

PreCalculus Honors '21-22

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Dr. Quattrin

Radical Functions Test

CALCULATOR ALLOWED

Score _____

Round to 3 decimal places. Show all work.

1. If $y = \frac{1}{\sqrt{x^2+6}}$, then $\frac{dy}{dx} =$

$$y = (x^2+6)^{-1/2} \quad y' = \frac{1}{2}(x^2+6)^{-3/2}(2x)$$

a) $\frac{-x}{(2x)^{1/2}(x^2+6)}$

(b)

$$\frac{-x}{(x^2+6)^{3/2}}$$

c) $\frac{-1}{2(x^2+6)^{3/2}}$

$$= \frac{-x}{\sqrt{x^2+6}}^{3/2}$$

d) $\frac{x}{(x^2+6)^{3/2}}$

e) $\frac{-x^2-6x}{(x^2+6)}$

2. If $y = \sqrt{\frac{3x}{x^2-9}}$, then $\frac{dy}{dx} = \frac{1}{2} \left(\frac{3x}{x^2-9} \right)^{-1/2} \left[\frac{(x^2-9)(3) - (3x)(2x)}{(x^2-9)^2} \right]$

~~(a)~~ $\sqrt{\frac{3}{2x}}$

~~(b)~~ $\frac{1}{2} \left(\frac{3x}{x^2-9} \right)^{-1/2}$

~~(c)~~ $\frac{-3x^2-27}{(3x)^{1/2}(x^2-9)^{3/2}}$

~~(d)~~ $\frac{(3x)^{1/2}(-3x^2-27)}{(x^2-9)^{5/2}}$

~~(e)~~ $\frac{-3x^2-27}{x^2-9}$



Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that have values given on the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	8	8	2	3
8	2	-12	4	4

Given that $h(x) = g(g(x))$, $h'(8) = g'(4) \cdot g'(8) = 3(4) = 12$

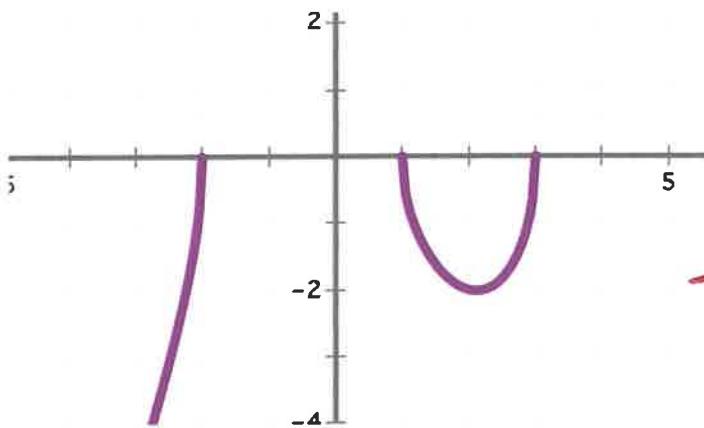
- a) 16 b) 32 c) -8 d) 4 e) 1

$$3x^2 - 2y \frac{dy}{dx} + 6 \frac{dy}{dx} - 8 = 0$$

4. If $x^2 - y^2 + 6y - 5 = 0$, then $\frac{dy}{dx} = \cancel{2x}$ $(6-2y) y' = \cancel{8-2x}$

- a) $3x^2 - 2y$ b) $\frac{3x^2 + 6}{-2y}$ c) $\frac{-2y + 6}{3x^2}$
- d) $\frac{3x^2}{2y - 6}$ e) $3x^2 - 2y + 6$
- $y' = \frac{-3x^2}{6-2y}$

5. Which of the following equations matches the graph below?



$$\begin{aligned} & -(x+2)(x-1)(x-3) \\ & (x+2)(x^2-4x+3) \\ & -(x^3-4x^2+3x+2x^2) \end{aligned}$$

a) $y = -\sqrt{x^3 - 2x^2 - 5x + 6}$

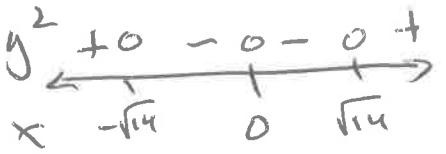
b) $\cancel{y = \sqrt{-x^3 + 2x^2 + 5x - 6}}$

c) $\cancel{y = \sqrt{x^3 - 2x^2 - 5x + 6}}$

d) $y = -\sqrt{-x^3 + 2x^2 + 5x - 6}$

6. What is the end behavior of $y = \sqrt{3x^2(x^2 - 14)}$?

- a) Up on the left and down on the right
 - b) None on the left and none on the right
 - c) Up on the left and none on the right
 - d) Up on both ends
 - e) Down on the left and none on the right
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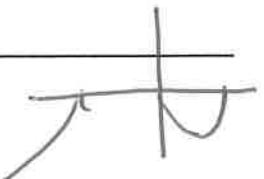


7. The x -value(s) of the relative maximum(s) of $y = -\sqrt{27x - x^3}$ is/are

- a) 3
- b) $3\sqrt{6}$
- c) -3
- d) $0, \pm 3\sqrt{3}$
- e) 0

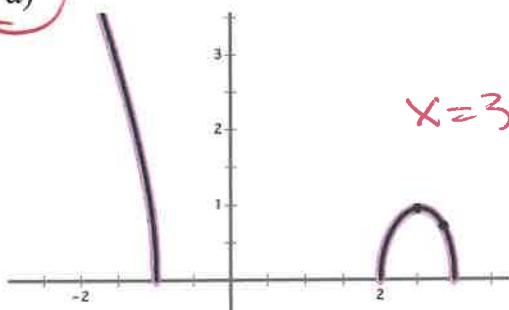
$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{2}(27x - x^3)^{-1/2} (27 - 3x^2) \\ &= \frac{-3(9 - x^2)}{2(27x - x^3)^{1/2}} \end{aligned}$$

i) $x = \pm 3$
ii) $x = 0, \pm \sqrt{27}$

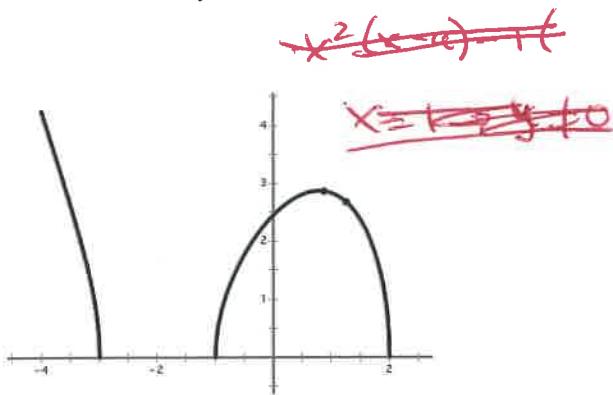


8. Which of the following graphs match the equation $y = \sqrt{-x^3 + 4x^2 - x - 6}$?

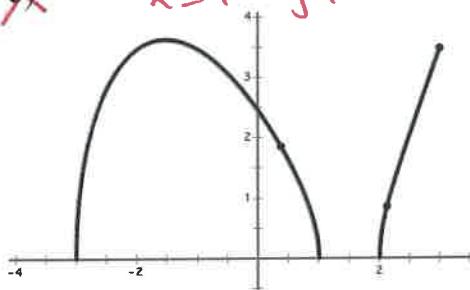
a)



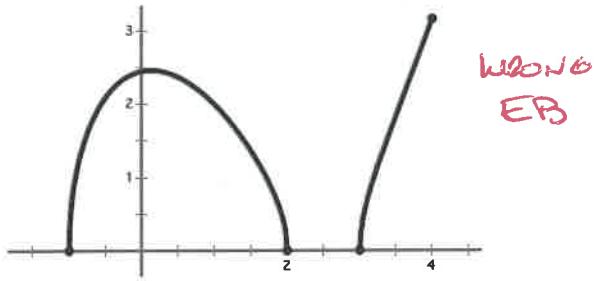
b)



c)



d)



9. A Golden Rectangle is one where the ratio (called ϕ) of the length to the short side w to the long side l is equal to the ratio of the long side to the sum of the two sides. In other words, $l = 1.236w$. If a Golden Rectangle changes such that w is growing at 2 in/min, how fast is the area changing when w is 5 inches?

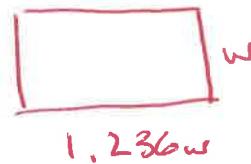
a) $1.236 \text{ in}^2/\text{min}$

b) $12.36 \text{ in}^2/\text{min}$

c) $2.472 \text{ in}^2/\text{min}$

d) $24.72 \text{ in}^2/\text{min}$

e) $30.9 \text{ in}^2/\text{min}$



$$\frac{d}{dt} [A = 1.236w^2]$$

$$\frac{dA}{dt} = 2.472w \frac{dw}{dt}$$

$$= 2.472(5)(2)$$

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Name: Solution Key

Dr. Quattrin

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Show all work.

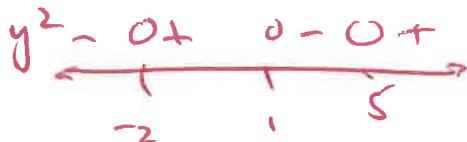
1. Find the zeros and Domain of $y = \sqrt{x^3 - 4x^2 - 7x + 10}$ on $x \in [-1, 6]$. Show the algebraic work to support the zeros.

$$(x+2)(x-1)(x-5)$$

Zeros: $(\cancel{-2,0}), (1,0), (5,0)$

Domain:

$$x \in [-2, 1] \cup [5, 6]$$



2. Find the Extreme Points of $y = \sqrt{x^3 - 4x^2 - 7x + 10}$ on $x \in [-1, 6]$. Show the algebraic work to support the critical values.

Extreme Points:

$$(-1, 3.424)$$

$$(6, 6.325)$$

$$\cancel{(2, 1, 0)}$$

$$(5, 0)$$

$$\text{i) } \frac{dy}{dx} = \frac{3x^2 - 8x - 7}{2(x^3 - 4x^2 - 7x + 10)^{1/2}} = 0 \rightarrow x = \frac{8 \pm \sqrt{8^2 - 4(3)(-7)}}{2(3)} = \begin{cases} 3.424 \\ -6.695 \end{cases} \quad (-6.695, 3.549)$$

$$\text{ii) } \frac{dy}{dx} \text{ DNE} \rightarrow x = -2, 1, 5$$

$$\text{iii) } x = -1, 6$$

3. Find the zeros, VAs, and domain of $y = -\sqrt{\frac{3x^2 + 11x - 4}{3x^2 + 5x - 2}}$. Show the Algebra that supports your answer.

Zeros: $x \neq 4, 0$

VAs: $x = -2$

Domain: $x \in (-\infty, -4] \cup (-2, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ POE $(\frac{1}{3} - \sqrt{3}/7)$

$$y = -\left(\frac{(3x-1)(x+4)}{(3x-1)(x+2)}\right)^{1/2}$$

$$\begin{array}{c} y^2 + 0 \\ \hline x \end{array} \quad \begin{array}{c} -\sqrt{A} + \text{POE} \\ -2 \end{array} \quad \begin{array}{c} + \\ \hline \frac{1}{3} \end{array}$$

4. Find the Extreme Points of $y = -\sqrt{\frac{3x^2 + 11x - 4}{3x^2 + 5x - 2}}$. Show the Algebra and derivative that supports your answer.

Extreme Points:

$$(-4, 0)$$

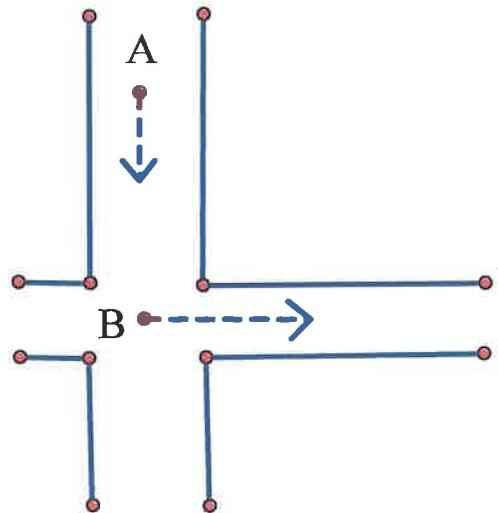
$$y \approx -\left(\frac{x+4}{x+2}\right)^{1/2}$$

$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{2} \left(\frac{x+4}{x+2}\right)^{-1/2} \left[\frac{(x+2)(1) - (x+4)(1)}{(x+2)^2} \right] \\ &= \frac{1}{(x+4)^{1/2} (x+2)^{3/2}} \end{aligned}$$

i) NONE

ii) $x = -2, -4$

iii) NONE



5. Person A is 220 feet north of an intersection and walking toward it at $10 \frac{ft}{sec}$. Person B starts at the intersection and walks east at $5 \frac{ft}{sec}$.

- a) At $t = 10$ seconds, how far is each person from the intersection?

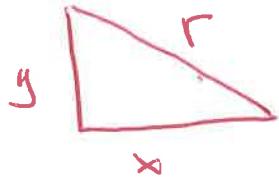
$$A = 220 - 10(10) = 120 \text{ ft}$$

$$B = 0 + 5(10) = 50 \text{ ft}$$

- b) At $t = 10$ seconds, how far apart are the two people?

$$50^2 + 120^2 = r^2 \quad r = 130$$

c) How fast is the distance between the two people changing at $t=10$ seconds?



$$x^2 + y^2 = r^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2r \frac{dr}{dt}$$

$$2(120)(-10) + 2(80)(5) = 2(130) \frac{dr}{dt}$$

$$\frac{dr}{dt} = -7.308 \frac{\text{ft}}{\text{sec}}$$

d) If person A looks at person B when $t=10$ seconds. How fast is the angle changing?

$$\frac{d}{dt} \left\{ \tan \theta = \frac{x}{r} \right\}$$

$$\left(\frac{r}{x} \right)^2 \frac{d\theta}{dt} \sec^2 \theta \frac{d\theta}{dt} = \frac{r \frac{dx}{dt} - x \frac{dr}{dt}}{r^2}$$

$$\frac{(130)^2}{120^2} \frac{d\theta}{dt} = \frac{120(5) - 50(-10)}{(120)^2}$$

$$\frac{d\theta}{dt} = 0.065 \frac{\text{rad}}{\text{sec}}$$

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Radical Functions Test – NO CALCULATOR ALLOWED

Show all work.

6. Find the traits and sketch $y = \sqrt{x^3 - 4x^2 - 7x + 10}$ on $x \in [-1, 6]$.

Domain: $x \in [-1, 1] \cup [5, 6]$

Range: $y \in [0, 6.325]$

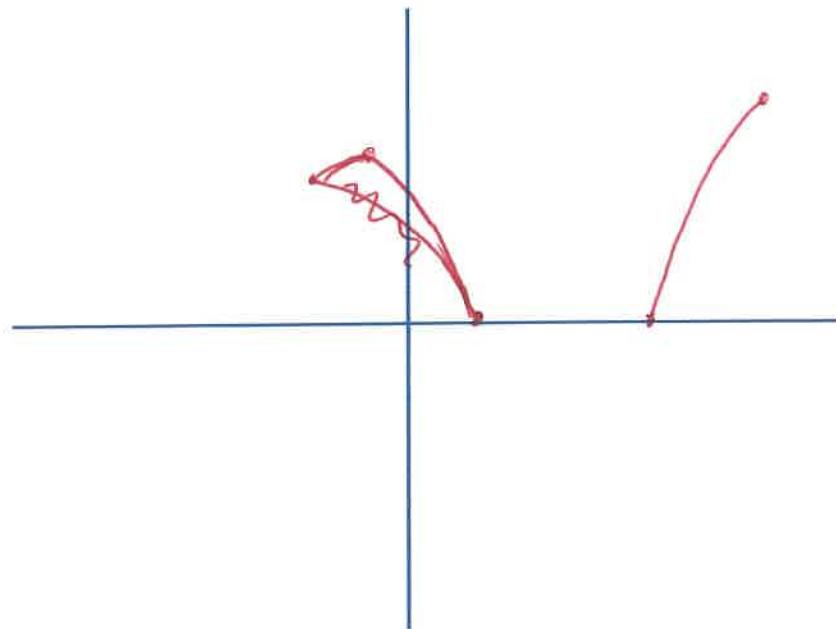
Y-Int: $(0, \sqrt{10})$

Zeros: $(+1, 0)$ $(5, 0)$

Extreme Points: See # 2

End Behavior (Left): None

End Behavior (Right): None



7. List the traits and sketch of $y = -\sqrt{\frac{3x^2 + 11x - 4}{3x^2 + 5x - 2}}$.

Domain: $\leq 2 \epsilon \#3$

Y-Int: $(0, \sqrt{2})$

Zeros: $(-4, 0)$

End Behavior (Left): $y = -1$

End Behavior (Right): $y = -1$

Range: $y \in (-\infty, -\sqrt{\frac{13}{7}}) \cup (-\sqrt{\frac{13}{7}}, -1) \cup (-1, 0]$

VAs: $x = -2$

Extreme Points: $(-4, 0)$

