

PreCalculus Honors '17-18

Dr. Quattrin

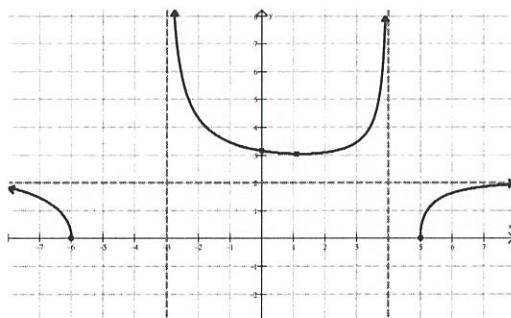
Radical Functions Test

CALCULATOR ALLOWED

Name: SOLUTION KEY

Score

Round to 3 decimal places. Show all work.



1. Which of the following sign patterns apply to the graph above?

~~I.~~
$$\begin{array}{ccccccccc} y & \leftarrow & + & 0 & - & \text{DNE} & + & \text{DNE} & - & 0 & + \\ x & & & & & & & & & & \end{array}$$

-6 -3 4 5

$$\text{II. } \begin{array}{ccccccccc} y^2 & + & 0 & - & \text{DNE} & + & \text{DNE} & - & 0 & + \\ x & \leftarrow & -6 & -3 & & 4 & & 5 \end{array}$$

$$\text{III. } \frac{dy}{dx} \begin{array}{c} - \quad \text{DNE} \quad \quad \text{DNE} \quad - \quad 0 \quad + \quad \text{DNE} \quad \quad \text{DNE} \quad + \\ \leftarrow \qquad \qquad \qquad \qquad \qquad \qquad \qquad \rightarrow \\ -6 \qquad \qquad -3 \qquad \qquad 1 \qquad \qquad 4 \qquad \qquad 5 \end{array}$$

2. At what approximate rate (in cubic meters per minute) is the volume of a sphere changing at the instant when the surface area is 2 square meters and the radius is increasing at the rate of $1/3$ meters per minute?

- a) 0.667 b) 1.080 c) 0.700 d) 2.128 e) 1.714
-

3. The absolute minimum of $y = \sqrt{25 - x^2}$ on $x \in [-2, 4]$ is

- a) -2 b) 0 c) 5 d) $\sqrt{21}$ e) 3
-

$$2) V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 2\left(\frac{1}{3}\right) = \frac{2}{3}$$

$$3) \frac{dy}{dx} = \frac{-2x}{2\sqrt{25-x^2}}$$

i) $x=0 \rightarrow y=5$
ii) $x \neq \pm 5$ NOT in domain

iii) $x \in [-2, 4]$

$$f(-2) = \sqrt{21}$$

$$f(4) = 3$$

4. Use Implicit Differentiation to find the points on $x^3 - y^2 + x^2 - 1 = 0$ which have horizontal tangent lines.

a) $(0, \pm 1)$

b) $(-1.5, 0)$ only

c) $\left(-\frac{2}{3}, \pm 1.072\right)$ only

d) $(0, \pm 1)$ and $\left(-\frac{2}{3}, \pm 1.072\right)$

e) The tangent line is never horizontal

$$3x^2 - 2y \frac{dy}{dx} + 2x = 0$$

$$-2y \frac{dy}{dx} = 3x^2 + 2x$$

$$\frac{dy}{dx} = \frac{3x^2 + 2x}{-2y}$$

$$m=0 \Rightarrow 0 = 3x^2 + 2x$$

$$x=0, -2/3$$

5. Let $f(x)$ be the function with $f(1)=2$ and $f'(x)=\sqrt{x^2+3}$. Using the tangent line approximation to the graph of $f(x)$ at $x=1$, estimate $f(0.98)$.

a. 1.99 b. 1.98

c. 1.97

d. 1.96 e. 1.95

$$f'(1) = 2$$

$$y - 2 = 2(x - 1)$$

$$f(0.98) \approx 2(0.98 - 1) + 2$$

6. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

Given that $h(x) = f(g(x))$, $h'(2) = f'(g(2)) \cdot g'(2)$
 $= f'(8) \cdot g'(2) = (-12)(1)$

- a) -16 b) -12 c) -1 d) 2 e) 10
-

7. Find the absolute maximum value of $y = \sqrt{36 - x^2}$ on the interval $x \in [-2, 2]$.

- a) -2 b) 0 c) 2 d) $4\sqrt{2}$ e) 6
-

$$\frac{dy}{dx} = \frac{-x}{(36-x^2)^{1/2}}$$

i) $x = 0 \Rightarrow y = 6$

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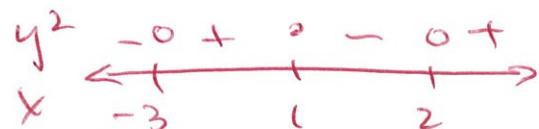
1. Find the zeros and Domain of $y = -\sqrt{x^3 - 7x + 6}$. Show the algebraic work to support the zeros and critical values.

Zeros: $(-3, 0)$, $(1, 0)$, $(2, 0)$

Domain:

$$x \in [-3, 1] \cup [2, \infty)$$

$$(x-1)(x^2+x-6)$$



2. Find the Extreme Points of $y = -\sqrt{x^3 - 7x + 6}$. Show the algebraic work to support the zeros and critical values.

Extreme Points: $(3, 0), (1, 0), (2, 0), (-1.528, -3.623)$

$$\frac{dy}{dx} = -\frac{1}{2}(x^3 - 7x + 6)^{-1/2}(3x^2 - 7)$$

$$= -\frac{(3x^2 - 7)}{2(x^3 - 7x + 6)^{1/2}}$$

$$\Rightarrow 3x^2 - 7 = 0 \Rightarrow x = \pm\sqrt{7/3} = \pm1.528 \text{ BUT } -1.528 \text{ is NOT in domain}$$

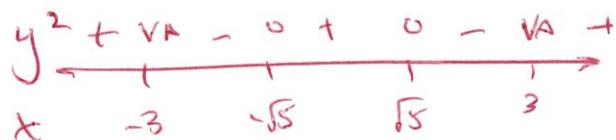
ii) $\frac{dy}{dx}$ DNE $\Rightarrow x = -3, 1, 2$

3. Find the zeros, VAs, and domain of $y = \sqrt{\frac{x^2-5}{x^2-9}}$ on $x \in [-4, 5]$. Show the Algebra that supports your answer.

Zeros: $(\pm\sqrt{5}, 0)$

VAs: $x = \pm 3$

Domain: $x \in [-4, -3) \cup [-\sqrt{5}, \sqrt{5}] \cup (3, 5]$



4. Find the Extreme Points of $y = \sqrt{\frac{x^2-5}{x^2-9}}$ on $x \in [-4, 5]$. Show the Algebra and derivative that supports your answer.

Extreme Points: $(0, \frac{\sqrt{5}}{3})$, $(\pm\sqrt{5}, 0)$, $(-4, \sqrt{\frac{9}{7}})$, $(5, \sqrt{\frac{20}{17}})$

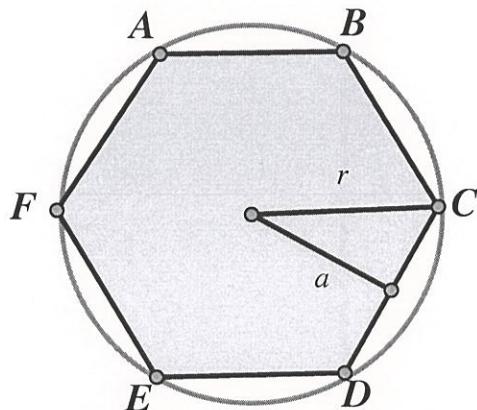
$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} \left(\frac{x^2-5}{x^2-9} \right)^{-1/2} \left[\frac{(x^2-9)(2x) - (x^2-5)(2x)}{(x^2-9)^2} \right] \\ &= \frac{(x^2-9)^{1/2} (-8x)}{2(x^2-5)^{1/2} (x^2-9)^2} = \frac{-4x}{(x^2-5)^{1/2} (x^2-9)^{3/2}} \end{aligned}$$

i) $x = 0$

ii) $x = \pm\sqrt{5}$

iii) $x = -4, 5$

5. A regular hexagon is inscribed in a circle. The circle's circumference is expanding at 4π in/sec and the hexagon maintains the contact of its corners with the circle.



Given that the area of a regular hexagon is equal to half the apothem a times the perimeter p , find out how fast the area inside the circle but outside the hexagon is expanding when the area of the circle is 64π in². [Hint: Find p and a in terms of r .]

1) Find $\frac{dA}{dt}$ where A is the white area

2) $A = \pi r^2 - \frac{1}{2}ap = \pi r^2 - \frac{1}{2}\left(\frac{\sqrt{3}}{2}r\right)(6r) = \pi r^2 - \frac{3\sqrt{3}}{2}r^2$

3) $A_0 = 64\pi = \pi r^2 \rightarrow r = 8 \quad \frac{dr}{dt} = 2\pi \frac{dr}{dt} = 4\pi \rightarrow \frac{dr}{dt} = 2$

$$\underline{\underline{A}} = \frac{dA}{dt} = 2\pi r \frac{dr}{dt} - 3\sqrt{3} \frac{dr}{dt}$$

$$= 2\pi(8)(2) - 3\sqrt{3}(8)(2)$$

$$= 32\pi - 48\sqrt{3} = 17.393 \frac{\text{in}^2}{\text{sec}}$$

6. A particular velocity function is given by the equation

$v(t) = 4 \sqrt{\left[\left(\frac{E(t)}{3} + 3t \right)^{\frac{3}{7}} - 4 \right]}.$ What is the equation for the acceleration?

$$= 2 \left(\left(\frac{E(t)}{3} + 3t \right)^{\frac{3}{7}} - 4 \right)^{-\frac{1}{2}} \left(\frac{3}{7} \left(\frac{E(t)}{3} + 3t \right)^{-\frac{4}{7}} \right) \left(\frac{1}{3} t + 3 \right)$$

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Radical Functions Test – NO CALCULATOR ALLOWED

Show all work.

Name: Solution Key

7. Find the traits and sketch $y = -\sqrt{x^3 - 7x + 6}$.

Domain: $\text{SEE } \#1$

Range: $y \notin (-\infty, 0]$

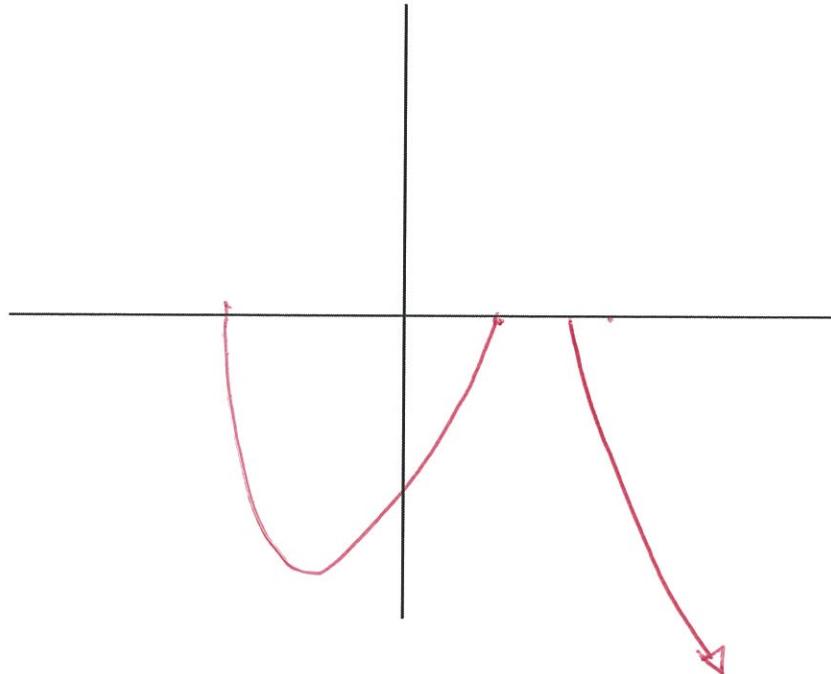
$Y - \text{Int: } (0, \sqrt{6})$

Zeros: $\text{SEE } \#1$

Extreme Points: $\text{SEE } \#2$

End Behavior (Left): NONE

End Behavior (Right): DOWN



8. List the traits and sketch of $y = \sqrt{\frac{x^2 - 5}{x^2 - 9}}$ on $x \in [-4, 5]$.

Domain: See #3

Y-Int: $(0, \sqrt{\frac{5}{4}})$

Zeros: See #3

End Behavior (Left): None

End Behavior (Right): None

Range: $y \in [0, \sqrt{\frac{5}{4}}] \cup [\sqrt{\frac{5}{4}}, \infty)$

VAs: ± 3

Extreme Points: See #4

