

1. Given this sign pattern $f'(x) \begin{array}{c} - & 0 & + & 0 & - & 0 \\ \leftarrow & -4 & -1 & 2 & \rightarrow \end{array}$, at what value(s) of x does f has a critical value?

- a) -4 b) -1 c) 2 d) -4 and 1 e) -4, -1, and 2
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2. The maximum value of $f(x) = \frac{5}{\sqrt{x}} + 2\sqrt{x}$ is
- a) $\frac{5}{2}$ b) $\frac{2}{5}$ c) $\frac{\sqrt{10}}{5}$ (d) $2\sqrt{10}$ e) No such value

$$f' = -\frac{5}{2}x^{-3/2} + x^{-1/2} = x^{-3/2} \left[-\frac{5}{2} + x \right] \Rightarrow x = \frac{5}{2} \Rightarrow y = 2\sqrt{10}$$

3. What are all values of x for which the function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 2$ is increasing?

- a) $-2 < x < 3$
 b) $-3 < x < 2$
 c) $x < -3$ or $2 < x$
 d) $x < -2$ or $3 < x$
 e) All real numbers

$$\begin{aligned} f' &= x^2 - x - 6 \\ &= (x-3)(x+2) \end{aligned}$$

$f' \begin{array}{c} + & 0 & - & 0 & + \\ \leftarrow & -2 & & 3 & \rightarrow \end{array}$

4. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (4 - x^2)g(x)$, which of the following is true?



- a) $f(x)$ has a relative maximum at $x = 2$ and a relative minimum at $x = -2$.
 - b) $f(x)$ has a relative minimum at $x = 2$ and a relative maximum at $x = -2$.
 - c) $f(x)$ has relative minima at $x = -2$ and $x = 2$.
 - d) $f(x)$ has relative maxima at $x = -2$ and $x = 2$.
 - e) It cannot be determined if f has any relative extrema.
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5.

- Consider a particle moving such that its position is described by the function $x(t) = \frac{t^4}{2} - \frac{t^5}{10}$. When does the particle attain its maximum acceleration?

- a) $t = 0$
- b) $t = 1$
- c) $t = 2$
- d) $t = 3$
- e) $t = 4$

$$x' = 2t^3 - \frac{1}{2}t^4$$

$$x'' = 6t^2 - 2t^3$$

$$a' = 12t - 6t^2 = 0$$

$$12t = 6t^2$$

$$t = 2$$

6. Find the x -value of the absolute maximum of $y = x^2 + 3x + 2$ on $x \in [-2, 3]$.

- a) $x = -\frac{2}{3}$
- b) $x = \frac{2}{3}$
- c) $x = -2$

d) $x = -\frac{3}{2}$

e) $x = 3$

$$y(-2) = 0$$

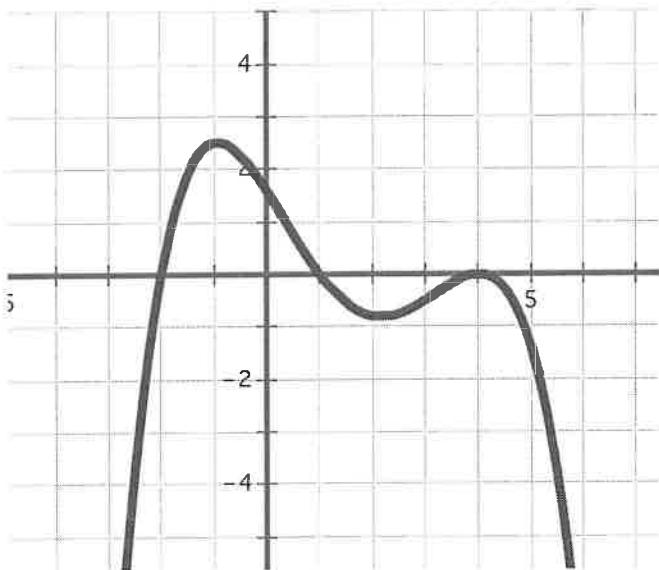
$$y(-\frac{3}{2}) =$$

$$y(3) = 20$$

7. Given this sign pattern $\begin{array}{c} f(x) \\ \hline x \\ - & 0 & + & 0 & + \\ \leftarrow & -3 & & 1 & \rightarrow \end{array}$, on which interval(s) is $f(x)$ increasing?

- a) $-3 < x < 1$
 - b) $-3 < x < 1$ and $x > 1$
 - c) $x < -3$
 - d) $x > 1$
 - e) It cannot be determined from this sign pattern
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8. Which of the following equations matches this graph:

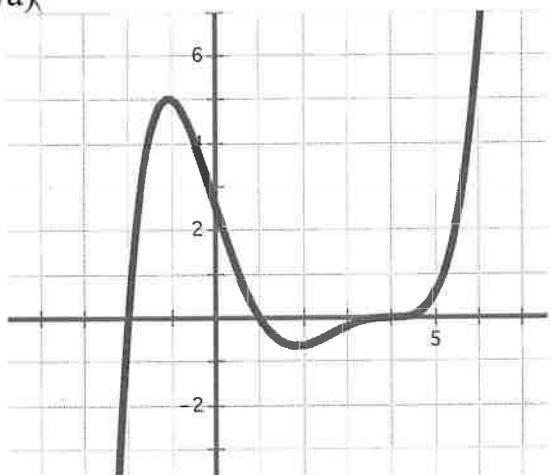


- a) $y = -.07(x+2)(x-1)^3(x-4)$
 - b) $y = -.3(x+2)(x-1)(x-4)$
 - c) $y = -.05(x+2)(x-1)(x-4)^2$
 - d) $y = -.02(x+2)(x-1)(x-4)^3$
-

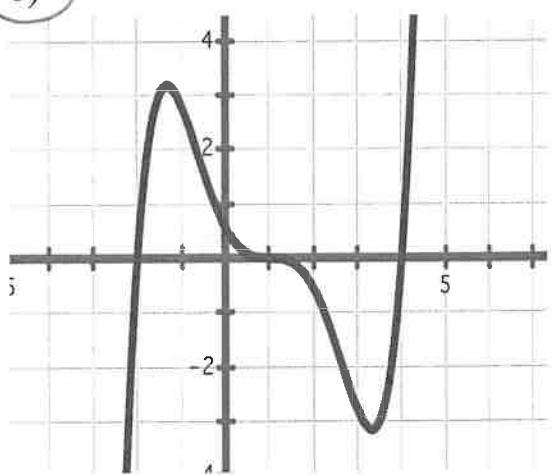
9. Which of the following graphs matches the equation

$$y = .07(x+2)(x-1)^3(x-4)$$

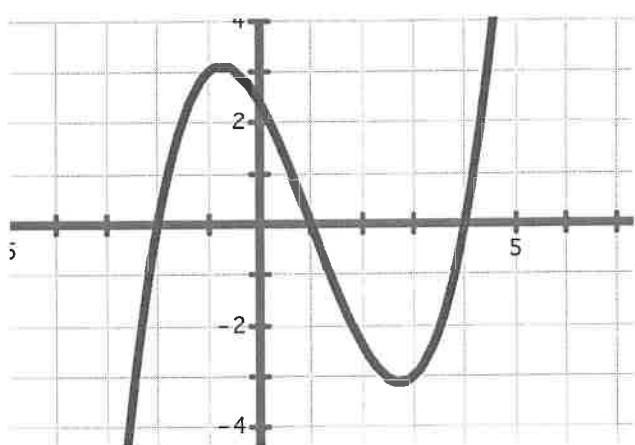
(a)



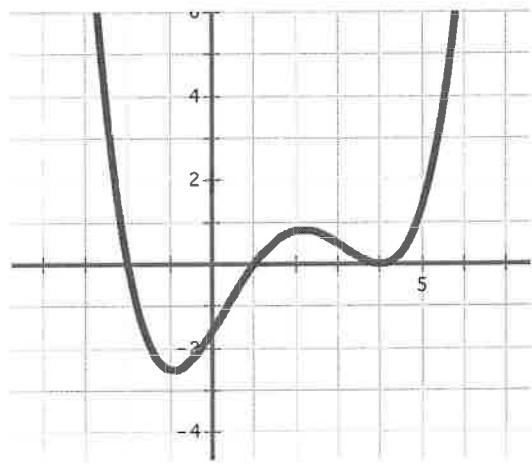
(b)



(c)



(d)



Honors PreCalculus '21-22

Name: Solutions Key

Dr. Quattrin

Polynomials Test-- CALCULATOR ALLOWED

Round to 3 decimal places.

Score _____

Show all work.

1. Find the zeros and extreme points of $y = -3x^3 + 19x^2 - 16x - 20$. Show the algebraic work to support the zeros and critical values.

$$\begin{array}{r} 2 \mid -3 & 19 & -16 & -20 \\ & -4 & 26 & 20 \\ \hline & -3 & 13 & +10 & \emptyset \end{array}$$

Zeros: $(2, 0)$, $(-\frac{2}{3}, 0)$, $(5, 0)$

$$-(x-3)(3x^2 - 13x - 10)$$

$$-(x-3)(3x+2)(x-5)$$

$$\frac{dy}{dx} = -9x^2 + 38x - 16$$

i) $\frac{dy}{dx} = 0 \rightarrow x = 2.222, 4$

$$(4, 28)$$

ii) $\frac{dy}{dx}$ DNE \Rightarrow no sol

$$(\frac{2}{3}, -22.650)$$

iii) Endpoints: none given

2. Find the zeros and extreme points of $f(x) = -x^4 + 6x^2 - 8$ on $x \in [-1.5, 3]$.
 Show the derivative and algebra to support the critical values.

$$f(x) = -(x^2 - 2)(x^2 - 4)$$

~~Zeros:~~ ~~$\pm\sqrt{2}, 0$~~ $(\pm 2, 0)$ BUT -2 is not in domain

$$\therefore (\pm\sqrt{2}, 0), (2, 0)$$

$$f'(x) = -4x^3 + 12x = -4x^3(x^2 - 3) = 0$$

i) $f' = 0 \rightarrow x = 0, \pm\sqrt{3}$ BUT sign of f' does not switch @ $x=0$ so that is NOT AN EXTREME

ii) f' DNE \Rightarrow NEVER

iii) ENDPOINTS $x = -1.5, 3$

EXTREME POINTS $(3, -35)$ $(-1.5, -438)$

$(0, -8)$ $(\sqrt{3}, 1)$

3a. Find the zeros, algebraically, of $f(x) = x^3 - x^2 - 16x + 16$.

$$\begin{aligned}f(x) &= x^2(x-1) - 16(x-1) \\&= (x^2 - 16)(x-1) = 0 \\x &= 1, \pm 4 \\(1, 0) &(\pm 4, 0)\end{aligned}$$

3b. Find the extreme points of $f(x) = x^3 - x^2 - 16x + 16$. Show the derivative before using your calculator.

$$\begin{aligned}f' &= 3x^2 - 2x - 16 \\&= (3x-8)(x+2)\end{aligned}$$

i) $f' = 0 \Rightarrow x = -2, \frac{8}{3}$

ii) f' DNE \Rightarrow no soln

iii) no endpoints

$$\begin{cases} (-2, 36) \\ (\frac{8}{3}, -14.815) \end{cases}$$

4. The sign pattern for the derivative of $H(x)$ is given. (a) Is $x = -4$ at a maximum, a minimum, or neither? Why? (b) Is $x = -1$ at a maximum, a minimum, or neither? Why?

$$\begin{array}{c} dH/dx \leftarrow + 0 - 0 - 0 + \\ x \quad -4 \quad -1 \quad 2 \end{array}$$

a) $H(b)$ HAS A RELATIVE MAXIMUM AT $x = -4$ BECAUSE

$\frac{dH}{dx}$ SWITCHES FROM POSITIVE TO NEGATIVE ABOUT
 $x = -4$

b) $H(x)$ HAS NEITHER A MAX NOR A MIN AT $x = -1$ BECAUSE

$\frac{dH}{dx}$ DOES NOT CHANGE SIGNS ABOUT $x = -1$

5. Find the traits and sketch $f(x) = -x^4 + 6x^2 - 8$ on $x \in [-1.5, 3]$.

Domain: $x \in [-1.5, 3]$

Range: $y \in [-\frac{35}{8}, 1]$

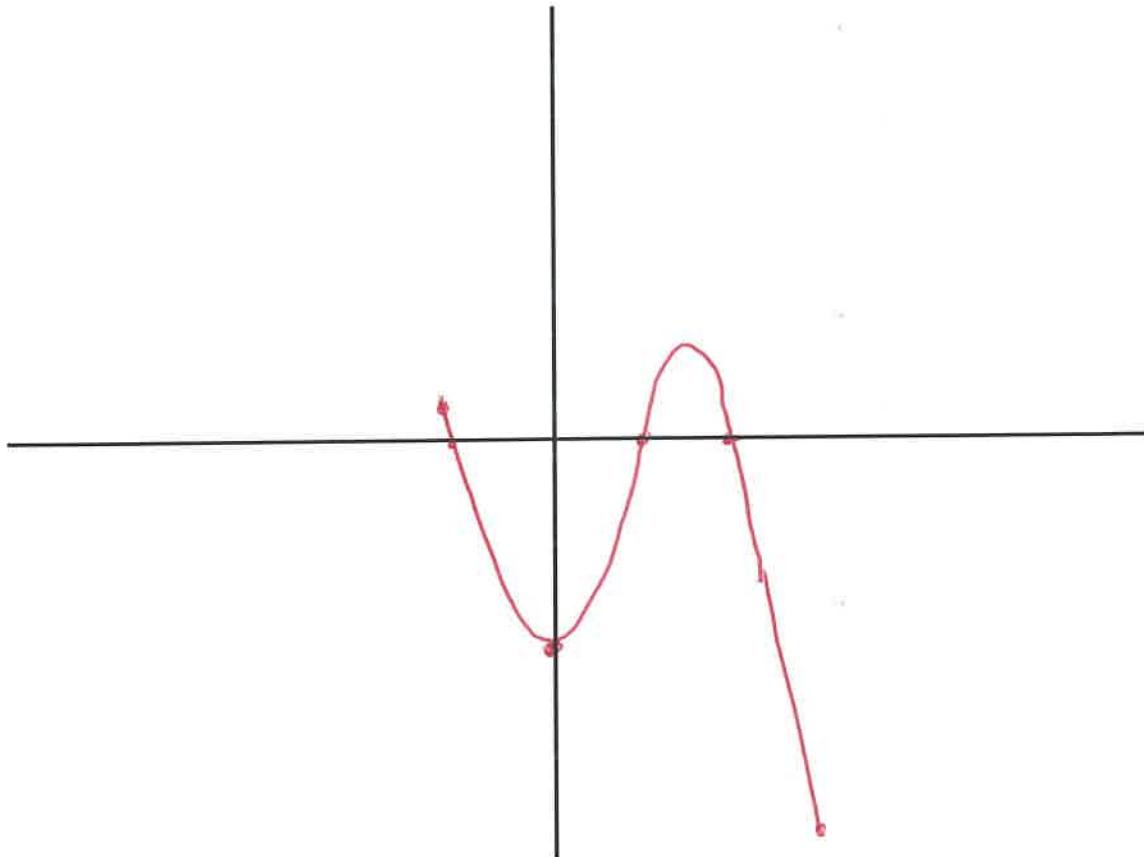
Zeros: $(\pm\sqrt{2}, 0), (2, 0)$

Y-Int: $(0, -8)$

End Behavior (left): none

Extreme Points: see # 2

End Behavior (right): none



6. Find the traits and sketch of $f(x) = x^3 - x^2 - 16x + 16$.

Domain: All Reals

Range: All Reals

Zeros: $(-4, 0)$ $(1, 0)$

Y-Int: $(0, 16)$

End Behavior (left): Down

Extreme Points: $(-2, 36)$

End Behavior (right): Up

$(8/3, -14.815)$

