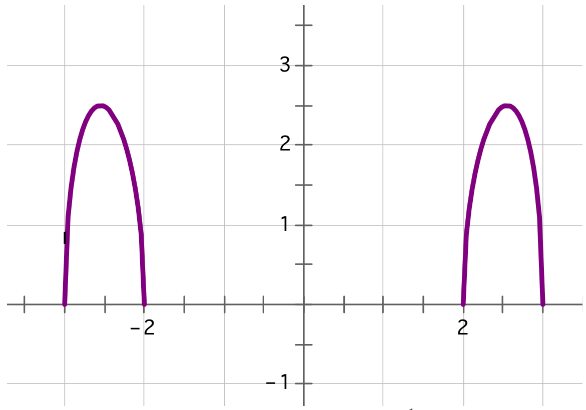


12. Domain: $x \in [-3, -2] \cup [2, 3]$ Range: $y \in [0, 2.5]$

Zeros: $(-3,0), (-2,0), (2,0), (3,0)$ y-int: None

VA: None POE: None EB: None

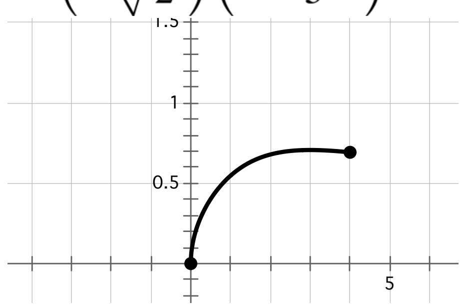
Extreme Points: $(-3,0)$, $(-2,0),(2,0),(3,0),\left(\sqrt{\frac{13}{2}},2.5\right),\left(-\sqrt{\frac{13}{2}},2.5\right)$



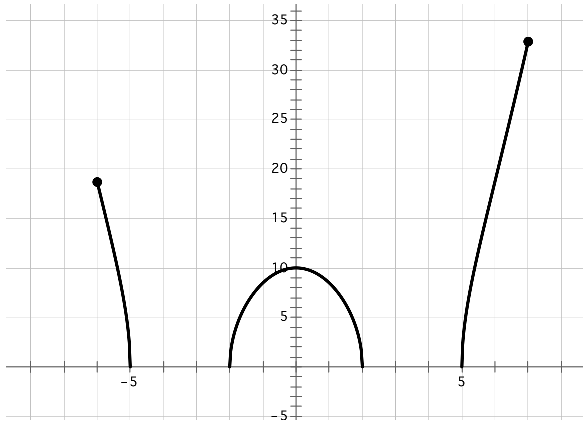
13. Domain: $x \in [0,4]$ Range: $y \in \left[0,\frac{1}{\sqrt{2}}\right]$ Zeros: $(0,0)$

y-int: $(0,0)$ VA: None POE: None EB: None

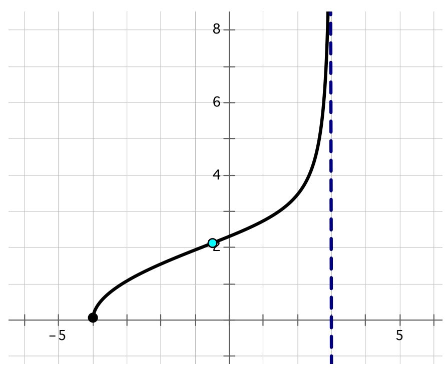
Extreme Points: $(0,0)$, $\left(3,\frac{1}{\sqrt{2}}\right),\left(4,\frac{2\sqrt{3}}{5}\right)$



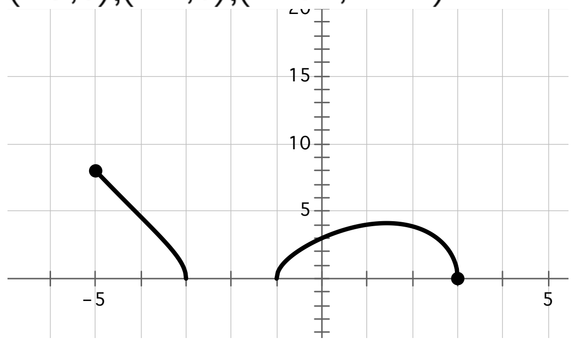
14. Domain: $x \in [-6,-5] \cup [-2,2] \cup [5,7]$ Range: $y \in [0,32.863]$
Zeros: $(-5,0),(-2,0),(2,0),(5,0)$ y-int: None
VA: None POE: None EB: None
Extreme Points: $(\pm 2,0),(\pm 5,0),(-6,18.762),(7,32.863)$



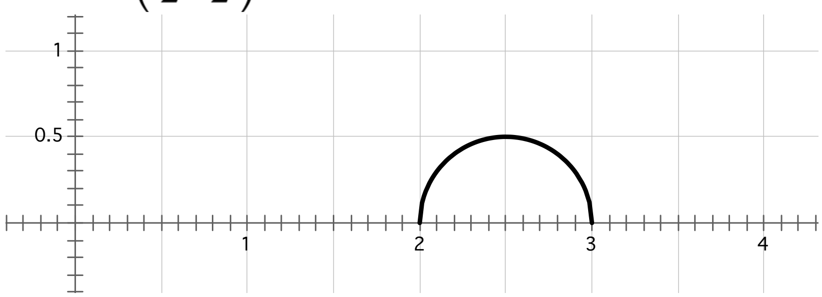
15. Domain: Range: $y \in [0,2.121) \cup (2.121, \infty)$
Zeros: $(-4,0)$ y-int: None
VA: $x = 3$ POE: $\left(-\frac{1}{2},2.121\right)$ EB: None
Extreme Points: $(-4,0)$



16. Domain: $x \in [-5,-3] \cup [-1,3]$ Range: $y \in [0, 4.111]$
Zeros: $(\pm 3,0),(-1,0)$ y-int: $(0,3)$
VA: None POE: None EB: None
Extreme Points: $(\pm 3,0),(-1,0),(1.431,4.111)$



17. Domain: $x \in [2,3]$ Range: $y \in [0,.5]$
Zeros: $(2,0),(3,0)$ y-int: None
VA: None POE: None EB: None
Extreme Points: $\left(\frac{5}{2},\frac{1}{2}\right),(2,0),(3,0)$

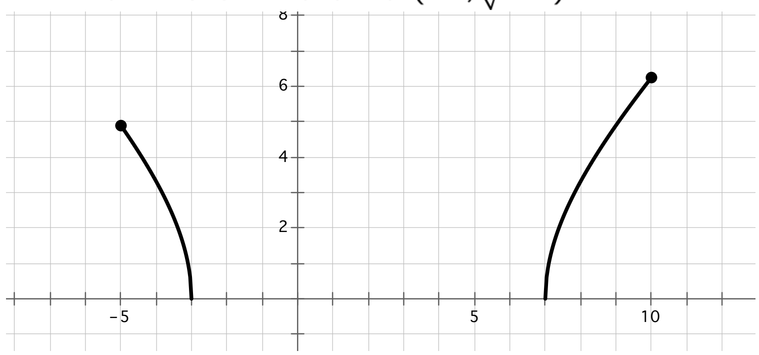


18. Domain: $x \in [-5,-3] \cup [7,10]$ Range: $y \in [0,,\sqrt{39}]$

Zeros: $(-3,0),(7,0)$ *y*-int: None

VA: None POE: None EB: None

Extreme Points: $(-5,5),(-3,0),(7,0),(10,\sqrt{39})$

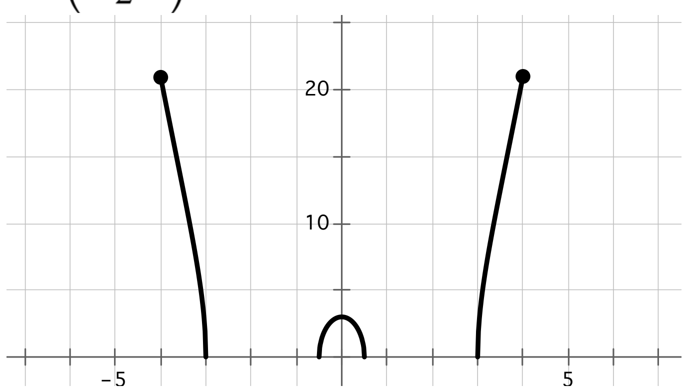


19. Domain: $x \in \left[-\frac{1}{2},\frac{1}{2}\right]$ Range: $y \in [0,21]$

Zeros: $\left(\pm\frac{1}{2},0\right)$, $(\pm 3,\ 0)$ *y*-int: $(0,3)$

VA: None POE: None EB: None

Extreme Points: $\left(\pm\frac{1}{2},0\right)$, $(\pm 3,\ 0)$, $(\pm 4,21),(0,3)$



<u>8-4 Multiple Choice Homework</u>

1. C 2. D 3. C 4. A 5. D 6. D

7. A

<u>8-5 Free Response Homework</u>

1. $\frac{5-2x}{2y+1}$ 2. $\frac{-2}{y+8}$ 3. $\frac{9(x+2)}{4(1-y)}$

4. $\frac{-3(x+3)}{2-y}$ 5. $\frac{1}{y(x+1)^2}$ 6. $\frac{x(x+y)^2-y}{-x}$

7. $\frac{dy}{dx}=\frac{y-2xy^2}{2y^3+x}$ 8. $\frac{dy}{dx}=\frac{y}{y(x+y)^2+x}$

9. $\frac{dy}{dx}=\frac{x^2+4x+1}{2y(x+2)^2}$ 10. $\frac{dy}{dx}=\frac{-2x^2+8x-1}{(x-4)^2}$

11. $y-8=-3(x-0)$ 12. $y=0$ 13. $y-3.6=-\frac{3}{20}(x-3)$

14. $y=\frac{\sqrt{3}}{3}\left(x-\sqrt{3}\right)=\frac{\sqrt{3}}{3}x-1$ 15. $y+2=\frac{3}{2}(x-0)$

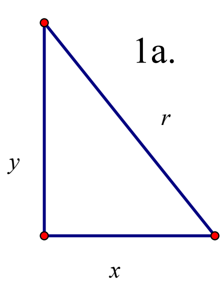
16. $y+3=-\frac{6}{5}(x+1)$

8-5 Multiple Choice Homework

1. D 2. D 3. C 4. B

5. D 6. D

8-6 Free Response Homework



1a.

b) $x, y, r, \frac{dx}{dt}, \frac{dy}{dt}, and \frac{dr}{dt}$

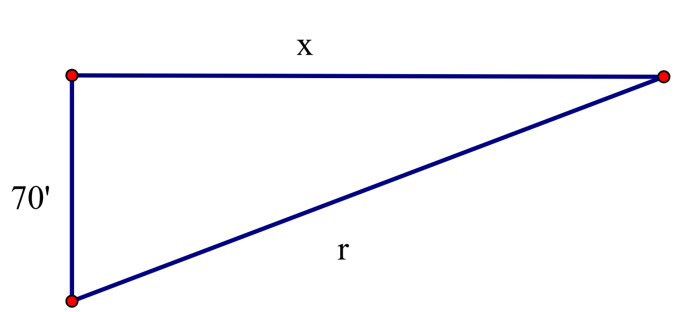
c) $x=1, y=1.4, \frac{dx}{dt}=5, \frac{dy}{dt}=12,$
$1^2+1.4^2=r^2 \rightarrow r=1.720.$

Find $\frac{dr}{dt}$

d) $x^2 + y^2 = r^2$

e) $\frac{288}{5} ft/\text{sec}$

2.



b) $x,\ r,\ \frac{dx}{dt},\ and\ \frac{dr}{dt}$

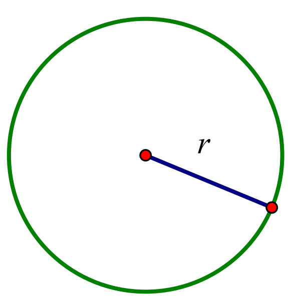
c) $\frac{dx}{dt} = 60$

$x|_{t=4} = \left(60\frac{ft}{\text{sec}}\right)(4\text{sec}) = 240ft$
$,\ r = 250$
Find $\frac{dr}{dt}$

d) $x^2 + 70^2 = r^2$

e) 12.674

3a.



b) $A,\ d,\ r,\ \frac{dA}{dt},\ and\ \frac{dr}{dt}$

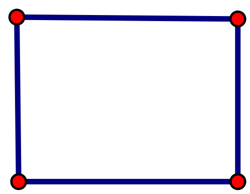
c) $\frac{dr}{dt} = \frac{1}{2}\text{in}/\text{min},\ d = 1\text{in} \rightarrow (r = \frac{1}{2}\text{ in})$

Find $\frac{dA}{dt}$

d) $A = \pi r^2$

e) $\frac{\pi}{2}\text{in}^2/\text{min}$

4a.



b) $l,\ s, p, A,\ \frac{dp}{dt},\ and\ \frac{dA}{dt}$

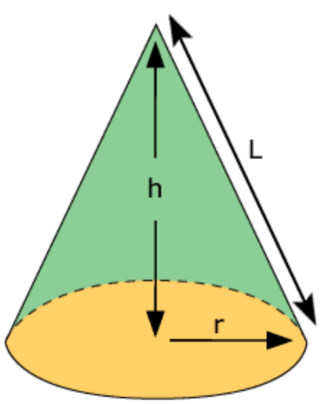
c) $\frac{dp}{dt} = .25\text{cm}/\text{sec},\ l = 3 \rightarrow s = 2$

Find $\frac{dA}{dt}$

d) $l = 1.5s,$
$p = 2l + 2s = 2(1.5s) + 2s = 5s,\ A = ls$

e) $1.25\text{cm}^2/\text{sec}$

5a.



b) $Volume,\ diameter,\ height,\ radius,\ \frac{dV}{dt},\ and\ \frac{dh}{dt}$

c) $\frac{dV}{dt} = 30\pi ft^3/\text{min},\ \text{h} = 5$
$\text{h} = 5 \rightarrow \text{r} = 2.5$

Find $\frac{dh}{dt}$

d) $\frac{1}{2}h = r; \quad V = \frac{\pi}{3}r^2h = \frac{\pi}{12}h^3;$
$V = \frac{\pi}{3}r^2h = \frac{\pi}{12}h^3$

e) $\frac{6}{5\pi}$

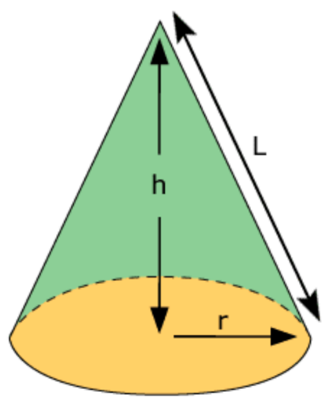
6a) $V,\ h,\ r,\ \frac{dV}{dt},\ and\ \frac{dh}{dt}$

b) $\frac{dV}{dt} = -300 ft^3/\text{min},\ h = 30,\ r = 10.$ Find $\frac{dh}{dt}$

c) $V = 100\pi h$

d) $-\frac{1}{\pi}\ ft/\text{min}$

7a)



b) $h,\ d,\ r,\ V,\ \frac{dh}{dt},\ \frac{dr}{dt},\ \frac{dV}{dt}$

c) $\frac{dV}{dt} = -5000\text{cm}^3/\text{min};\ h = 8;\ d = 4$

$d = 4 \rightarrow r = 2$

Find $\frac{dh}{dt}$ and $\frac{dr}{dt}$

d) $V = \frac{\pi}{3} r^2 h,\ d = 2r$

e) $-\frac{1250}{\pi}\ \text{m}^3/\text{min}$

f) $\frac{dr}{dt} = -\frac{625}{2\pi}\ \text{m}^3/\text{min}$

8. $\frac{6}{5\pi}$

9. $\frac{3}{200\pi}\text{ft}/\text{min}$

10 $162\text{in}^3/\text{sec}$

11. $-\frac{120}{7} ft/\sec$

12. $-\frac{55}{6}$

13. $65 {}^{mi}/_{h}$

14. $-80 in^3/\text{min}$

15a. Person A: $120ft$; Person B: $50ft$ 15b. $130ft$

15c. $-7.308 {}^{ft}/_{\text{sec}}$ 15d. $0.065\ {}^{\text{rad}}/_{\text{sec}}$

16a. $12.207ft$ 16b. 0.610 16c. $0.960\ rads$ 16d. $0.029 {}^{rads}/_{\text{min}}$

17a. $300.666ft$ 17b. $\frac{dr}{dt} = 19.825 {}^{ft}/_{\text{sec}}$

17c. 0.142 17d. $-0.015 {}^{\text{rad}}/_{\text{sec}}$

18a. $\frac{dC}{dt} = \pi x$ 18b. $\frac{dA}{dt} = 4\pi - 8$

19a. πx 19b. $4\pi - 8$

20. $143.002 in^2/\text{sec}$

8-6 Multiple Choice Homework

1. D 2. D 3. D 4. C 5. E 6. C

7. E 8. D

<u>Radical Functions Practice Test Answer Key</u>

<u>Multiple Choice</u>

1. A 2. B 3. C 4. E

<u>Free Response</u>

1. Zeros: $(-2,0),(-1,0),\left(-\frac{1}{2},0\right)$; Domain: $x \in [-2,-1] \cup [\frac{1}{2},\infty)$

2. Zeros: None; Domain: $x \in (3,\infty)$

3. $\begin{array}{ll} y' & \xrightarrow{\text{DNE}-} \\ x & \quad 3 \end{array}$ No extreme points; No intervals of increasing

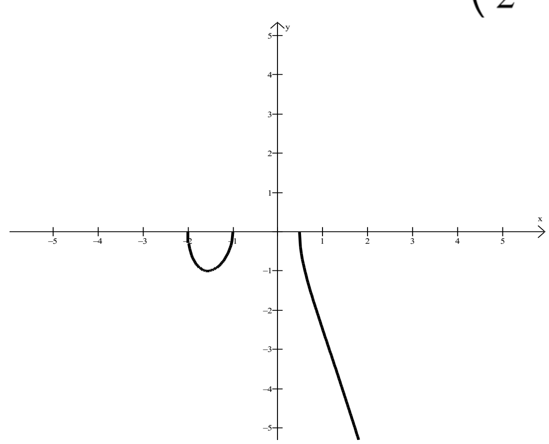
4. Domain: $x \in [-2,-1] \cup [\frac{1}{2},\infty)$ Range: $y \in (-\infty,0]$

Zeros: $(-2,0)$, $(-1,0)$, $\left(\frac{1}{2},0\right)$ y-int: None

EB: Left end none, right end down

VA: None POE: None

Extreme Points: $(-2,0),(-1.560,-1.008),(-1,0),\left(\frac{1}{2},0\right)$



5. Domain: $x \in (3, \infty)$ Range: $y \in (0, \infty)$
Zeros: None y-int: None
EB: Left end none, HA $y = 0$ on right
VA: $x = 3$ POE: None Extreme Points: None