Honors PreCalculus '21-22 Name: Solution Key

Dr. Quattrin

Rational Functions Test

CALCULATOR ALLOWED Score _____

Round to 3 decimal places. Show all work.

1. The equation of the line tangent to the graph of $y=\frac{2x+3}{2-3x}$ at the point (1, -5) is

a) $y=-13x+8$ b) $y=13x-18$ c) $y=-3x$

$\frac{dy}{dx}=\frac{(2-3x)(2)-(2x+3)(-3)}{(2-3x)^2}$ $m=\frac{13}{1}=13$

d) $y=-\frac{1}{13}x-\frac{64}{13}$ e) $y=\frac{1}{13}x-\frac{66}{13}$

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2. The end behavior of $y=\frac{3-x-x^2}{4x^2+x-1}$ is $\approx\frac{-x^2}{4x^2}=-\frac{1}{4}$

a) $y=0$ b) $y=\frac{3}{4}$ c) $y=\frac{1}{4}$

d) $y=-\frac{1}{4}$ e) DNE

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3. Let $f(x)$ and $g(x)$ be differentiable functions. The table below gives the values of $f(x)$ and $g(x)$, and their derivatives, at several values of x.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	-6
2	1	8	-5	7
3	7	-2	7	9

If $h(x)=\dfrac{g(x)}{f(x)}$, what is the value of $h'(2)$? $= \dfrac{f(2)g'(2)-g(2)f'(2)}{[f(2)]^2} = \dfrac{1(7)-8(-5)}{1}$

$= 47$

a) -1 b) -77 c) 77 d) $-\dfrac{77}{4}$ e) $\dfrac{11}{7}$

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4. Which of the following would be a rational function that has x–intercepts at (-5, 0), VA at $x = 6$, a POE at $x = 2$, and a HA at $y=\dfrac{3}{7}$?

a) $y=\dfrac{(x+5)(x-2)}{(x-2)(x-6)}$

b) $y=\dfrac{3(x-5)(x+2)}{7(x+2)(x+6)}$

c) $y=\dfrac{3(x-6)(x-2)}{7(x-2)(x+5)}$

d) $y=\dfrac{3(x-2)^2}{7(x+5)(x-6)}$

e) $y=\dfrac{3(x+5)(x-2)}{7(x-2)(x-6)}$

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f' $-$ 0 $+$ 0 $+$

x -1 2

5. Suppose $f'(x)=\dfrac{(x+1)^3(x-2)^2}{(x^4+1)}$. Which of the following statements must be **true**?

~~I.~~ The slope of the line tangent to $y=f(x)$ at $x=0$ is -4. $m=f'(0)=4$

II. $f(x)$ is increasing on $x\in(-1,\,2)$

~~III.~~ $f(x)$ has a POE at $x=-1$

a) I only b) II only c) III only

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6. If $y=\dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx}=$ $= 3(4+x^2)^{-1}$ $-3(4+x^2)^{-2}(2x)=\dfrac{-6x}{(4+x^2)^2}$

a) $y=\dfrac{-6x}{(4+x^2)^2}$ b) $y=\dfrac{3x}{(4+x^2)^2}$ c) $y=\dfrac{6x}{(4+x^2)^2}$

d) $y=\dfrac{-3x}{(4+x^2)^2}$ e) $\dfrac{3}{2x}$

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7. Which of the following functions has the horizontal asymptote $y=-1$?

Powers =

~~a)~~ $y=\dfrac{x^3-5x^2-2x+24}{2x^2+7x-15}$ ~~b)~~ $y=\dfrac{2x^3-9x^2+7x+6}{6-x-x^2}$

~~c)~~ $y=\dfrac{x^3-8}{x^4-13x^2+36}$ d) $y=\dfrac{x^2-1}{x^2+2x-3}$

e) $y=\dfrac{1-x^2}{x^2+2x-3}$

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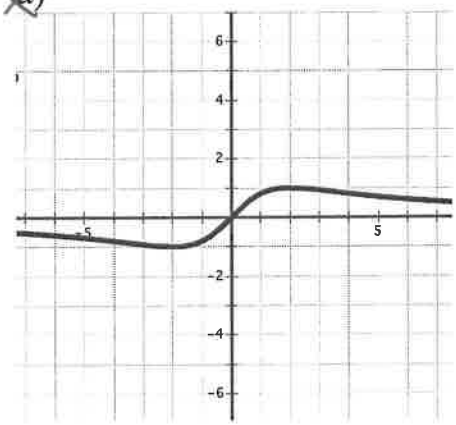
8. Which of the following graphs match the equation $y = \dfrac{4x^2 - 4}{x^2 + 4}$?

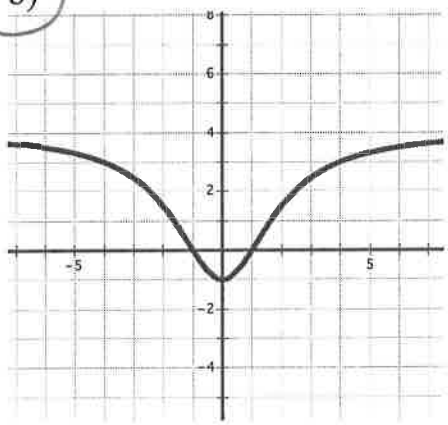
ZEROS @ $x = \pm 1$

HA @ $y = 4$

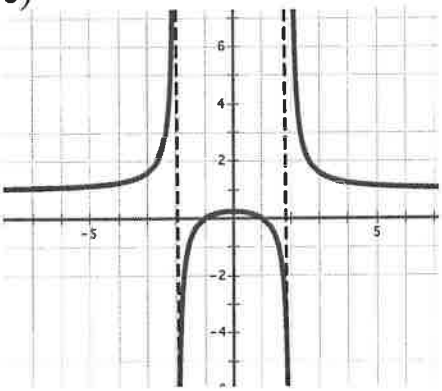
a)



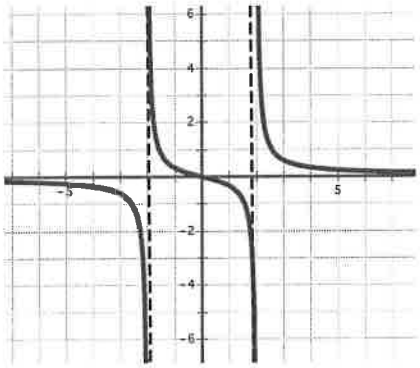
b)



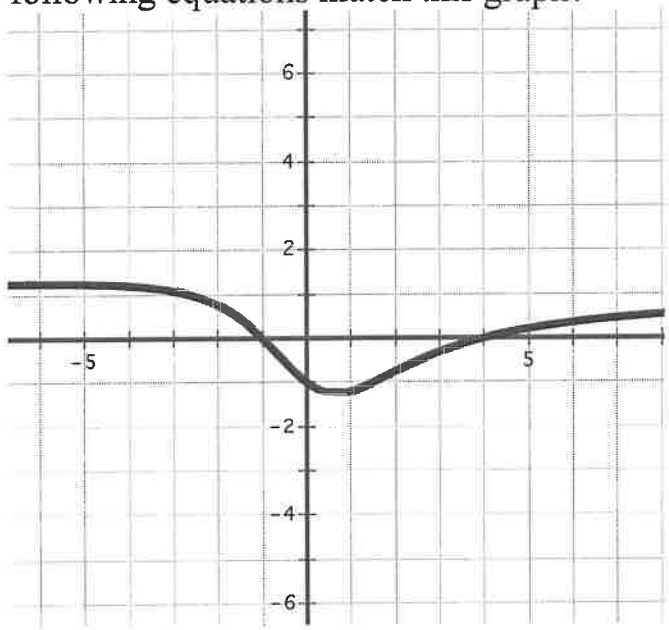
c)



d)



9. Which of the following equations match this graph?



$y = (x+1)(x-4)$

$= x^2 - 3x - 4$

a) $y = \dfrac{4x}{x^2+4}$

b) $y = \dfrac{x^2-3x-4}{x^2+4}$

c) $y = \dfrac{4x^2-4}{x^2+4}$

d) $y = \dfrac{4-4x^2}{x^2-4}$

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1. Find the zeros, VAs, and POEs of $y=\frac{2x^2-5x+2}{4-x^2}$ on $x\in(-\infty,5]$. Show the supporting algebraic work.

Zeros: $(-1/2, 0)$

$y=\frac{(2x+1)(x-2)}{(2-x)(2+x)} \approx -\frac{2x+1}{2+x}$

VAs: $x=-2$

POE: $(2, -\frac{5}{4})$

2. Find the extreme points of $y=\frac{2x^2-5x+2}{4-x^2}$ on $x\in(-\infty,5]$. Show the derivative and algebra to support the critical values.

$y \approx \frac{-2x-1}{x+2}$ $\quad \frac{dy}{dx} = \frac{(x+2)(-2)-(-2x-1)(1)}{(x+2)^2} = \frac{-2x-4+2x+1}{(x+2)^2}$

$= \frac{-3}{(x+2)^2}$

i) $\frac{dy}{dx}=0 \Rightarrow$ NO SOLUTION

ii) $\frac{dy}{dx}=$ DNE $\Rightarrow x=-2$ BUT NOT IN DOMAIN

iii) ENDPOINTS: $x=5$

$(5, \frac{-11}{7})$

3. Find the equations of the lines tangent to and normal to $y=\dfrac{x^2-13x+12}{-x^2+12x}$ at $x=-1$?

$y(-1)=-2$

$= \dfrac{\cancel{(x-12)}(x-1)}{-x\cancel{(x-12)}}$

Tangent: $y+2=-1(x+1)$

$\dfrac{dy}{dx} = \dfrac{-x(1)-(x-1)(-1)}{(-x)^2}$

Normal: $y+2=1(x+1)$

$= -\dfrac{1}{x^2}$

$m=-1$

4. Find the zeros, VAs, POEs and EB of $y=\dfrac{x^3-4x^2-x+4}{4x^2-16x}$. Show the supporting algebraic work.

$= \dfrac{x^2(x-4)-1(x-4)}{4x(x-4)} = \dfrac{(x^2-1)(x-4)}{4x(x-4)}$

Zeros: $(\pm 1, 0)$

VAs: $x=0$

POE: $\left(4, \frac{15}{16}\right)$

$$\begin{array}{r} \frac{1}{4}x \\ 4x\overline{)\,x^2-1} \\ -x^2 \\ \hline 0x \end{array}$$

End Behavior (left): $y=\frac{1}{4}x$

End Behavior (right): $y=\frac{1}{4}x$

5. Find the extreme points of $y=\dfrac{x^3-4x^2-x+4}{4x^2-16x}$. Show the derivative and algebra to support the critical values.

$$y \approx \frac{x^2-1}{4x} = \frac{1}{4}x - \frac{1}{4}x^{-1}$$

$$\frac{dy}{dx} = \frac{1}{4} + \frac{1}{4}x^{-2} = \frac{x^2+1}{4x^2}$$

i) $\frac{dy}{dx} = 0 \rightarrow$ NO SOLUTIONS

ii) $\frac{dy}{dx}$ DNE $\rightarrow$ $x=0$ BUT NOT IN DOMAIN

iii) ENDPOINTS : NONE GIVEN

THERE ARE NO EXTREMES

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NO CALCULATOR ALLOWED Score _____

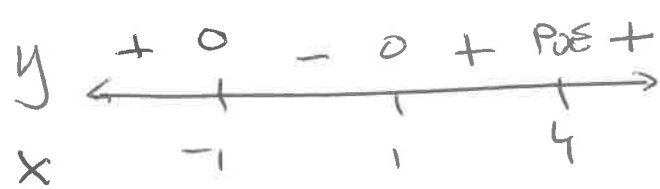
Show all work.

6. Write an equation of a rational function that has x–intercepts at (-5, 0) and (-6, 0), VA at $x = 9$ and $x = -1$, a POE at $x = 6$, and a HA at $y = \frac{5}{3}$.

$$y = \frac{5(x+5)(x+6)(x-6)}{3(x+9)(x+1)(x-6)}$$

7. Show the sign pattern and solve $\frac{4x^2 - 16x}{x^3 - 4x^2 - x + 4} > 0$.

$$\frac{4x(x-4)}{(x^2-1)(x-4)} \geq 0$$



$$x \in (-\infty, -1) \cup (1, 4) \cup (4, \infty)$$

8. Find the traits and **sketch** $y = \dfrac{x^3 - 4x^2 - x + 4}{4x^2 - 16x}$.

Domain: $x \neq 0, 4$

Y – Intercept: NONE

Zeros: $(\pm 1, 0)$

Range: $y \in (-\infty, \infty)$

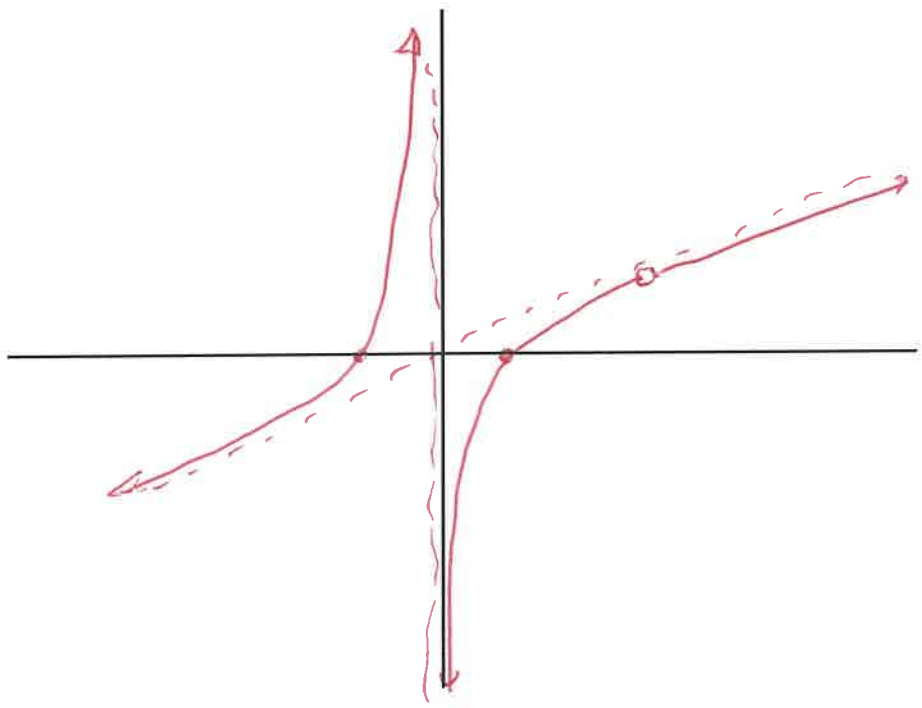
VAs: $x = 0$

POEs: $\left(4, \frac{15}{16}\right)$

End Behavior (left): $y = \frac{1}{4}x$

End Behavior (right): $y = \frac{1}{4}x$

Extreme Points: NONE



9. Find the traits and **sketch** of $y=\dfrac{2x^2-5x+2}{4-x^2}$ on $x\in\left(-\infty,5\right]$.

Domain: $x \neq \pm 2$

Y – Intercept: $(0, -1/2)$

Zeros: $(-1/2, 0)$

Range: $y \in (-\infty, -2) \cup \left(-\frac{11}{7}, \frac{-5}{4}\right) \cup \left(\frac{-5}{4}, \infty\right)$

VAs: $x = -2$

POEs: $2, -5/4$

End Behavior (left): $y = -2$

End Behavior (right): NONE

Extreme Points: $5, \frac{-11}{7}$