PreCalculus Honors '15-16 Name: SOLUTION KEY
Dr. Quattrin
Rational Functions Test
CALCULATOR ALLOWED Score ___________
Round to 3 decimal places. Show all work.

1. An equation of the line normal to the graph of $y=\dfrac{2x+3}{3x-2}$ at $(1, 5)$ is

a) $13x-y=8$

b) $13x+y=18$

[✓ c)] $x-13y=-64$

d) $x+13y=66$

e) $-2x+3y=13$

$$\frac{dy}{dx}=\frac{(3x-2)(2)-(2x+3)(3)}{(3x-2)^2}$$

$$m_{TAN}=\frac{-13}{1}$$

$$m_{NORM}=1/13$$

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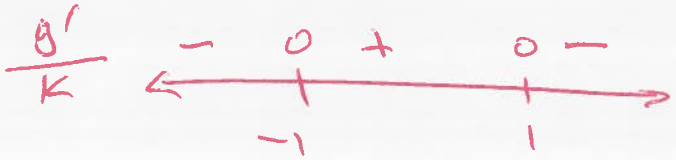
2. A function is defined as $g(x)=\dfrac{kx}{x^2+1}$, where k is a constant. For what values of k, if any, is f strictly increasing on the interval $(-1, 1)$?

(a) $k<0$
[✓ (b)] $k>0$
(c) $k>1$ only
(d) $-1<x<1$
(e) No such Values of k

$$g'=\frac{(x^2+1)k-kx(2x)}{(x^2+1)^2}$$

$$=\frac{k(1-x^2)}{(x^2+1)^2}$$



SIGN PATTERN WITHOUT K

INCREASING ON (−1, 1) MEANS + ∴

K > 0 TO MAINTAIN THE +

3. Let $f(x)$ and $g(x)$ be differentiable functions. The table below gives the values of $f(x)$ and $g(x)$, and their derivatives, at several values of x.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	-6
2	1	8	-5	7
3	7	-2	7	9

If $h(x)=\dfrac{f(x)}{g(x)}$, what is the value of $h'(3)$?

$$\frac{g^{*}(f)'-f^{*}g'}{g^{2}} = \frac{\begin{array}{c}-2(7)-9(7)\\ \cancel{9(7)-7(-2)}\end{array}}{(-2)^{2}}$$

a) -4 b) $\frac{11}{7}$ c) 1 [✓ d)] $-\frac{77}{4}$ e) $\frac{49}{4}$

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4. Suppose $f'(x)=\dfrac{(x+1)^3(x-4)^2}{(x^4+1)}$. Which of the following statements must be true?

[✓ I.] The slope of the line tangent to $y=f(x)$ at $x=1$ is 36.

[✓ II.] $f(x)$ is increasing on $x\in(1, 4)$

~~III.~~ $f(x)$ has a minimum at $x=4$

(a) I only (b) II only (c) III only [✓ (d)] I and II only (e) I, II and III

5. A particle moves along the x-axis at that its position at any time $t \geq 0$ is given by $x(t) = \frac{t}{4+t^2}$. The particle is at rest at $t =$

$$V = \frac{4+t^2 - t(2t)}{(4+t^2)^2}$$

$$\cancel{4-t^2} = 4 - t^2$$

$t = \pm$

a) 0 b) $\frac{1}{4}$ c) 1 [✓ d) 2] e) 4

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6. If $y = \frac{1-x}{x-1}$, then $\frac{dy}{dx} = \frac{(x-1)(-1)-(1-x)(1)}{\cancel{1-x}(x-1)^2} = 0$

OR $y = -1 \therefore \frac{dy}{dx} = 0$

a) -1 [✓ b) 0] c) $\frac{-1}{x-1}$ d) $\frac{-2}{x-1}$ e) $\frac{-2x}{(x-1)^2}$

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7. $\lim\limits_{x\to\infty} \frac{2x^2 - 5x + 3}{2x^3 + 5x^2 - 4x - 12} =$ 0 (DENOM POWER > NUM POWER)

[✓ a) 0] b) 1 c) 2 d) $-\frac{1}{4}$ e) DNE

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1. Find asymptotes, POEs, and zeros of $y=\frac{3x^3-x^2+12x-4}{3x^2-7x+2}$. Show the algebraic work to support the zeros.

$$= \frac{x^2(3x-1)+4(3x-1)}{(3x-1)(x-2)} \approx \frac{x^2+4}{x-2}$$

ZEROS : NONE

VA : $x=2$

POE : $\left(1/3, -\frac{37}{15}\right)$

$$\lim_{x\to 1/3} \frac{x^2+4}{x-2} = \frac{37/9}{-5/3} = -\frac{37}{15}$$

SA : $y = x+2$

$$\begin{array}{r} x+2 \\ x-2 \overline{)\, x^2 \quad +4} \\ -(x^2-2x) \\ 2x \end{array}$$

2. Find the extreme points of $y=\frac{3x^3-x^2+12x-4}{3x^2-7x+2}$ graphically, but show the algebraic work to support the critical values.

$$y \approx \frac{x^2+4}{x-2} \qquad \frac{dy}{dx} = \frac{(x-2)(2x)-(x^2+4)(1)}{(x-2)^2}$$

$$= \frac{x^2-4x-4}{(x-2)^2}$$

i) $\frac{dy}{dx} = 0 \rightarrow \boxed{x = \begin{cases} -.828 \\ 4.828 \end{cases}}$

ii) $\frac{dy}{dx}$ DNE $\Rightarrow x=2$ BUT IT IS NOT IN THE DOMAIN

iii) NO INTERVAL GIVEN

$$4x(x-4)$$

3. $$\frac{d}{dx}\left[\frac{4x^2-16x}{x^3-4x^2-x+4}\right] = \frac{d}{dx}\left(\frac{4x}{x^2-1}\right)$$

$$x^2(x-4)-1(x-4)$$

$$= \frac{(x^2-1)(4)-4x(2x)}{(x^2-1)^2}$$

$$= \frac{-4x^2-4}{(x^2-1)^2}$$

$[-4, 4]$

4. Find the Extreme Points of $y=\frac{-6x}{x^2+9}$ on $x\in \cancel{[-3,3]}$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx} = \frac{(x^2+9)(-6)-(-6x)(2x)}{(x^2+9)^2} = \frac{6x^2-54}{(x^2+9)^2}$$

i) $\frac{dy}{dx} = 0 \rightarrow 6x^2-54=0 \rightarrow x = \pm 3$

ii) $\frac{dy}{dx} =$ DNE → NEVER

iii) ENDPOINTS $x = \pm 4$

$(3, -1)$

$(-3, 1)$

$\left(4, \frac{-24}{25}\right)$

$\left(-4, \frac{24}{25}\right)$

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5. Write an equation of a rational function that has x–intercepts at (-3, 0), VA at $x = 5$, a POE at $x = -2$, and a HA at $y = \frac{6}{5}$.

$$y = \frac{6(x+3)(x+2)}{5(x+2)(x-5)}$$

6. Show the sign pattern and solve $\frac{4x^2 - 16x}{x^3 - 4x^2 - x + 4} > 0$.

$$= \frac{4x(x-4)}{(x-4)(x^2-1)} > 0$$

y	−	VA	+	0	−	VA	+	POE	+
x		−1		0		+1		4	

$x \in (-1, 0) \cup (1, 4) \cup (4, \infty)$

7. Find the traits and **sketch** $y = \frac{-6x}{x^2+9}$ on $x \in$ ~~$[-3, 3]$~~ $[-4, 4]$.

Domain: $[-4, 4]$

Range: $y \in [-1, 1]$

Y – Int: $(0, 0)$

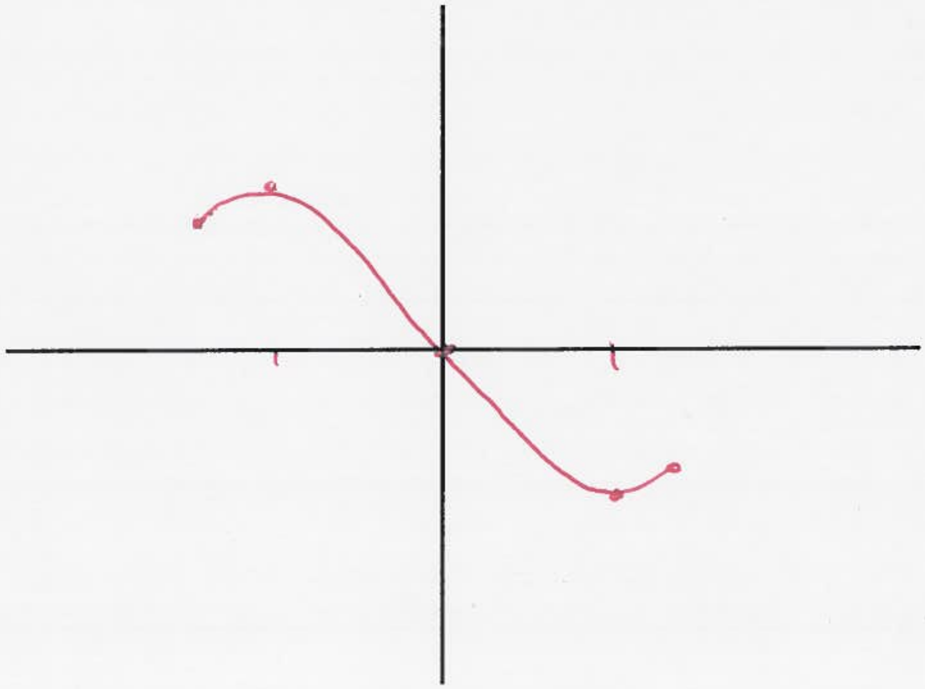
End Behavior: NONE

Vas: NONE

POEs: NONE

Zeros: $(0, 0)$

Extreme Values: $(+3, -1)$ $(-3, 1)$ $(4, \frac{-24}{25})$ $(-4, \frac{24}{25})$



8. Find the traits and **sketch** of $y=\dfrac{3x^3-x^2+12x-4}{3x^2-7x+2}$.

Domain: $x \neq 1/3, 2$

Range: $y \in (-\infty, -1.657] \cup [9.657, \infty$

Y – Int: $0, -2$

End Behavior: $y = x + 2$

Zeros: NONE

Extreme Values: $y \{-1.657, \{9.657)$

POEs: $\left(\frac{1}{3}, \frac{-37}{15}\right)$

VAs: $x = 2$