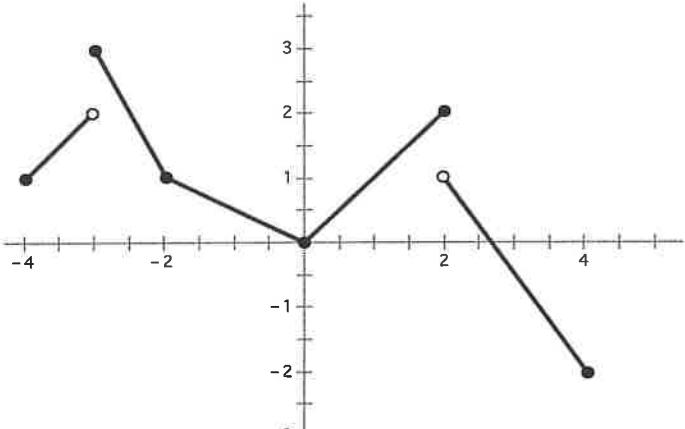


Honors PreCalculus '21-22
Piece-Wise Defined Functions Test
Dr. Quattrin
Calculator allowed

Name: Sophia Kay
score _____

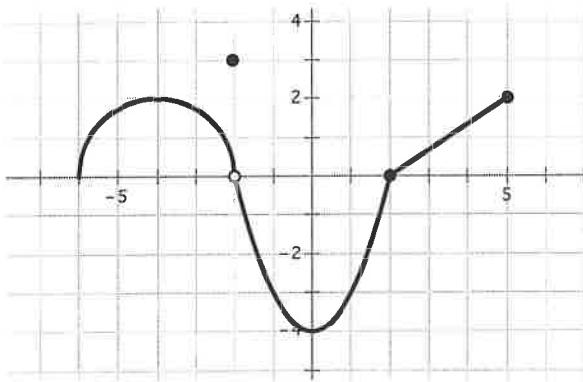
1. The function f is defined on the interval $x \in (-4, 4)$ and has the graph shown below.



For which of the following values is f not differentiable?

- a) -3 and 2 only b) 0 only c) -2 and 0 only
d) -4, -2, and 0 only e) -3, -2, 0, and 2
-

2. The function f is shown below. Which of the following statements about the function f , shown below, is **false**?



- (a) $\lim_{x \rightarrow 0} f(x)$ does not exist F
- b) $\lim_{x \rightarrow 2} f(x)$ exists
- c) f is continuous at $x = -2$
- d) $\lim_{h \rightarrow 0} \frac{f(1-h)+3}{h}$ exists
-

3. The function f defined on all the Reals such that $f(x) = \begin{cases} 2-mx, & \text{if } x \leq 2 \\ k\sqrt{x^2-3}, & \text{if } x > 2 \end{cases}$.

For which of the following values of k and m will the function f be both continuous and differentiable on its entire domain?

- a) $m = -2, k = -2$
- b) $m = 2, k = -2$
- c) $m = -2, k = 2$
- d) $m = 2, k = 2$

$$f \Rightarrow 2 - 2m = k$$

$$f' \Rightarrow -m = k$$

$$\Rightarrow 2 - 2(-m) = m - m$$

$$2 = 0$$

$$m = 2 \Rightarrow k = -2$$

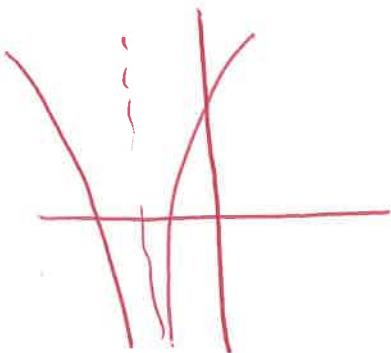
4. Let f be defined by $f(x) = \begin{cases} 4x^2 + 10, & \text{if } x < 1 \\ mx^3 + 8, & \text{if } 1 < x \end{cases}$. Determine the value of m for which f is continuous for all real x .

- a) 6 b) -2 c) 8 d) 14 e) None of these

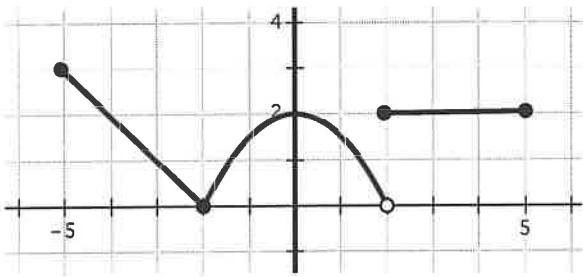
$$14 = m + 8 \rightarrow m = 6 \text{ but } f(1) \neq$$

5. A function $f(x)$ has a vertical asymptote at $x = -2$. The derivative of $f(x)$ is negative for all $x < -2$ and positive for all $-2 < x$. Which of the following statements is **true**?

- a) $\lim_{x \rightarrow -2^-} f(x) = -\infty$ and $\lim_{x \rightarrow -2^+} f(x) = -\infty$
- b) $\lim_{x \rightarrow -2^-} f(x) = -\infty$ and $\lim_{x \rightarrow -2^+} f(x) = +\infty$
- c) $\lim_{x \rightarrow -2^-} f(x) = +\infty$ and $\lim_{x \rightarrow -2^+} f(x) = +\infty$
- d) $\lim_{x \rightarrow -2^-} f(x) = -\infty$ and $\lim_{x \rightarrow -2^+} f(x) = -\infty$



6. Given this graph of $f(x)$, which of the following might be the equation?



a) $f(x)=\begin{cases} -x, & \text{if } -4 \leq x < -2 \\ \sqrt{4-x^2}, & \text{if } -2 \leq x \leq 2 \\ 1, & \text{if } 2 < x \leq 4 \end{cases}$

Semi-circle

b) $f(x)=\begin{cases} -x-2, & \text{if } -4 \leq x < -2 \\ 2-\frac{1}{2}x^2, & \text{if } -2 \leq x \leq 2 \\ 2, & \text{if } 2 < x \leq 4 \end{cases}$

c) $f(x)=\begin{cases} x+2, & \text{if } -4 \leq x < -2 \\ \sqrt{4-x^2}, & \text{if } -2 \leq x \leq 2 \\ 1, & \text{if } 2 \leq x \leq 4 \end{cases}$

d) $f(x)=\begin{cases} -x-2, & \text{if } -4 \leq x < -2 \\ 2-\frac{1}{2}x^2, & \text{if } -2 \leq x < 2 \\ 2, & \text{if } 2 \leq x \leq 4 \end{cases}$

$$f(x) = \begin{cases} \frac{x^2 - 7x + 10}{b(x-2)} & \text{for } x \neq 2 \\ b & \text{for } x = 2 \end{cases}.$$

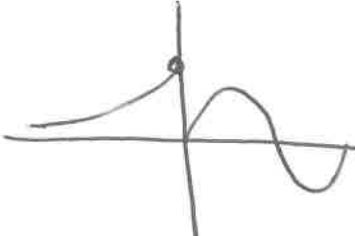
7. For which value of b will the function be continuous throughout its domain?

- a) -3 b) $\sqrt{2}$ c) 3 d) 5 e) None of these

$$\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{b(x-2)} = \lim_{x \rightarrow 2} \frac{(x-5)(x-2)}{b(x-2)} = b \rightarrow b^2 = -3$$

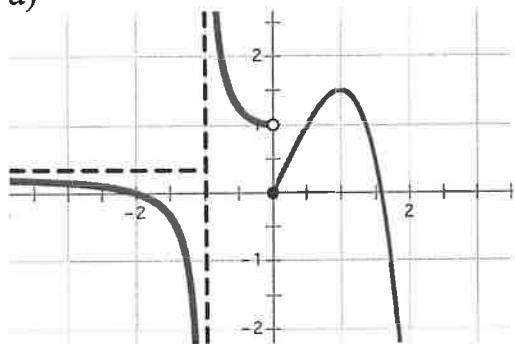
8. At $x=0$, the function given by $f(x) = \begin{cases} e^x, & \text{if } x \leq 0 \\ \sin x, & \text{if } 0 < x \end{cases}$ is

- a) Undefined
 b) Continuous but not differentiable
 c) Differentiable but not continuous
 d) Neither continuous nor differentiable
 e) Both continuous and differentiable

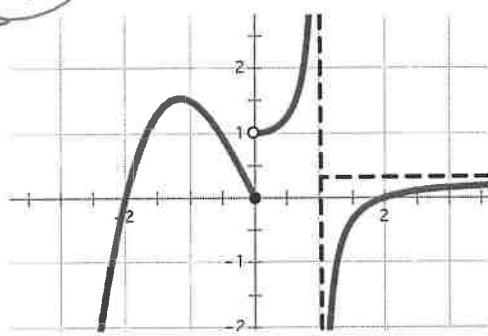


9. Which of the following is the graph of $f(x) = \begin{cases} \frac{x^2-4}{4(x^2-1)}, & \text{if } 0 < x \\ 0.5x^3 - 2x, & \text{if } x \leq 0 \end{cases}$?

a)



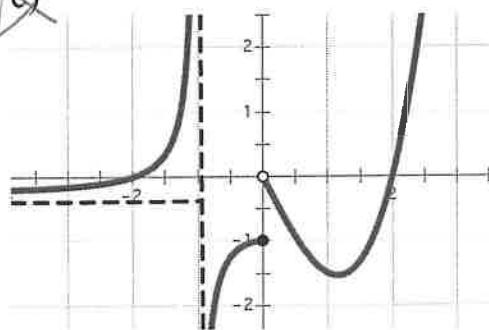
b)



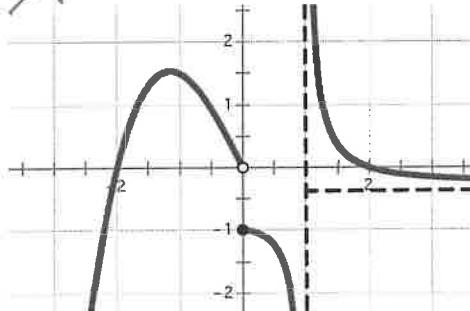
HA @ $y = \frac{1}{4}$

RATIONAL ON RIGHT

c) ~~(X)~~



d) ~~(X)~~



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$$1. \quad f(x) = \begin{cases} \sqrt{-x^2 - 8x - 12}, & \text{if } -6 \leq x < -2 \\ 3, & \text{if } x = -2 \\ x^2 - 4, & \text{if } -2 < x < 2 \\ \frac{2}{3}x - \frac{4}{3}, & \text{if } 2 \leq x < 5 \end{cases}$$

i) Is $f(x)$ continuous at $x = -2$? Why or why not?

i) $f(-2) = 3$

ii) $\lim_{x \rightarrow -2} \sqrt{-x^2 - 8x - 12} = 0 \quad \Rightarrow \quad \lim_{x \rightarrow -2} f(x) = 0$
 $\lim_{x \rightarrow -2} x^2 - 4 = 0$

iii) $\lim_{x \rightarrow -2} f(x) \neq f(-2) \quad \therefore \text{DISCONTINUOUS}$

ii) Is $f(x)$ differentiable at $x = -2$? Why or why not?

NOT DIFFERENTIABLE BECAUSE $f(x)$ IS NOT
 CONTINUOUS

$$2. \quad f(x) = \begin{cases} \sqrt{-x^2 - 8x - 12}, & \text{if } -6 \leq x < -2 \\ 3, & \text{if } x = -2 \\ x^2 - 4, & \text{if } -2 < x < 2 \\ \frac{2}{3}x - \frac{4}{3}, & \text{if } 2 \leq x < 5 \end{cases}$$

i) Is $f(x)$ continuous at $x=2$? Why or why not?

$$\text{i)} \quad f(2) = \frac{2}{3}(2) - \frac{4}{3} = 0$$

$$\text{ii)} \quad \lim_{x \rightarrow 2^-} f(x) = 2^2 - 4 = 0 = \frac{2}{3}(2) - \frac{4}{3} = \lim_{x \rightarrow 2^+} f(x)$$

so $\lim_{x \rightarrow 2} f(x)$ exists

$$\text{iii)} \quad \lim_{x \rightarrow 2} f(x) = f(2) \therefore \text{CONTINUOUS}$$

ii) Is $f(x)$ differentiable at $x=2$? Why or why not?

$$f'(x) = \begin{cases} 2x & \text{if } -2 < x < 2 \\ 2/3 & \text{if } x > 2 \end{cases}$$

i) CONTINUOUS

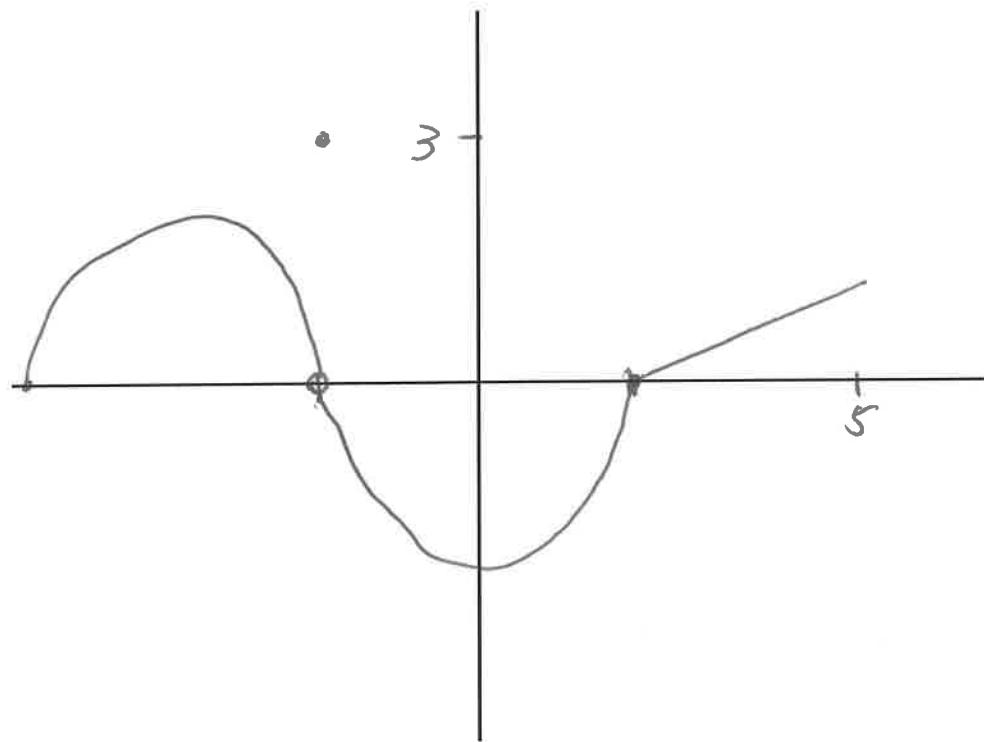
$$\text{ii)} \quad \lim_{x \rightarrow 2^+} f(x) \text{ and } \lim_{x \rightarrow 2^-} f(x) \text{ exist}$$

$$\text{iii)} \quad \lim_{x \rightarrow 2^-} f(x) = f(2) = 4 \neq 2/3 = \lim_{x \rightarrow 2^+} f(x)$$

so $f(x)$ is NOT DIFFERENTIABLE AT $x=2$

$$3. \text{ Sketch } f(x) = \begin{cases} \sqrt{-x^2 - 8x - 12}, & \text{if } -6 \leq x < -2 \\ 3, & \text{if } x = -2 \\ x^2 - 4, & \text{if } -2 < x < 2 \\ \frac{2}{3}x - \frac{4}{3}, & \text{if } 2 \leq x < 5 \end{cases} . \text{ State the Traits listed.}$$

$\sqrt{-(x+6)(x+2)}$



Domain: $x \in [-6, 5]$

Range: $y \in [-4, 3]$

Zeros: $(-6, 0), (2, 0)$

Y-int: $(0, 3)$

VAs: ~~None~~

EB (Left): ~~None~~

EB (Right): ~~None~~

x -values of discontinuities: $x = -2$

x -values of non-differentiability: $x = \pm 2$

Extreme Points (provide non-graphical evidence):

$$\frac{dy}{dx} = \begin{cases} \frac{-x^2 - 4}{(-x^2 - 8x - 12)^{1/2}} & \text{if } -6 < x < -2 \\ -2x & \text{if } -2 < x < 2 \\ \frac{2}{3} & \text{if } 2 < x < 5 \end{cases}$$

- i) $\frac{dy}{dx} = 0 \rightarrow x = -4$ $(-4, z)$
- ii) $\frac{dy}{dx}$ DNE $\rightarrow x = 2, -2$ $(2, z)$
- iii) END POINTS $x = -6, 5$ $(-6, 0)$
 $(5, z)$