

Honors PreCalculus '17-18

Name: Solutions Key

Dr. Quattrin

Polynomials Test

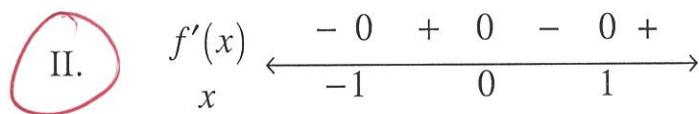
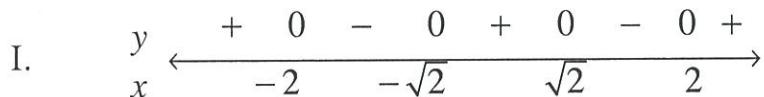
CALCULATOR ALLOWED

Score \_\_\_\_\_

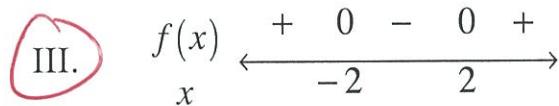
Round to 3 decimal places. Show all work.

1. Which of the following sign patterns apply to the equation

$$f(x) = x^4 - 2x^2 - 8 ? \quad = (x^2 - 4)(x^2 + 2)$$



$$\begin{aligned} f' &= 4x^3 - 4x \\ &= 4x(x^2 - 1) \end{aligned}$$



- a) II only      b) II only      c) III only  
d) I and II only      e) II and III only

- 
2. Give the approximate location of a local maximum for the function

$$f(x) = 2x^3 + 2x^2 - 4x$$

Graph on Calc

- a) (-1.215, -1.262)    b) (-1.215, 4.174)    c) (0.549, -1.211)  
d) (-1.215, 4.224)    e) (0.549, -1.262)
-

3. A particle is moving such that its velocity is described by  $v(t) = t^2 - t^4$ .

When does the particle reach its maximum acceleration?

a)  $t = 0$

b)  $t = \frac{1}{\sqrt{2}}$

c)  $t = -\frac{1}{\sqrt{2}}$

d)  $t = \frac{1}{\sqrt{6}}$

e)  $t = -\frac{1}{\sqrt{6}}$

$$a(t) = 2t - 4t^3$$

$$a'(t) = 2 - 12t^2 = 0$$

$$t = \pm \frac{1}{\sqrt{6}}$$

$$\begin{array}{c} a' \\ \hline - \leftarrow \textcircled{o} + \textcircled{o} - \rightarrow \\ -\frac{1}{\sqrt{6}} \quad \frac{1}{\sqrt{6}} \end{array}$$

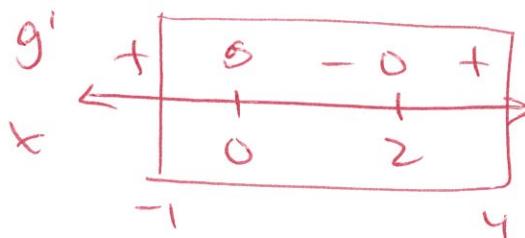

---

4. Find the locations of the **absolute** extreme values of  $g(x) = x^3 - 3x^2 + 1$  on the interval  $x \in [-1, 4]$ .

- a) Absolute maximum at  $x = 0$ , Absolute minimum at  $x = 2$   
 b) No absolute maximum, Absolute minimum at  $x = 1$   
 c) Absolute maximum at  $x = 0$  and  $4$ , Absolute minimum at  $x = -1$  and  $2$   
 d) Absolute maximum at  $x = 4$ , Absolute minimum at  $x = 0, 2$   
 e) Absolute maximum at  $x = 4$ , Absolute minimum at  $x = -1$  and  $2$

$$g'(x) = 3x^2 - 6x = 0$$

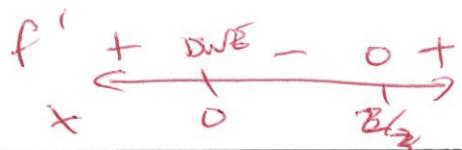
$$3x(x-2) = 0$$



x	g
-1	-3
0	1
2	-3
4	23

5. The minimum value of  $f(x) = \frac{2}{\sqrt{x}} + 3\sqrt{x}$  is  $= 2x^{-1/2} + 3x^{1/2}$

- a)  $\frac{2\sqrt{6}}{3}$       b)  $\frac{2}{3}$       c)  $\frac{\sqrt{6}}{3}$       d)  $2\sqrt{6}$       e) dne



$$\begin{aligned}f' &= -x^{-3/2} + \frac{3}{2}x^{-1/2} \\&= x^{-3/2}\left(-1 + \frac{3}{2}x\right) = 0 \\x &= \frac{2}{3} \rightarrow y = 2\sqrt{6}\end{aligned}$$

6. Find the  $x$ -value of the point on the graph of  $y = x^2 + x + 2$  on  $x \in [1, 3]$  at which the tangent to the graph has the same slope as the line through  $(1, 4)$  and  $(3, 15)$ .

$$f'(x) = 2x + 1 = \frac{15 - 4}{3 - 1} = \frac{11}{2}$$

- (a)  $x = \frac{1}{2}$   
 (b)  $x = \frac{5}{4}$   
 (c)  $x = \frac{3}{2}$   
 (d)  $x = 2$   
 (e)  $x = \frac{9}{4}$

$$\begin{aligned}2x &= \frac{9}{2} \\x &= \frac{9}{4}\end{aligned}$$

7. Given this sign pattern  $f'(x) \begin{array}{c} -0 \\[-3] x \\ +0- \end{array}$ , which of the following might be the sign pattern of  $f(x)$ ?

- a)  $f(x) \begin{array}{c} +0-0+0- \\[-5] x \\ -5-12 \end{array}$
- b)  ~~$f(x) \begin{array}{c} -0+0-0+ \\[-5] x \\ -5-12 \end{array}$~~
- c)  ~~$f(x) \begin{array}{c} -0+0-0+ \\[-3] x \\ -3-11 \end{array}$~~
- d)  ~~$f(x) \begin{array}{c} +0-0+0- \\[-3] x \\ -3-11 \end{array}$~~
- e)  ~~$f(x) \begin{array}{c} +0-0-0+ \\[-3] x \\ -3-11 \end{array}$~~
-

## Honors PreCalculus '17-18

Name: Solution Key

Dr. Quattrin

Polynomials Test-- CALCULATOR ALLOWED

Round to 3 decimal places.

Score \_\_\_\_\_

Show all work.

1. Find the zeros and extreme points of  $y = 2x^3 - 5x^2 - 14x + 35$ . Show the algebraic work to support the zeros and critical values.

$$\text{Zeros: } y = x^2(2x-5) - 7(2x-5) - \\ = (x^2-7)(2x-5) = 0 \\ (\pm\sqrt{7}, 0) (5/2, 0)$$

$$\text{Ext Points: } \frac{dy}{dx} = 6x^2 - 10x - 14 = 0$$

$$3x^2 - 5x - 7 = 0$$

$$x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 3 \cdot (-7)}}{2 \cdot 3} = \begin{cases} 2.573 \\ -.907 \end{cases}$$

$$(2.573, -0.655)$$

$$(-.907, 42.092)$$

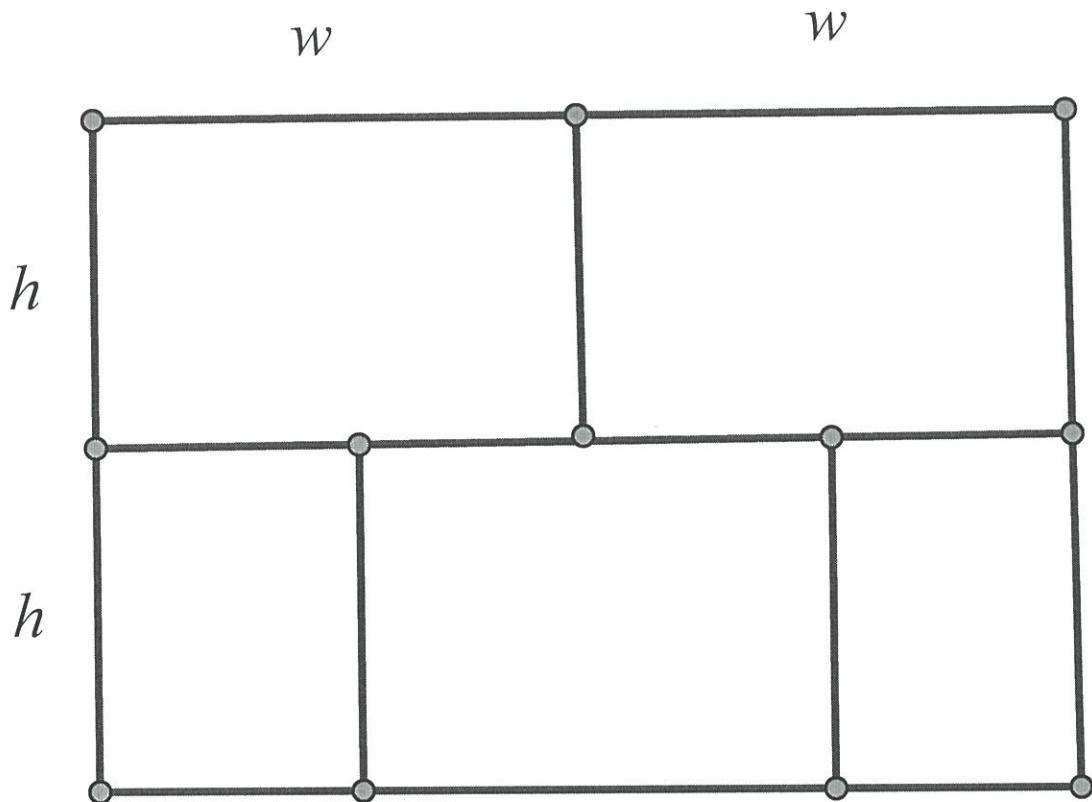
2. Find the zeros and extreme points of  $y = 4x^3 - 12x^2$  on  $x \in [-1, 4]$ . Show the derivative and algebra to support the critical values.

Zeros:  $y = 4x^2(x-3) = 0$   
 $x = 0, 3$   
 $(3, 0), (0, 0)$

Extreme Points:  $\frac{dy}{dx} = 12x^2 - 24x$   
 $= 12x(x-2) = 0$   
 $x = 0, 2$   
 $(0, 0), (2, -16)$

Endpoint:  $(-1, -16), (4, 64)$

3. A field with an area equal to 160 square yards in partitioned as the diagram below.



What is the minimum amount of fencing required for the portioning?

$$F = 6w + 7h$$

$$A = 2w(2h) = 4wh = 160$$

$$h = \frac{40}{w}$$

$$F = 6w + 7\left(\frac{40}{w}\right)$$

$$= 6w + 280w^{-1}$$

$$F' = 6 - 280w^{-2} = 0$$

$$6 = \frac{280}{w^2}$$

$$\begin{array}{c} F' + 0 - 0 + \\ \hline -6.831 & 6.831 \\ \hline \text{min} \end{array}$$

$$w^2 = \frac{140}{3}$$

$$w = \pm 6.831$$

$$F(6.831) = 81.976$$

4a. Find the zeros, algebraically, of  $y = 4 + 8x - x^2 - 2x^3$ .

$$4(1+2x) - x^2(1+2x) = 0$$

$$(4-x^2)(1+2x) = 0$$

$$x = \pm 2, -\frac{1}{2}$$

$$(\pm 2, 0), \left(-\frac{1}{2}, 0\right)$$

---

4b. Find the extreme points of  $y = 4 + 8x - x^2 - 2x^3$ . Show the derivative before using your calculator.

$$\frac{dy}{dx} = 8 - 2x - 6x^2 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(-6)(8)}}{-12} = \begin{cases} 1 \\ -\frac{4}{3} \end{cases}$$

$$(1, 9), \left(-\frac{4}{3}, -3.704\right)$$

---

5. The sign pattern for the derivative of  $H(x)$  is given. (a) Is  $x = -4$  at a maximum, a minimum, or neither? Why? (b) Is  $x = -1$  at a maximum, a minimum, or neither? Why?

$$\begin{array}{c} dH/dx \\ \hline x \end{array} \leftarrow \begin{matrix} - & 0 & - & 0 & + & 0 & - \end{matrix} \rightarrow \begin{matrix} -4 & -1 & 2 \end{matrix}$$

a) NEITHER; no sign change

b) Minimum;  $\frac{dH}{dx}$  switches from ~~-~~ - to +  
 (or h switches from DECREASING to INCREASING)

---

6. Find the traits and sketch  $y = 4x^3 - 12x^2$  on  $x \in [-1, 4]$ .

Domain:  $x \in [-1, 4]$

Zeros:  $(0, 0), (3, 0)$

End Behavior (left):  $\text{NONE}$

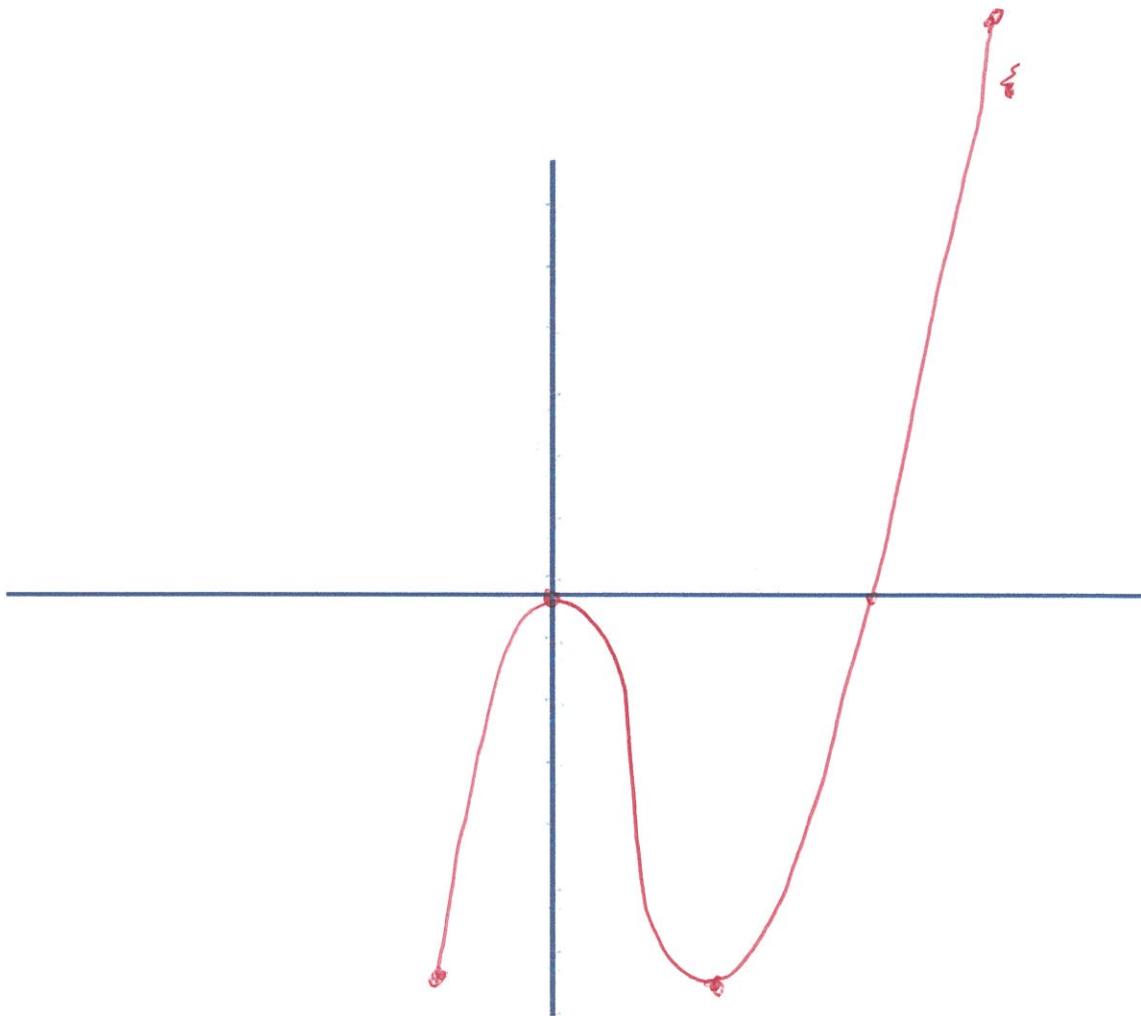
End Behavior (right):  $\text{NONE}$

Range:  $y \in [-16, 24)$

Y-Int:  $(0, 0)$

Extreme Points:  $(4, 64) (0, 0)$

$(-1, -16) (2, -16)$



7. Find the traits and sketch of  $y = 2x^3 - 5x^2 - 14x + 35$ .

Domain: All Reals

Y-Int: (0, 35)

Zeros:  $(\pm \sqrt{7}, 0), (9, 0)$

Range: All Reals

End Behavior: Right Up  
Left Down

Extreme Points:  $(2.573, -0.055)$   
 $(-0.907, 42.092)$

