

1. Find the equation of the line tangent to the function $f(x) = 2(\ln x)^2$ when $x = e^3$.

$$f(e^3) = 2(3)^2 = 18$$

(a) $y - 18 = \frac{12}{e^3}(x - e^3)$

$$f'(x) = \frac{4 \ln x}{x}$$

~~(b)~~ $y - e^3 = \frac{12}{e^3}(x - 3)$

$$m = f'(e^3) = \frac{12}{e^3}$$

c) $y - 18 = 12(x - e^3)$

~~d)~~ $y - 18 = 12(x - 3)$

~~e)~~ $y - e^3 = 12(x - 3)$

2. The slope of the line tangent to the curve $xy^3 + x^2y^2 = 6$ at $(2, 1)$ is

$$x(3y^2)\frac{dy}{dx} + y^3(1) + x^2(2y\frac{dy}{dx}) + y^2(2) = 0$$

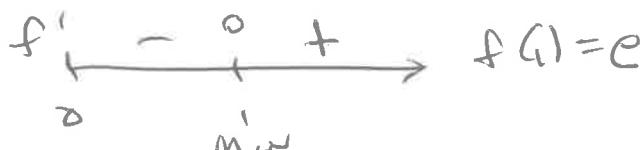
- a) -1 b) $-\frac{3}{5}$ c) $-\frac{5}{14}$ d) $-\frac{3}{14}$ e) 0

$$6\frac{dy}{dx} + 1 + 8\frac{dy}{dx} - 4 = 0$$

3. Let $f(x) = \frac{e^x}{x}$ on $x \in (0, \infty)$. The minimum value attained by f is

- a) 1 b) e c) $\frac{1}{e}$ d) $e-1$ e) undefined

$$f' = \frac{xe^x - e^x(1)}{x^2} = \frac{e^x(x-1)}{x}$$



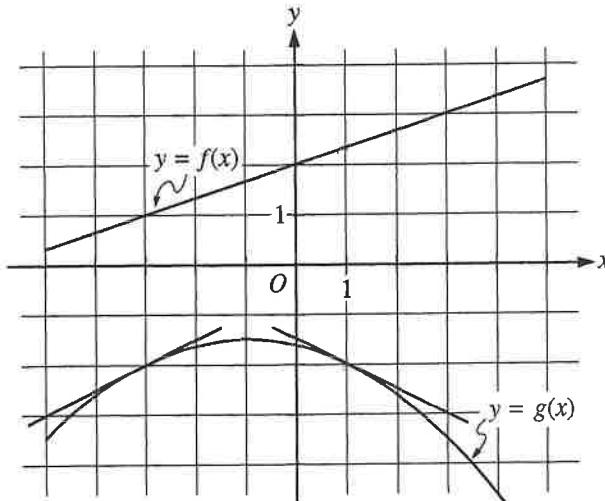
4. If $h(t) = e^{2t}(t+1)$, then $h'(0) = 1 + 2 = 3$

$$h' = e^{2t}(1) + (t+1)e^{2t}(2)$$

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
-

5. The figure below shows the graph of the functions f and g . The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = f(x) \cdot g(x)$, what is $B'(1)$?

- a) $-\frac{11}{6}$
 b) $-\frac{1}{2}$
 c) $\frac{1}{6}$
 d) $\frac{1}{3}$
 e) $\frac{1}{2}$



$$B = g \cdot f + g \cdot f'$$

$$\left(-\frac{1}{2}\right)\frac{7}{3} + (-2)\left(\frac{1}{3}\right) = -\frac{7}{6} - \frac{2}{3}$$

6. $\lim_{x \rightarrow 3} \frac{\ln\left(\frac{x-1}{2}\right)}{3-x} \stackrel{L'H}{=} \lim_{x \rightarrow 3} \frac{\frac{1}{x-1} \cdot \left(\frac{1}{2}\right)}{-1} = \frac{-1 \left(\frac{1}{2}\right)}{-1}$

- a) -1 b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) 1

7. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below, find $h'(2)$, given that $h(x) = g(x) \cdot f(x)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

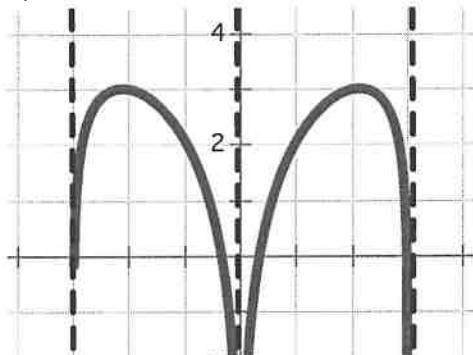
- a) -12 b) 24 c) 0 d) -48 e) 62

$$f(2) \cdot g'(2) + g(2) f'(2)$$

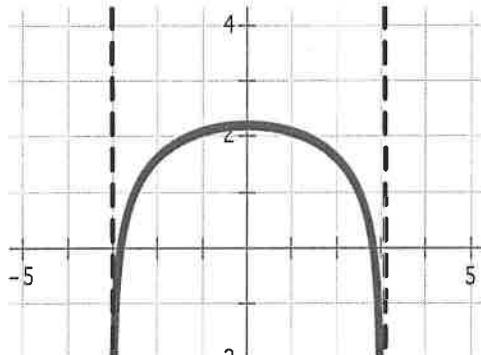
$$= 4(1) + 8(-2)$$

8. Which if the following is the graph of $y = -\ln(9x^2 - x^4)$?

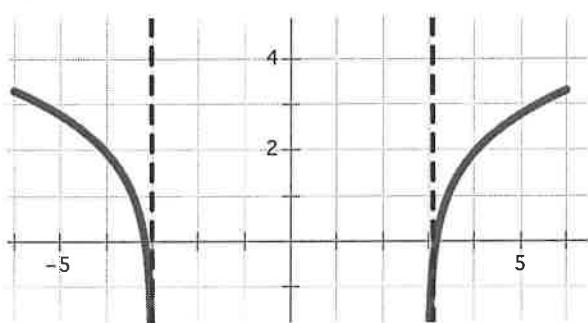
a)



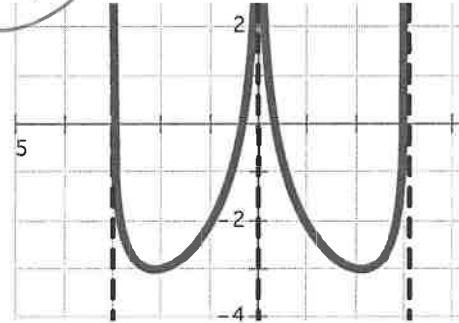
b)



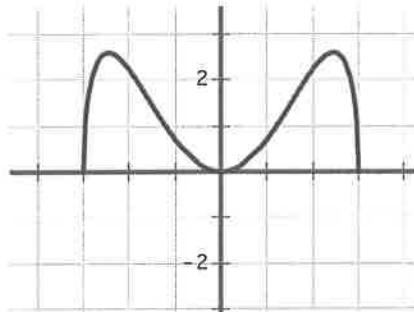
c)



d)



9. Which of the following is the equation of this graph?



a) $y = .125(9x^2 - x^4)$

(b)

$y = (.25x^2)\sqrt{9-x^2}$

~~✓~~ b) $y = \frac{9x^2 - x^4}{.4x^4 + 5}$

~~✓~~

$y = \ln(9x^2 - x^4)$

1. Find domain and zeros of $K(x) = (2x)\sqrt{16-x^2}$.

$$x \in [-4, 4]$$

$$\star (\pm 4, 0) (0, 0)$$

2. Find the extreme points of $K(x) = (2x)\sqrt{16-x^2}$. Show the algebraic work to support the critical values.

$$\begin{aligned} K' &= 2x \left(\frac{1}{2}(16-x^2)^{-\frac{1}{2}}(-2x) + (16-x^2)^{\frac{1}{2}}(2) \right) \\ &= \frac{-2x^2}{(16-x^2)^{\frac{1}{2}}} + 2(16-x^2)^{\frac{1}{2}} = \frac{-4x^2+32}{(16-x^2)^{\frac{1}{2}}} \end{aligned}$$

i) $\frac{dy}{dx} = 0 \Rightarrow x = \pm 2\sqrt{2}$

ii) $\frac{dy}{dx}$ DNE $\rightarrow x = \pm 4$ $(\pm 4, 0)$

iii) none $(2\sqrt{2}, 16)$
 $(-2\sqrt{2}, -16)$

3. Find domain and zeros of $g(x) = e^{-x} \sqrt{x+3}$.

Domain $x \in [-3, \infty)$

Zeros: $(-3, 0)$

4. Find the extreme points of $g(x) = e^{-x} \sqrt{x+3}$. Show the algebraic work to support the critical values.

$$\begin{aligned}\frac{dy}{dx} &= e^{-x} \left(\frac{1}{2}(x+3)^{-\frac{1}{2}} \right) + (x+3)^{\frac{1}{2}} \cdot (e^{-x}(-1)) \\ &= e^{-x} \left[\frac{1}{2(x+3)^{\frac{1}{2}}} - \cdot (x+3)^{\frac{1}{2}} \right] = e^{-x} \left[\frac{2(2-x) - 5 - 2x}{2(x+3)^{\frac{1}{2}}} \right]\end{aligned}$$

i) $\frac{dy}{dx} = 0 \Rightarrow x = -\frac{5}{2}$ $(-\frac{5}{2}, 8.614)$

ii) $\frac{dy}{dx}$ DNE $\Rightarrow x = -3$ $(-3, 0)$

iii) none

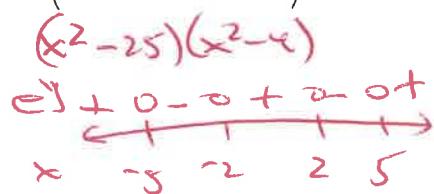
5. Find domain, VAs, and zeros of $h(x) = \ln(x^4 - 29x^2 + 100)$.

$$\text{Domain } x \in (-\infty, -5) \cup (-2, 2) \cup (5, \infty)$$

$$\text{VA: } x = \pm 5, \pm 2$$

$$\text{Zeros } (\pm 5, 0.05, 0)$$

$$(\pm 1.985, 0)$$



6. Find the extreme points of $h(x) = \ln(x^4 - 29x^2 + 100)$ on $x \in [-1, 7]$. Show the algebraic work to support the critical values.

$$\frac{dy}{dx} = \frac{4x^3 - 58}{x^4 - 29x^2 + 100}$$

i) $\frac{dy}{dx} = 0 \rightarrow x = 0, \pm \sqrt[3]{\frac{29}{2}}$ $(0, \ln 100)$

ii) $\frac{dy}{dx} \text{ DNE} \rightarrow x = \pm 5, \pm 2$ $(-1, 4.277)$

iii) $x = 7, -1$ $(7, 6.985)$

Honors PreCalculus '21-22

Name: SOLUTION KEY

Chapter 10 Test

NO CALCULATOR ALLOWED

Score _____

7. $y = (5x^2 - 3)^7 (7x^4 + 4)^{10}$. Find $\frac{dy}{dx}$ in factored form.

$$\begin{aligned}\frac{dy}{dx} &= (5x^2 - 3)^7 \cdot 10(7x^4 + 4)^9 (28x^3) + (7x^4 + 4)^{10} (7(5x^2 - 3)^6 (10x)) \\ &= 70 \times (5x^2 - 3)^6 (7x^4 + 4)^9 \left[\cancel{5}x^2(5x^2 - 3) + \cancel{4}(7x^4 + 4) \right] \\ &= 70 \times (5x^2 - 3)^6 (7x^4 + 4)^9 [27x^4 - 12x^2 + 4]\end{aligned}$$

DO TWO OF THE FOLLOWING THREE SKETCHING PROBLEMS

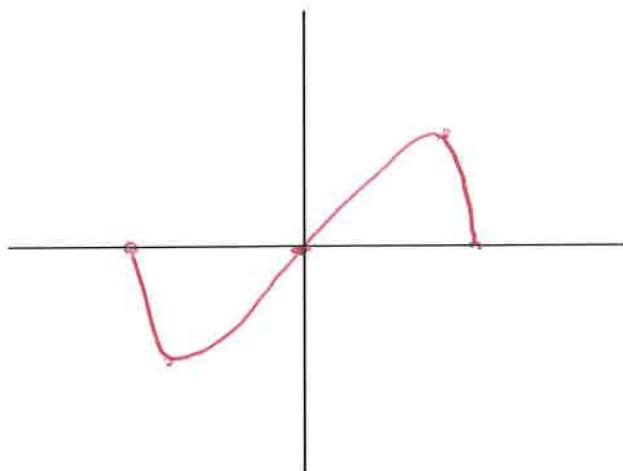
8. Find the traits and sketch $K(x) = (2x)\sqrt{16 - x^2}$.

Y-intercept: $(0, 0)$

Range: $y \in [-8, 8]$

End Behavior (Left): NONE

End Behavior (Right): NONE



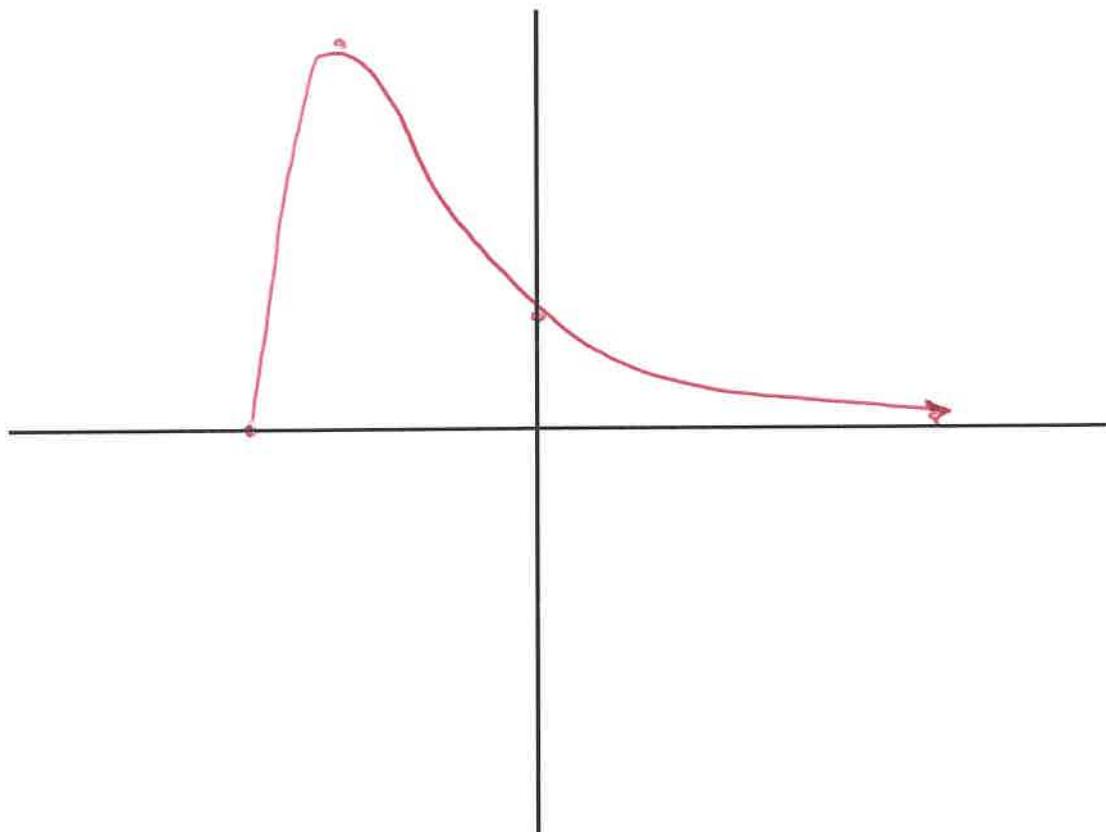
9. Find the traits and sketch of $g(x) = e^{-x}\sqrt{x+3}$.

Y-intercept: $(0, \sqrt{3})$

Range: $y \in [0, 2.614]$

End Behavior (Left): No SE

End Behavior (Right): $y = 0$



10. Find the traits and sketch of $h(x) = \ln(x^4 - 29x^2 + 100)$ on $x \in [-1, 7]$.

Y-intercept: $(0, 4.605)$

Range: $y \in (-\infty, 6.985)$

End Behavior (Left): NONE

End Behavior (Right): NONE

