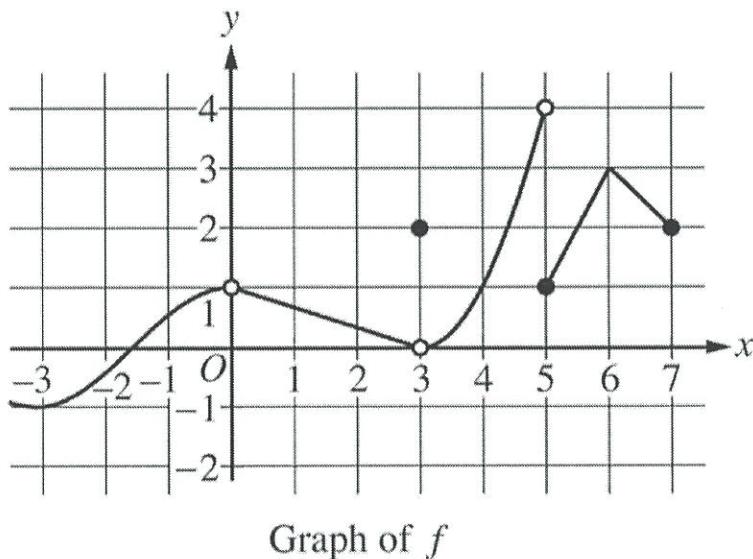


Honors PreCalculus '16-17
Piece-Wise Defined Functions Test
Dr. Quattrin
Calculator allowed

Name: Savanna Kay

1. The function f is defined on the interval $x \in (-4, 7)$ and has the graph shown below.



For which of the following statements are **false**?

- I. $\lim_{x \rightarrow 5^+} f(x) = 4$. **F**
- II. f is differentiable at $x = 6$. **F**
- III. f has a local maximum at $x = 3$. **T**

- (a) I only (b) II only (c) III only
(d) I and II only (e) I and III only

2. The end behavior of $g(x) = \sqrt{\frac{x^2 - 9}{x^2 + 4}}$

$$\lim_{x \rightarrow \pm\infty} g(x) = 1$$

- a) $y = 0$ on both ends
 - b) $y = 1$ on the left and none on the right
 - c) $y = 1$ on the left and none on the right
 - d) $y = 1$ on both ends
 - e) None on the left and $y = 0$ on the right
-

3. The function f is defined on all the Reals such that

$$f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \leq 1 \\ 3x + b & \text{for } x > 1 \end{cases}. \quad \text{For which values of } k \text{ and } b \text{ will the function be}$$

continuous and differentiable throughout its domain?

- a) $k = -1$ and $b = -3$
- b) $k = 1$ and $b = 3$
- c) $k = 1$ and $b = 4$
- d) $k = 1$ and $b = -4$
- e) $k = -1$ and $b = 6$

$$@ x=1 \quad k-2 = 3+b$$

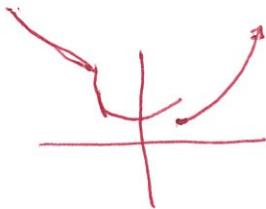
$$f'(x) = \begin{cases} 2x+k \\ 3 \end{cases}$$

$$@ x=1 \quad 2+k=3 \rightarrow k=1$$

$$b = -4$$

4. Let $f(x) = \begin{cases} -x+5, & \text{if } x < -2 \\ x^2+3, & \text{if } -2 \leq x \leq 1 \\ 2x^3, & \text{if } 1 < x \end{cases}$. Which of the following statements is false about f ?

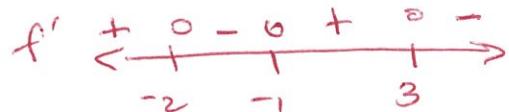
- I. f is continuous at $x = -2$. T
- II. f is differentiable at $x = 1$. F
- III. f has a local minimum at $x = -2$. F



-
- (a) I only
 - (b) II only
 - (c) III only
 - (d) I and III only
 - (e) II and III only

5. If the derivative of the function f is $f'(x) = -3(x+2)^4(x+1)(x-3)^3$, then f has a local minimum at $x =$

- (a) -2 only (b) -1 only (c) 3 only (d) -2 and 3 (e) -1 and 3

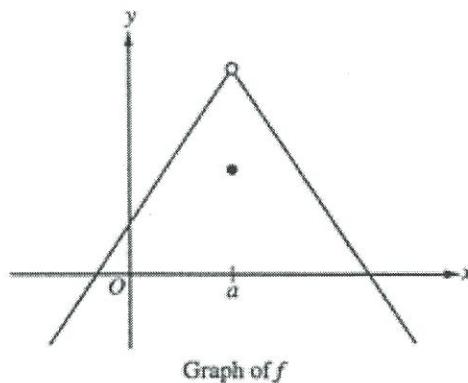


6. Let f be defined by $f(x) = \begin{cases} 4x^2 + 10, & \text{if } x < 1 \\ mx^3 + 8, & \text{if } 1 < x \end{cases}$. Determine the value of m for which f is continuous for all real x .

- a) -6 b) -2 c) 8 d) 14 e) None of these

7. The graph of the function $f(x)$ is shown below. Which of the following statements **must** be false?

- a) $f(a)$ exists.
 b) $f(x)$ is defined for $0 < x < a$
 c) $f(x)$ is not continuous at $x = a$.
 d) $\lim_{x \rightarrow a} f(x)$ exists.
 e) $\lim_{x \rightarrow a} f'(x)$ exists.



8. A function $f(x)$ has a vertical asymptote at $x = 2$. The derivative of $f(x)$ is positive for all $x \neq 2$. Which of the following statements are true?

I. $\lim_{x \rightarrow 2} f(x) = +\infty$

II. $\lim_{x \rightarrow 2^-} f(x) = +\infty$

III. $\lim_{x \rightarrow 2^+} f(x) = +\infty$

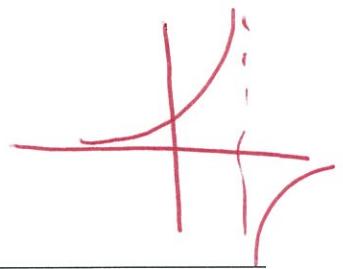
(a) I only

(b) II only

(c) III only

(d) I and II only

(e) I, II and III



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Name: Solution Key

$$1. \quad h(x) = \begin{cases} 2-x, & \text{if } x < -2 \\ 3, & \text{if } x = -2 \\ \sqrt{3x^2 + 4}, & \text{if } -2 < x < 0 \\ 5-x, & \text{if } 0 \leq x < 3 \\ 3x-7, & \text{if } x \geq 3 \end{cases}$$

i) Is $h(x)$ continuous at $x = -2$? Why or why not?

i) $f(-2)$ exists

ii) $\lim_{x \rightarrow -2} f(x)$ ~~exists~~ because $\lim_{x \rightarrow -2} (2-x) \neq \lim_{x \rightarrow -2} \sqrt{3x^2 + 4}$
 $4 \neq$

iii) $\lim_{x \rightarrow -2} f(x) \neq f(-2)$

\therefore DISCONTINUOUS

ii) Is it differentiable at $x = -2$? Why or why not?

NOT DIFF BECAUSE NOT CONT.

$$2. h(x) = \begin{cases} 2-x, & \text{if } x < -2 \\ 3, & \text{if } x = -2 \\ \sqrt{3x^2+4}, & \text{if } -2 < x < 0 \\ 5-x, & \text{if } 0 \leq x < 3 \\ 3x-7, & \text{if } x \geq 3 \end{cases}$$

i) Is $h(x)$ continuous at $x=3$? Why or why not?

i) $f(3)$ exists

ii) $\lim_{x \rightarrow 3^-} f(x)$ exists because $\lim_{x \rightarrow 3^-} (5-x) = \lim_{x \rightarrow 3^+} (3x-7) = 2$

iii) $\lim_{x \rightarrow 3} f(x) = f(3)$

\therefore Continuous

ii) Is it differentiable at $x=3$? Why or why not?

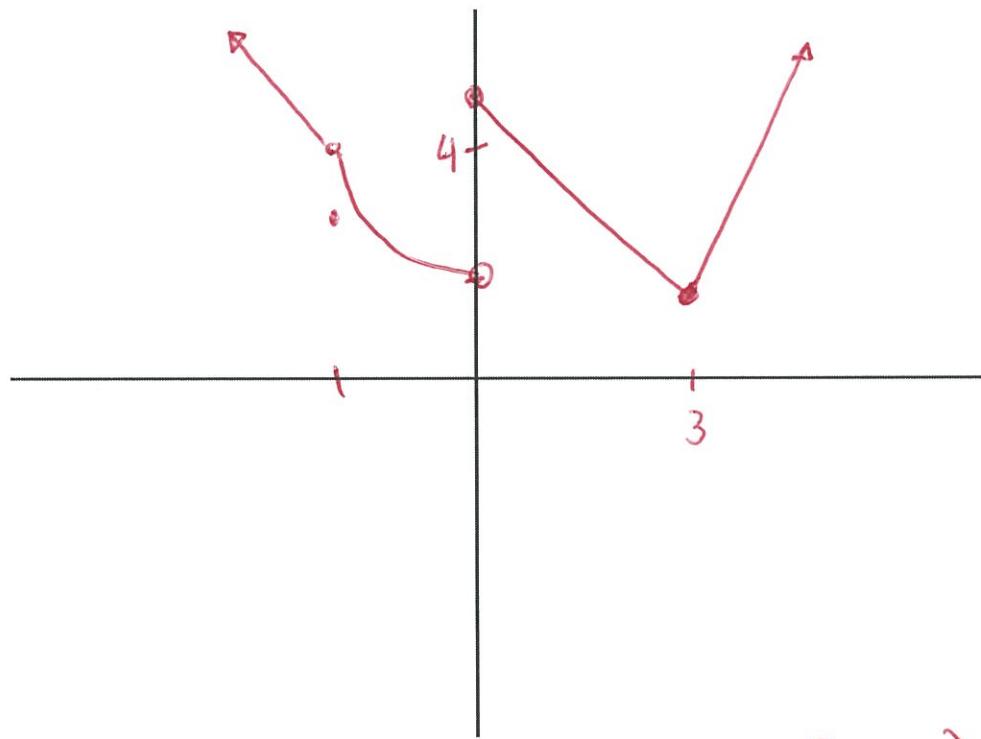
$$f' = \begin{cases} -1 & \text{IF } x < 3 \\ 3 & \text{IF } 3 < x \end{cases}$$

i) $f(x)$ is continuous

ii) $\lim_{x \rightarrow 3^-} f'(x)$ AND $\lim_{x \rightarrow 3^+} f'(x)$ BOTH EXIST

iii) $\lim_{x \rightarrow 3^-} f'(x) \neq \lim_{x \rightarrow 3^+} f'(x) \therefore f(x)$ IS NOT DIFFERENTIABLE

3. Sketch $h(x) = \begin{cases} 2-x, & \text{if } x < -2 \\ 3, & \text{if } x = -2 \\ \sqrt{3x^2+4}, & \text{if } -2 < x < 0 \\ 5-x, & \text{if } 0 \leq x < 3 \\ 3x-7, & \text{if } x \geq 3 \end{cases}$. State the Traits listed. Provide proof for the extreme points.



Domain: All Reals

Range: $y \in [2, \infty)$

Zeros: None

Y-int: $(0, 5)$

VAs: None

EB (Left): $\cup\leftarrow$

EB (Right): $\cup\uparrow$

Discontinuities: $x = -2, 0$

Points of non-differentiability: $x = -2, 0, 3$

Extreme Points: $(0, 5)$, $(3, 2)$, $(-2, 3)$

$$h'(x) = \begin{cases} -1 & \text{IF } x < -2 \\ \frac{3x}{\sqrt{3x^2+4}} & \text{IF } -2 \leq x < 0 \\ -1 & \text{IF } 0 < x < 3 \\ 3 & \text{IF } 3 < x \end{cases}$$

i) $h' = 0$ NO

ii) h' DNE: $x = -2, 0, 3$

iii) NO ENDPOINTS

$\therefore (-2, 3), (0, 5) \text{ & } (3, 2)$