

Dr. Quattrin

Limits and Derivatives Test

CALCULATOR ALLOWED

Score _____

Round to 3 decimal places. Show all work.

1. Find the instantaneous rate of change of $f(x) = 3x^2 - \frac{1}{2x^3}$ at $x = -1$.

$$\cancel{f(t) = \frac{5}{2}}$$

$$f' = 6x + \frac{3}{2}x^{-4}$$

$$m = -6 + \frac{3}{2}(-1)^{-4} = -\frac{9}{2}$$

- a) dne b) $-\frac{9}{2}$ c) $\frac{7}{2}$ d) $-\frac{15}{2}$ e) 0

$$\frac{f(x) - f(2)}{x - 2}$$

2. If $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x - 2}$ is the numerical derivative of the function $f(x)$, then

$$f(x) =$$

- a) x^2 b) -4 c) $x^2 - 4x$

- d) $2x - 4$ e) None of these

3. Suppose f is a differentiable function such that $f(3) = 5$ and $f'(3) = -4$.

Using the line tangent to the graph of $f(x)$ at $x = -1$, find the approximation of

$$f(2.9)$$

$$y - 5 = -4(x - 3)$$

- a) -3.5 b) 11.4 c) 4.6 d) -4.6 e) 5.4

4. Find an equation of the normal line to the curve $f(x) = \frac{2}{x^3}$ at $x=1$.

a) $y = 6x - 4$

b) $y = -\frac{1}{6}x + \frac{13}{6}$

c) $y = \frac{1}{6}x + \frac{11}{6}$

d) $y = -6x + 8$

e) $y = -6x + 13$

~~yzx~~

$$f' = -6x^{-4}$$

$$m_{\text{Tan}} = -6 \quad m_{\text{Norm}} = \frac{1}{6}$$

$$y - 2 = \frac{1}{6}(x - 1)$$

5. Let $f(x) = \sqrt[3]{x^4} - x^2$. Find $f''(x)$.

a) $\frac{3}{\sqrt[3]{x}} - 2$

$$= x^{4/3} - x^2$$

b) $\frac{4}{3}\sqrt[3]{x} - 2$

c) $\frac{4}{3}\sqrt[3]{x} - 2x$

d)

$$\frac{4}{9\sqrt[3]{x^2}} - 2$$

e) None of these

6. A particle moves in the xy-plane so that its coordinates at time t are $x = t^2$ and $y = 4 + t^3$. At $t = 1$, the acceleration vector is

$$v = \langle 2t, 3t^2 \rangle$$

a) $\langle 2, -3 \rangle$

b) $\langle 2, -6 \rangle$

c) $\langle 1, 6 \rangle$

d)

$$\langle 2, 6 \rangle$$

e) $\langle 1, -2 \rangle$

$$a = \langle 2, 6t \rangle$$

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1. Use the equation of the line tangent to $y = 2x^3 - 4x^2 + 3x - 1$ at $x = 2$ to approximate $f(2.1)$

$$y(2) = 5$$

$$\frac{dy}{dx} = 6x^2 - 8x + 3$$

$$m = 11$$

$$y - 5 = 11(x - 2)$$

$$f(2.1) \approx y(2.1) = 5 + (-1.1) \\ = 3.9$$

2. The motion of a particle is described by $y(t) = 8t^3 + 33t^2 - 18t + 15$.
- When the particle is stopped?
 - Which direction it is moving at $t = -1$? $V = 24t^2 + 66t - 18$
 - Where is it when $t = -1$?
 - Find $a(-1)$.

$$a) V = 6(4t^2 + 11t - 3) \\ = 6(4t - 1)(t + 3) = 0$$

$$t = 4/4, -3$$

$$b) V(-1) = -48 \Rightarrow \text{DOWN}$$

$$c) x(-1) = 58$$

$$d) a(t) = 48t + 66$$

$$a(-1) = 18$$

3. A particle's position $\langle x(t), y(t) \rangle$ at time t is described by
 $\langle t^2 - 2t - 3, 16t - t^3 \rangle$. $v = \langle 2t-2, 16-3t^2 \rangle$

a) Find the speed at $t = 3$.

$$v(3) = \langle 4, -11 \rangle$$

$$s = \sqrt{4^2 + 11^2} = \sqrt{137} \approx 11.705$$

b) When, if ever, is the particle stopped? Prove it.

$$v_x = 2t - 2 = 0 \Rightarrow t = 1$$

$v_y(1) = 16 - 3 \neq 0 \quad \therefore \text{THE PARTICLE DOES NOT}$
 $\text{STOP BECAUSE BOTH } v_x \text{ AND } v_y$
 $\text{NEED TO BE ZERO AT THE}$
 SAME TIME

Honors PreCalculus

Name: SOLUTION KEY

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5. Set up, but do not solve, the limit definition of the derivative for
 $y = 3x^4 - 7x^3 + 4x^2 + 5$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{[3(x+h)^4 - 7(x+h)^3 + 4(x+h)^2 + 5] - [3x^4 - 7x^3 + 4x^2 + 5]}{h}$$

6. Use the Power Rule to find:

a) Find $\frac{dy}{dx}$ if $y = 2x^3 - 3x^3 + 4x - 6$

$$\frac{dy}{dx} = 6x^2 - 9x + 4$$

b) $\frac{d}{dx} \left[\sqrt[8]{x^3} + \frac{1}{3x^3} - \sqrt[7]{x} + \pi^4 \right] = \frac{d}{dx} \left[x^{3/8} + \frac{1}{3}x^{-3} - x^{1/7} + \pi^4 \right]$

$$= \frac{3}{8}x^{-5/8} - x^{-4} - \frac{1}{7}x^{-6/7}$$

7. Evaluate the following limits:

a) $\lim_{x \rightarrow 5} \frac{2x^2 - 7x + 5}{6x^2 - 11x - 10}$

$$= \lim_{x \rightarrow 5/2} \frac{(2x-5)(x+1)}{(2x-5)(3x+2)}$$

$$= \frac{5/2 - 1}{3(5/2) + 2} = \frac{3/2}{19/2} = \frac{3}{19}$$

b) $\lim_{x \rightarrow 1} \frac{x^4 + 2x^3 + x^2 - x - 3}{x^3 + 7x^2 - 6}$

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^3+3x^2+4x+3)}{(x-1)^2} = 0$$

$$\begin{array}{r} \textcolor{red}{1} \\ \textcolor{red}{1} \ 2 \ 1 \ -1 \ 3 \\ \hline 1 \ 3 \ 4 \ 3 \\ 1 \ 3 \ 4 +3 \ 0 \end{array}$$

$$\begin{array}{r} 5 \\ \times 7 \\ \hline 35 \end{array}$$

$$\text{c) } \lim_{x \rightarrow 7} \frac{x^3 - 7x^2 - 3x + 21}{3x^3 - 20x^2 - 11x + 28} =$$

$$\lim_{x \rightarrow 7} \frac{(x-7)(x^2-3)}{(x-7)(3x^2+x-4)}$$

$$= \frac{46}{144} = \frac{23}{72}$$

$$\begin{array}{r}
 7) \quad 3 \quad -20 \quad -11 \quad 28 \\
 \underline{-} \quad \quad \quad \quad \quad -28 \\
 \quad \quad 21 \quad \quad 7 \quad \quad 28 \\
 \underline{-} \quad \quad \quad \quad \quad 28 \\
 \quad \quad \quad \quad 3 \quad \quad 1 \quad \quad -14
 \end{array}$$