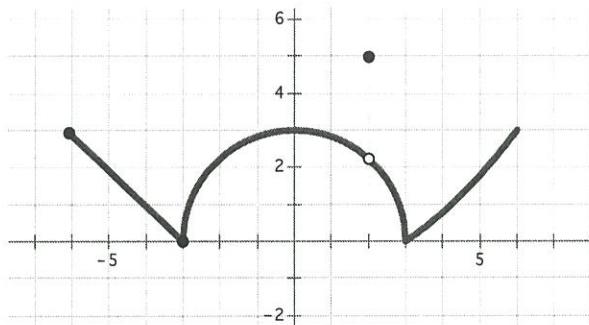


Honors PreCalculus '20-21  
Piece-Wise Defined Functions Test  
Dr. Quattrin  
Calculator allowed

Name: SOLUTION KEY

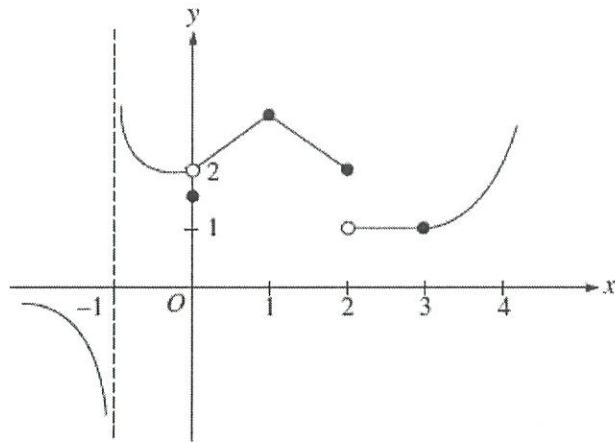
1. The function  $f$  is defined on the interval  $x \in [-6, 6]$  and has the graph shown below.



For which of the following values is  $f$  not differentiable?

- (a) -3 only      (b) 3 only      (c) 3 only  
(d) -3 and 3 only      (e) -3, 2, and 3

2. The function  $f$  is shown below. Which of the following statements about the function  $f$ , shown below, is false?



- a)  $\lim_{x \rightarrow 0} f(x)$  exists T
- b)  $\lim_{x \rightarrow 2} f(x)$  exists F
- c)  $f$  is continuous at  $x = 1$  T
- d)  $\lim_{x \rightarrow 3} \frac{f(x) - 5}{x - 3}$  does not exist T
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3. The function  $f$  defined on all the Reals such that

$$f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \leq 1 \\ 3x + b & \text{for } x > 1 \end{cases}$$
. For which of the following values of  $k$  and  $b$  will

the function  $f$  be both continuous and differentiable on its entire domain?

- (a)  $k = -1, b = -3$        $f(1) \Rightarrow k - 2 = b$   
b)  $k = 1, b = 3$   
c)  $k = 1, b = 4$   
d)  $k = 1, b = -4$   
e)  $k = -1, b = 6$
- 

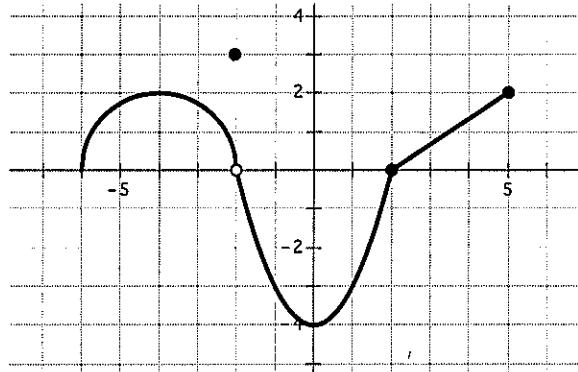
4. Let  $f(x) = \begin{cases} -x + 5, & \text{if } x < -2 \\ x^2 + 3, & \text{if } -2 \leq x \leq 1 \\ 2x^3, & \text{if } 1 < x \end{cases}$ . Which of the following statements is

true about  $f$ ?

- I.  $f$  is continuous at  $x = 1$ . F  
II.  $f$  is differentiable at  $x = 1$ . F  
III.  $f$  has a local maximum at  $x = 0$ . T

- (a) I only      (b) II only      (c) III only  
(d) I and III only      (e) II and III only
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5. Given this graph of  $f(x)$ , which of the following might be the equation?



a)  $f(x)=\begin{cases} \sqrt{-x^2-8x-12}, & \text{if } -6 \leq x < -2 \\ 3, & \text{if } x = -2 \\ x^2-4, & \text{if } -2 < x < 2 \\ \frac{2}{3}x-\frac{4}{3}, & \text{if } 2 \leq x < 5 \end{cases}$

b)  $f(x)=\begin{cases} -x^2-8x-12, & \text{if } -6 \leq x < -2 \\ 3, & \text{if } x = -2 \\ \sqrt{4-x^2}, & \text{if } -2 < x < 2 \\ \frac{2}{3}x-\frac{4}{3}, & \text{if } 2 \leq x < 5 \end{cases}$

c)  $f(x)=\begin{cases} \sqrt{-x^2-8x-12}, & \text{if } -6 \leq x < -2 \\ 3, & \text{if } x = -2 \\ x^2-4, & \text{if } -2 < x < 2 \\ \frac{3}{2}x-\frac{4}{3}, & \text{if } 2 \leq x < 5 \end{cases}$

d)  $f(x)=\begin{cases} -x^2-8x-12, & \text{if } -6 \leq x < -2 \\ \sqrt{4-x^2}, & \text{if } -2 < x < 2 \\ \frac{2}{3}x-\frac{4}{3}, & \text{if } 2 \leq x < 5 \end{cases}$

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6. Let  $f$  be defined by  $f(x) = \begin{cases} 5x^2 + 10, & \text{if } x < -5 \\ 10x, & \text{if } x = -5 \\ Ax + 10, & \text{if } -5 < x \end{cases}$ . Determine the value of  $A$  for which  $f$  is continuous for all real  $x$ .

- (a)  $-5$  (b)  $5$  (c)  $-15$  (d)  $-25$  (e) None of these

$$x = -5 \rightarrow 5(-5)^2 + 10 = 135$$

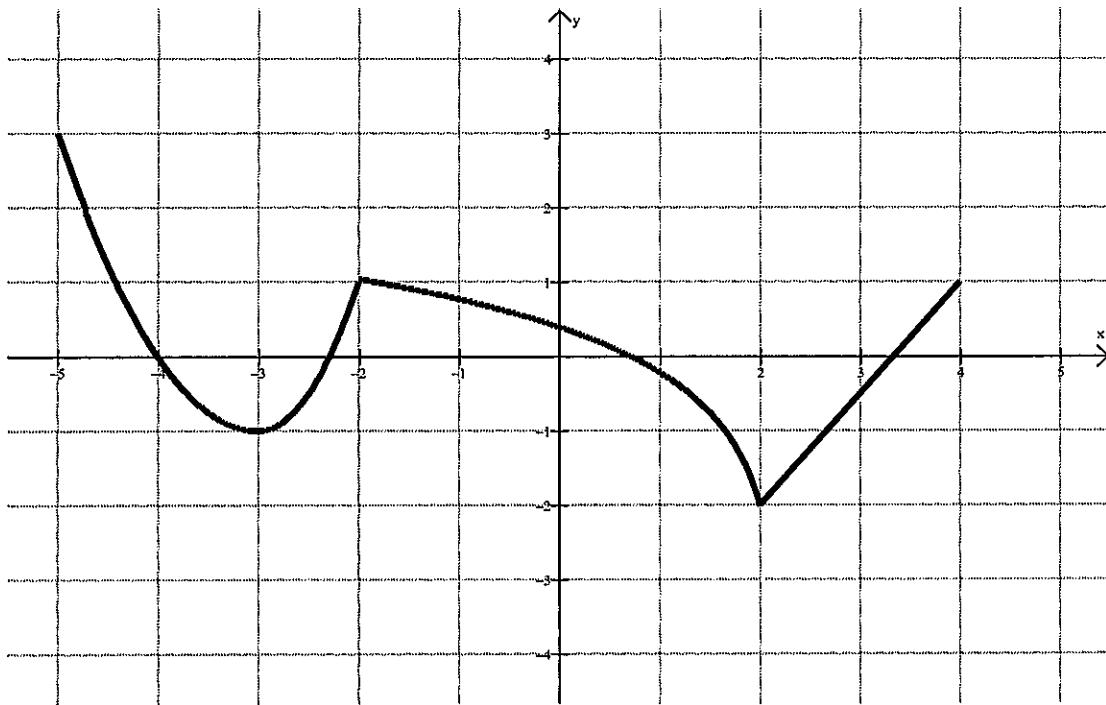
$$-5A + 10 = 135$$

$$-5A = 125$$

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$$A = -25$$

7. At what point is the graph of  $f$  below continuous but not differentiable?



- (a) -3      (b) -2      (c) 2      (d) -3 and 2      (e) -2 and 2

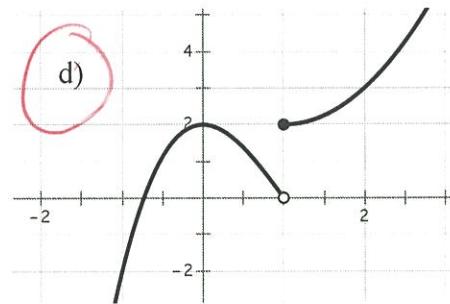
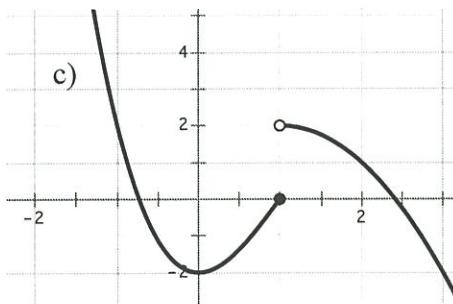
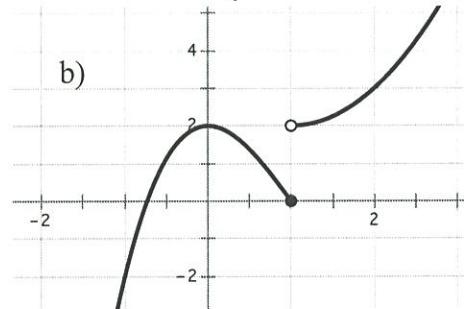
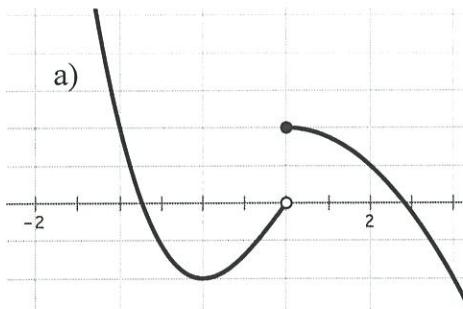
8. A function  $f(x)$  has a vertical asymptote at  $x = 2$ . The derivative of  $f(x)$  is negative for all  $x < -2$  and positive for all  $-2 < x$ . Which of the following statements are **false**?

I.  $\lim_{x \rightarrow -2} f(x) = -\infty$  F      II.  $\lim_{x \rightarrow -2^-} f(x) = -\infty$  T      III.  $\lim_{x \rightarrow -2^+} f(x) = +\infty$  T

- (a) I only      (b) II only      (c) III only  
 (d) I and II only      (e) II and III

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9. Which of the following is the graph of  $f(x) = \begin{cases} x^2 - 2x + 3, & \text{if } 1 \leq x \\ x^3 - 3x^2 + 2, & \text{if } x < 1 \end{cases}$  ?



$$1. \quad f(x) = \begin{cases} \sqrt{-x^2 - 8x - 12}, & \text{if } -6 \leq x \leq -2 \\ x^2 - 4, & \text{if } -2 < x < 2 \\ \frac{2-x}{x}, & \text{if } 2 \leq x \end{cases}$$

i) Is  $f(x)$  continuous at  $x = -2$ ? Why or why not?

1)  $f(-2)$  exists

$$2) \lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2} \sqrt{-x^2 - 8x - 12} = 0$$

$$\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2} \cancel{\frac{x^2 - 4}{x}} = \cancel{\frac{4}{2}} = -2 \quad \lim_{x \rightarrow -2} f(x) \text{ does not exist}$$

$$3) \lim_{x \rightarrow -2} f(x) = f(-2) \therefore$$

$\therefore f(x)$  is ~~not~~ continuous

ii) Is  $f(x)$  differentiable at  $x = -2$ ? Why or why not?

~~$f(x)$  is not differentiable because it is not continuous~~

i)  $f(x)$  is continuous

ii)  $\lim_{x \rightarrow -2^\pm} f'(x)$  ~~exists~~

~~iii)  $f'(x)$  is not diff~~

$$f' = \begin{cases} \frac{-x - 4}{\sqrt{-x^2 - 8x - 12}} & \text{if } x < -2 \end{cases}$$

$$2. \quad f(x) = \begin{cases} \sqrt{-x^2 - 8x - 12}, & \text{if } -6 \leq x \leq -2 \\ x^2 - 4, & \text{if } -2 < x < 2 \\ \frac{2-x}{x}, & \text{if } 2 \leq x \end{cases}$$

i) Is  $f(x)$  continuous at  $x=2$ ? Why or why not?

i)  $f(2)$  exists

$$\text{ii) } \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} \sqrt{-x^2 - 8x - 12} = 0$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} \frac{2-x}{x} = 0 \quad \therefore \lim_{x \rightarrow 2} f(x) \text{ exists}$$

$$\text{iii) } \lim_{x \rightarrow 2} f(x) = f(2) \therefore \text{continuous}$$

ii) Is  $f(x)$  differentiable at  $x=2$ ? Why or why not?

$$f' = \begin{cases} 2x & x < 2 \\ \frac{2}{x} & x > 2 \end{cases}$$

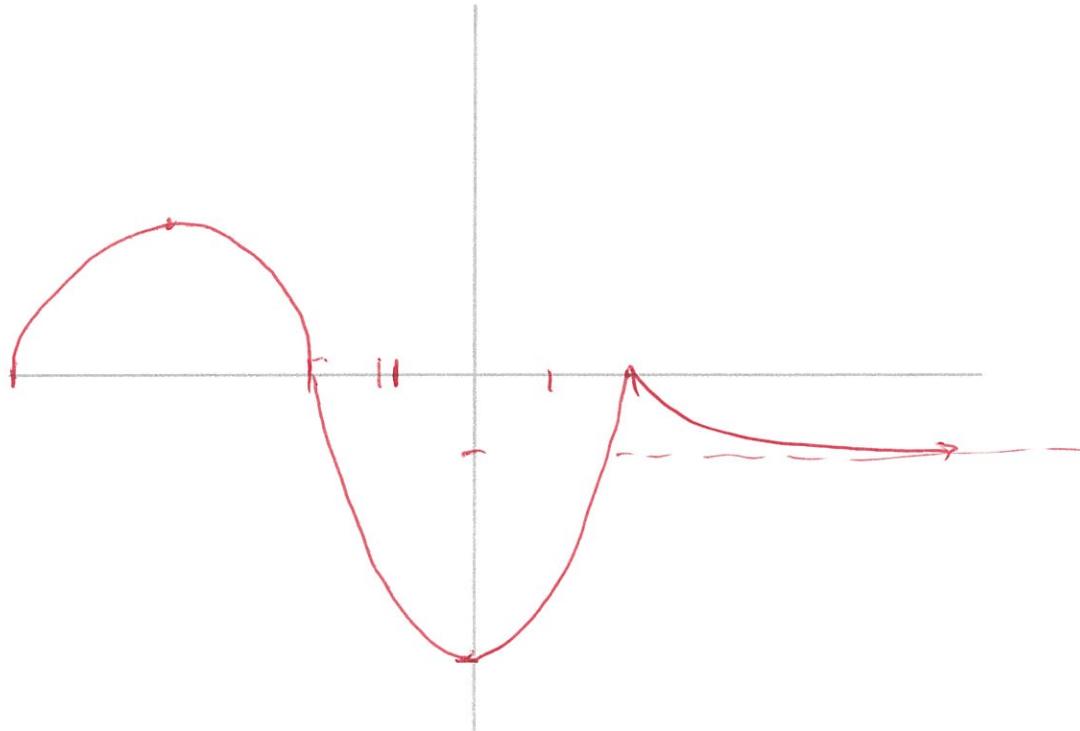
i) continuous

$$\text{ii) } \lim_{x \rightarrow 2^\pm} f'(x) \text{ exist}$$

$$\text{iii) } \lim_{x \rightarrow 2^-} f'(x) = 4 \neq \lim_{x \rightarrow 2^+} f'(x) = 1$$

$\therefore f(x)$  is not differentiable

3. Sketch  $f(x) = \begin{cases} \sqrt{-x^2 - 8x - 12}, & \text{if } -6 \leq x \leq -2 \\ x^2 - 4, & \text{if } -2 < x < 2 \\ \frac{2-x}{x}, & \text{if } 2 \leq x \end{cases}$ . State the Traits listed.



Domain:  $x \in [-6, \infty)$

Range:  $y \in [-4, 2]$

Zeros:  $(-6, 0), (\pm 2, 0)$

Y-int:  $(0, -4)$

VAs: NONE

EB (Left): NONE

EB (Right):  $y = -1$

$x$ -values of discontinuities: NONE

$x$ -values of non-differentiability:  $x = \pm 2$

Extreme Points (provide non-graphical evidence):

$$\text{X} \Rightarrow (-4, 2)$$

$$(0, 4)$$

$$(2, 0)$$

$$f'(x) = \begin{cases} \frac{-x-4}{-x^2-8x-12} & \text{if } -6 < x < -2 \\ 2x & \text{if } -2 < x < 2 \\ \frac{-2}{x^2} & \text{if } x > 2 \end{cases}$$

i)  $-x-4=0 \rightarrow x=-4 \quad (-4, 2)$

$2x=0 \rightarrow x=0 \quad (0, 4)$

ii)  $-x^2-8x-12=0 \rightarrow x=-6, \cancel{x=2}$

iii)  $x=-6 \rightarrow (-6, 0)$

$x=2$  NOT AN Extreme

$x=2 \rightarrow (2, 0)$