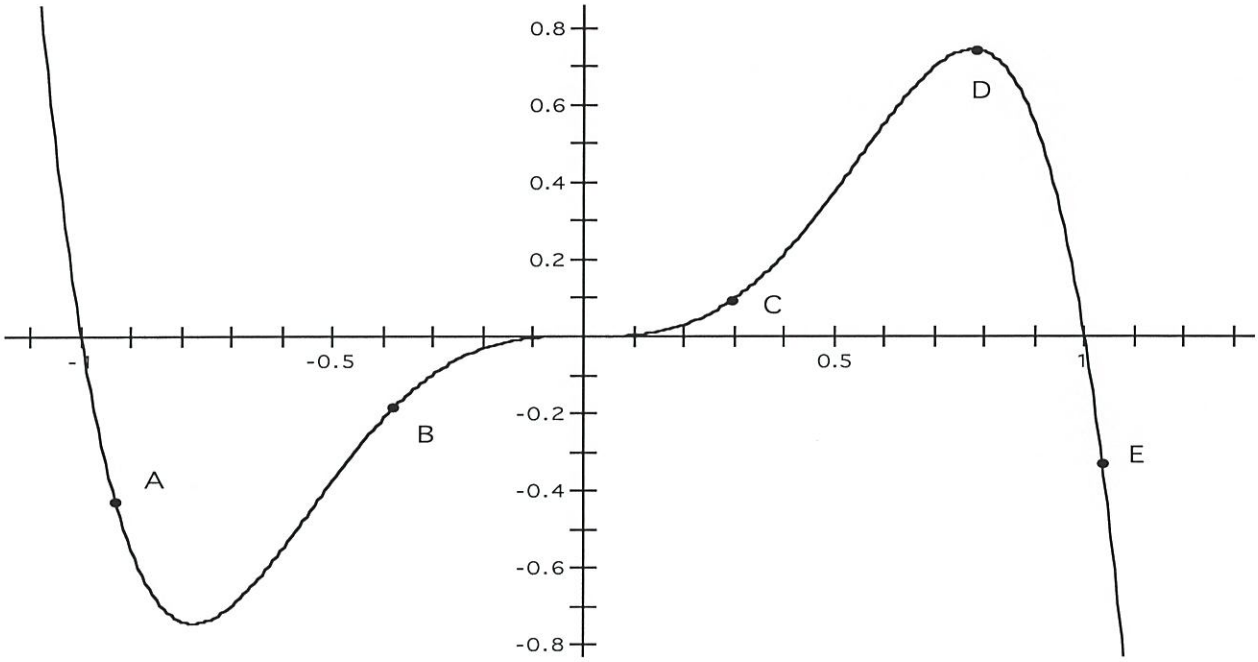
Honors PreCalculus '15-16
Advanced Curve Sketching

Name: Solution Key

score_____

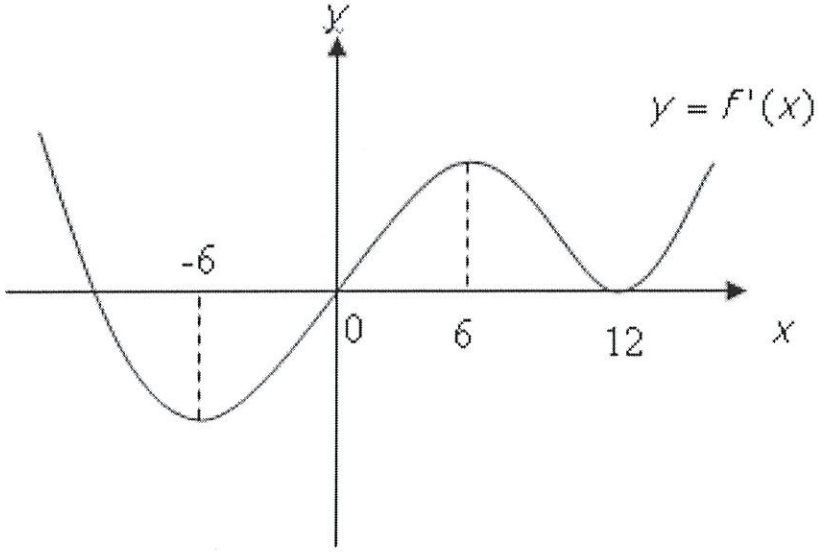


1. The graph of the function $f(x)$ is shown above. At which point on the graph of $f(x)$ is $f'(x)<0$ and $f''(x)<0$?

DEC CONCAVE ↓

(a) A (b) B (c) C (d) D (e) E

2. The graph of the derivative of f is shown below. Which of the following must be true?



a) f is concave down on [0, 12].
b) f is increasing on [-6, 6].
c) f has a local maximum at $x = 0$.
d) f has a local minimum at $x = -6$.
e) f has a point of inflection at $x = 12$.

3. Given this sign pattern, $\begin{array}{c} F''(x) \\ x \end{array} \begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & + \\ & -1 & & 0 & & 2 & \end{array}$ and that $f'(-2) = f'(1) = f'(2) = 0$, which of the following statements must be true?

I. $F(x)$ has a local minimum at $x = 1$ F''<0 ∴ MAX
II. $F(x)$ has a local maximum at $x = -2$
III. $F(x)$ has a point of inflection at $x = 2$ SIGN CHANGE ON F''

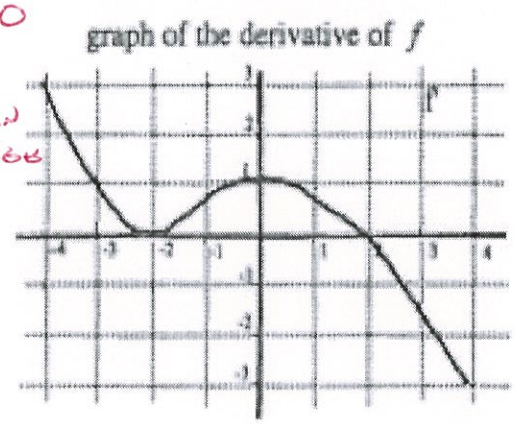
(a) I only
(b) II only
(c) III only
(d) I and III only
(e) II and III only

4. If $f''(x) = x^2(x+1)(x-2)$, then the graph of $f(x)$ has inflection points when $x =$

(a) -1 only
(b) 2 only
(c) -1 and 0 only
(d) -1 and 2 only
(e) -1, 0, and 2 only

5. The graph of the *derivative* of $f(x)$ is shown at the right. Which of the following is true about the function $f(x)$?

I. $f(x)$ is decreasing at $x = 0$ $F'(0) > 0$
II. $f(x)$ has a local maximum at $x = -2$
III. $f(x)$ is concave up at $x = -1$ NO SIGN CHANGE



(a) I only
(b) II only
(c) III only
(d) I and III only
(e) I, II, and III

6. If $f(x) = 6\sin^2(5x)$, then $f''\left(\frac{\pi}{30}\right) =$

(a) 0 (b) 150 (c) $30\sqrt{2}$ (d) 30 (e) $30\sqrt{3}$

$f' = 12 \sin 5x \cos 5x \ (5) = \cancel{60 \sin} \ 60 \sin 5x \cos 5x$

$= 30 \sin 10x$

$f'' = 300 \cos 10x$

$f''(\pi/30) = 300 \cos \frac{\pi}{3} = 150$

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Round to 3 decimal places. score_____

Show all work.

1. Given $y = x^2 e^{-\frac{1}{2}x}$, find the sign pattern for $\frac{dy}{dx}$ and determine if the critical values are at a maximum, minimum, or neither.

$$y' = x^2 e^{-\frac{1}{2}x}\left(-\tfrac{1}{2}\right) + 2xe^{-\frac{1}{2}x^2} = xe^{-\frac{1}{2}x}\left(-\tfrac{1}{2}x+2\right) = 0$$

$$= e^{-\frac{1}{2}x}\left(-\tfrac{1}{2}x^2+2x\right)$$

y': $-$ 0 $+$ 0 $-$

x: 0 (min), 4 (max)

$(0,0)$ $(4, 2.165)$

2. Given $y = x^2 e^{-\frac{1}{2}x}$, find the sign pattern for $\frac{d^2y}{dx^2}$ and name the points of Inflection.

$$y'' = e^{-\frac{1}{2}x}(-x+2) + \left(-\tfrac{1}{2}x^2+2x\right)e^{-\frac{1}{2}x}\left(-\tfrac{1}{2}\right)$$

$$= e^{-\frac{1}{2}x}\left(\tfrac{1}{4}x^2 - 2x + 2\right) = 0$$

$$x = \frac{2 \pm \sqrt{4-2}}{2(1/4)} = 4 \pm 2\sqrt{2} = \begin{cases} 6.828 \\ 1.172 \end{cases}$$

f'': $+$ 0 $-$ 0 $+$

1.172, 6.828

$(1.172, .764)$

$(6.828, 1.534)$

3. Find the domain, zeros and VAs of $y=\ln\left(x^3-36x\right)$.

x^3-36x $\quad - \; 0 \; + \; 0 \; - \; 0 \; +$

x $\quad -6 \quad 0 \quad 6$

DOMAIN $x \in (-6, 0) \cup (6, \infty)$

VAs: $x = 0, \pm 6$

ZEROS: $(-.025, 0)$

$(-5.986, 0)$

$(6.014, 0)$

4. Find the extreme points and end behavior of $y=\ln\left(x^3-36x\right)$.

$$\frac{dy}{dx} = \frac{3x^2-36}{x^3-36x}$$

i) $\frac{dy}{dx} = 0 \rightarrow x^2 - \cancel{18}\,12 = 0$ $\quad (-2\sqrt{3}, 4.421)$

$x = \pm 2\sqrt{3}$

ii) $\frac{dy}{dx}$ DNE $\rightarrow x = \cancel{\pm 6, 0}$

EB LEFT: NONE

RIGHT: UP

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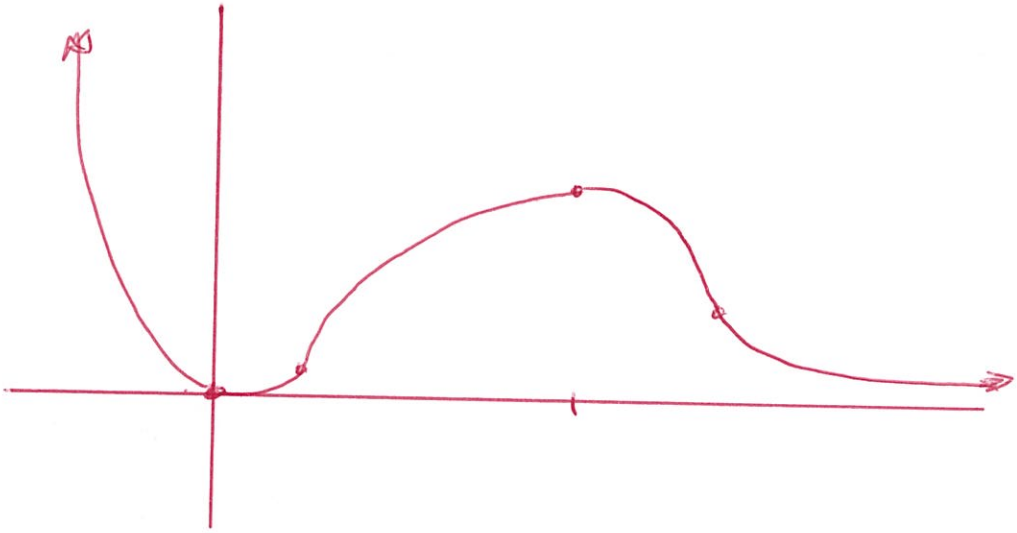
NO CALCULATOR ALLOWED

Show all work.

5. Set up a Key Trait table and sketch $y = x^2 e^{-\frac{1}{2}x}$

x		0		1.772		4		6.828		
y'	−	0	+	+	+	0	−	−	−	
y''	+	+	+	0	−	−	−	0	+	
CONC	∪	MIN	∪	POI	∩	MAX	∩	POI	∪	



6. Show the Algebraic Traits and sketch $y = \ln\left(x^3 - 36x\right)$.

Domain: SEE #3

Range: ALL REALS

Y – Int: NONE

VAs: SEE #3

Zeros: SEE #3

Extreme Points: SEE # 3

End Behavior (Left):

End Behavior (Right):

SEE #4

POEs: NONE