

Dr. Quattrin

Limits and Derivatives Test

CALCULATOR ALLOWED

Score \_\_\_\_\_

Round to 3 decimal places. Show all work.

1. Find the instantaneous rate of change of  $f(x) = x^3 - \frac{1}{3x^2}$  at  $x = -1$ .

$$f' = 3x^2 + \frac{2}{3}x^{-3} \rightarrow f'(-1) = 3 - \frac{2}{3}$$

- a) dne    b)  $\frac{-8}{3}$     c)  $\frac{7}{3}$     d)  $-\frac{11}{3}$     e)  $\frac{11}{3}$
- 

2. If  $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}$  is the numerical derivative of the function  $f(x)$ , then

$$f(x) = \frac{f(x) - f(2)}{x - 2} \neq \frac{0}{0} \therefore \text{NOT A DERIVATIVE}$$

- a)  $x^2$     b) -4    c)  $x^2 - 4x$   
 d)  $x^2 - 4x - 4$     e) None of these
- 

3. Suppose  $f$  is a differentiable function such that  $f(-1) = 2$  and  $f'(-1) = \frac{1}{2}$ .

Using the line tangent to the graph of  $f(x)$  at  $x = -1$ , find the approximation of

$$f(-1.1)$$

$$y - 2 = \frac{1}{2}(x + 1) \rightarrow y(-1.1) = 2 + \frac{1}{2}(-.1)$$

- a) -3.05    b) -1.95    c) .95    d) 1.95    e) 3.05
-

4. Find an equation of the normal line to the curve  $f(x) = \frac{2}{x}$  at  $x=1$ .

(a)  $y = \frac{1}{2}x + \frac{3}{2}$

(b)  $y = -\frac{1}{2}x$

(c)  $y = \frac{1}{2}x + 2$

(d)  $y = -\frac{1}{2}x + 2$

(e)  $y = 2x + 5$

$f'(x) = -2x^{-2}$

$m_{tan} = -2$

$m_{normal} = \frac{1}{2}$

$y - 2 = \frac{1}{2}(x - 1)$

5. Let  $f(x) = \frac{1}{x^2} - x^2$ . Find  $f''(x)$ .

$f'(x) = -2x^{-3} - 2x$ ;  $f''(x) = 6x^{-4} - 2$

a)  $-\frac{2}{x^3} - 2x$

b)  $\frac{6}{x^4} - 2$

c)  $\frac{3}{2} - 2$

d)  $-\frac{6}{x^4} - 2$

e) None of these

6. A particle moving in a straight line such that  $x(t) = -80t + 3t^2 - 4t^3$ . When is the particle at rest?

$v(t) = -12t^2 + 6t - 80 = 0$

a)  $t = 0$

b)  $t = 3$

c)  $t = \frac{5}{2}$   $6t^2 - 3t + 40 = 0$

d)  $t = \frac{5}{2}, 3$

e)  $t = 0, \frac{5}{2}, 3$

THIS PROBLEM  
DOES NOT WORK

Honors PreCalculus '21

Dr. Quattrin

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Name: SOLUTION KEY

Score \_\_\_\_\_

1. Use the equation of the line tangent to  $y = 6x^3 - 3x^2 + 5x - 4$  at  $x = 1$  to approximate  $f(0.9)$

$$y(1) = 4$$

$$\frac{dy}{dx} = 18x^2 - 6x + 5$$

$$m = 18 - 6 + 5 = 17$$

$$y - 4 = 17(x - 1)$$

$$f(0.9) \approx y(0.9) = 4 + 17(-0.1) = 2.3$$

2. The motion of a particle is described by  $y(t) = 2t^3 + 5t^2 - 4t + 3$ .

- a) When the particle is stopped?  
b) Which direction it is moving at  $t = 4$ ?  
c) Where is it when  $t = 4$ ?  
d) Find  $a(4)$ .

$$\begin{aligned} v(t) &= 6t^2 + 10t - 4 \\ &= 2(3t^2 + 5t - 2) \\ &= 2(t+2)(3t-1) \end{aligned}$$

a)  $t = -2, 1/3$

$$a(t) = 12t + 10$$

b)  $v(4) = 2(6)(11) > 0 \therefore \text{up}$

c)  $x(4) = 2(4)^3 + 5(4)^2 - 4(4) + 3 = 195$

d)  $a(4) = 12(4) + 10 = 58$

3. A particle's position  $\langle x(t), y(t) \rangle$  at time  $t$  is described by  $\langle t^3 + t^2 - 2t + 1, -35t + 2 \rangle$ .

- a) Find the speed at  $t = 3$ .

$$\mathbf{v}(t) = \langle 3t^2 + 2t - 2, -35 \rangle$$

$$\mathbf{v}(3) = \langle 31, -35 \rangle$$

$$S(3) = \sqrt{31^2 + 35^2} = \sqrt{2186}$$

$$= 46.755$$

- b) When, if ever, is the particle stopped? Prove it.

$$v_y(t) = -35$$

$\therefore$  THE PARTICLE IS NEVER STOPPED

Honors PreCalculus

Name: Sowton Kay

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NO CALCULATOR ALLOWED

5. Set up, but do not solve, the limit definition of the derivative for

$$y = 5x^4 - x^3 + 7x^2 + 3^4$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{(5(x+h)^4 - (x+h)^3 + 7(x+h)^2 + 81) - (5x^4 - x^3 + 7x^2 + 81)}{h}$$

6. Use the Power Rule to find:

a)  $\frac{dy}{dx}$  if  $y = 6x^7 - 19x^4 + 3x^2 - 12x - 13$

$$\frac{dy}{dx} = 42x^6 - 76x^3 + 6x - 12$$

b)  $\frac{d}{dx} \left[ x^7 - 4\sqrt[3]{x^7} + 7^3 - \frac{1}{\sqrt[7]{x^4}} + \frac{1}{5x} \right] = \frac{d}{dx} \left[ x^7 - 4x^{7/3} + 7^3 - x^{-4/7} + \frac{1}{5}x^{-1} \right]$

$$= 7x^6 - \frac{14}{3}x^{4/3} + 4x^{-11/7} - \frac{1}{5}x^{-2}$$

7. Evaluate the following limits:

$$a) \lim_{x \rightarrow 3} \frac{x^2 - 9}{2x^2 - 7x + 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(2x-1)(x-3)}$$

$$= \frac{6}{5}$$

$$b) \lim_{x \rightarrow -1} \frac{3x^2 + 2x - 1}{x^3 + x^2 + 4x + 4} = \lim_{x \rightarrow -1} \frac{(3x-1)(x+1)}{x^2(x+1) + 4(x+1)}$$

$$= \frac{3(-1)-1}{(-1)^2+4} = -\frac{4}{5}$$

$$EC) \lim_{x \rightarrow 3} \frac{\sqrt{x^2+1} - \sqrt{8}}{(x^2 - x - 6)} \cdot \frac{\sqrt{x^2+1} + \sqrt{8}}{\sqrt{x^2+1} + \sqrt{8}} = \frac{\sqrt{10} - \sqrt{8}}{0} = \text{DNE}$$

$$= \cancel{\frac{\sqrt{x^2+1} - \sqrt{8}}{x^2 - x - 6}} \cdot \cancel{\frac{\sqrt{x^2+1} + \sqrt{8}}{\sqrt{x^2+1} + \sqrt{8}}}$$