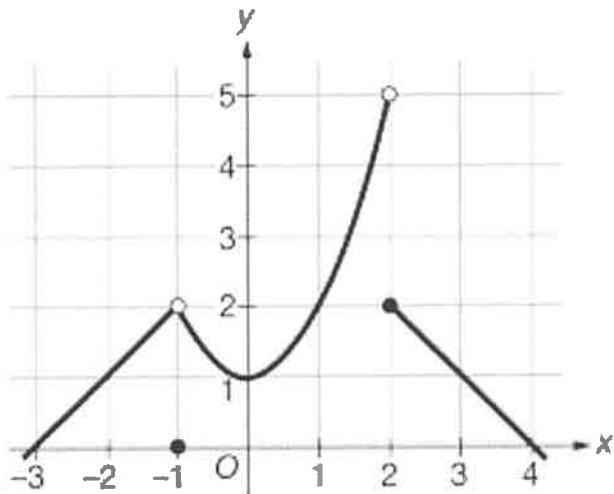


1. The graph of  $f$  is shown below. Which of the statements is/are true about  $f$ ?



Graph of  $f$

- I.  $\lim_{x \rightarrow 2^-} f(x) = 5$       II.  $\lim_{x \rightarrow 2} f(x) = 2$       III.  $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow 2^+} f(x) = 2$
- T                          F                          T

- (A) I only      (B) I and II only      (C) III only  
(D) II and III only      (E) I and III only

---

$$2. \quad \frac{d}{dx} \left[ \ln \sqrt{4x+1} \right] = \frac{d}{dx} \frac{1}{2} \ln(4x+1) = \frac{1}{2} \cdot \frac{1}{4x+1} \quad (4)$$

- (A)  $\frac{1}{\sqrt{4x+1}}$       (B)  $\frac{1}{2} \ln(4x+1)$       (C)  $\frac{4}{\sqrt{4x+1}}$   
 (D)  $4 \ln \sqrt{4x+1}$       (E)  $\frac{2}{4x+1}$
- 

3. The functions  $f(x)$  and  $g(x)$  are continuous and differentiable, and have values given in the table

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	4	-2	8	1
4	10	8	4	3
8	6	-12	2	4

below.

$$\text{Given that } k(x) = \frac{f(x)}{g(x)}, \text{ find } k'(4). = \frac{g(4) f'(4) - f(4) g'(4)}{g(4)^2} = \frac{4(8) - 10(3)}{4^2}$$

- (A)  $\frac{5}{2}$       (B)  $\frac{8}{3}$       (C)  $\frac{1}{2}$       (D)  $\frac{1}{8}$       (E)  $\frac{3}{5}$
-

4. The volume of a cube is given by  $V = s^3$ , where  $s$  is the length of one side of the cube in *inches* and  $V$  is the volume of the cube in  $\text{in}^3$ . An ice cube melts, maintaining its cube shape, in such a way that its volume decreases at a rate of  $0.04 \text{ in}^3 / \text{min}$ . Which of these describes the change of the side length when the side is 0.5 inches?

(A) the side length decreases at  $0.003 \text{ in/sec}$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

(B) the side length increases at  $0.320 \text{ in}^3/\text{sec}$

$$-0.04 = 3(0.5)^2 \frac{ds}{dt}$$

(C) the side length decreases at  $0.053 \text{ in/sec}$

$$\frac{ds}{dt} = -0.053$$

(D) the side length decreases at  $0.04 \text{ in}^3/\text{sec}$

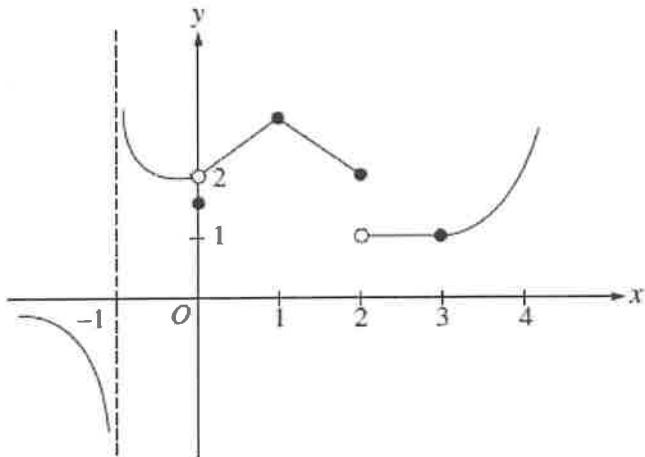
5. Which of the following statements must be true?

I.  $\frac{d}{dx} \sqrt{e^x + 3} = \frac{1}{2\sqrt{e^x + 3}}$

II.  $\frac{d}{dx} (\ln \sin x) = \cot x$

III.  $\frac{d}{dx} \left( 4x^4 - e + \sqrt[7]{x^2} - \frac{2}{x^3} \right) = 16x^3 + \frac{2}{7\sqrt[7]{x^5}} + \frac{6}{x^4}$

- a) I only
- b) II only
- c) III only
- d) II and III only
- e) I, II, and III



6. The graph of a function  $f$  is shown above. If  $\lim_{x \rightarrow b} f(x)$  exists and  $f(x)$  is not continuous at  $b$ , then  $b =$

- a) -1
- b) 0
- c) 1
- d) 2
- e) 3

Not cont @ -1, 0, 2

$\lim$  exists @  $\infty, 0$

7. Consider the closed curve in the  $x$ - $y$  plane given by  $4x^3 + 7x + y^3 + 4y = 19$ . Which of the following is correct?

a)  $\frac{dy}{dx} = -\frac{12x^2 + 7}{3y^2 + 4}$       b)  $\frac{dy}{dx} = \frac{12x^2 + 7}{3y^2 + 4}$       c)  $\frac{dy}{dx} = -\frac{12x^2 + 7}{19x + 3y^2 + 4}$

d)  $\frac{dy}{dx} = -\frac{12x^2 + 7}{3y^2 + 19x}$       e)  $\frac{dy}{dx} = \frac{12x^2 + 7}{3y^2 + 19x}$

$$12y^3 + 7 + 3y^2 \frac{dy}{dx} + 4y \frac{dy}{dx} = 0$$

$$(3y^2 + 4) \frac{dy}{dx} = -12y^3 - 7$$

8. Find the equation of the line tangent to the curve  $f(x) = 4x^4 - 5x^2 + x$  at the point where  $x = -1$ .

$$y_1 = f(-1) = 4 - 5 - 1 = -2$$

(A)  $y + 2 = -5(x + 1)$

$$m = f'(-1) = 16(-1)^3 - 10(-1) + (-1) = -5$$

(B)  $y + 2 = -6(x + 1)$

(C)  $y - 5 = -2(x + 1)$

(D)  $y - 5 = -6(x + 1)$

---

9.  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\sin(x)} = \lim_{x \rightarrow 0} \frac{e^{2x}(2)}{\cos x} = \frac{2}{1}$

- (A) 0      (B) 1      (C) 2      (D) 3      (E) DNE
- 

10.  $\lim_{x \rightarrow \infty} \frac{1 - 5x^3}{2x + 15x^3} = -\frac{1}{3}$

- (A) 0      (B)  $\frac{1}{2}$       (C)  $-\frac{1}{3}$       (D)  $-\frac{4}{17}$       (E)  $\infty$
-

11. Which of these functions has a point of exclusion at  $(1, 3)$  and a vertical asymptote at  $x = \frac{1}{2}$ ?

~~(A)~~  $g(x) = \frac{x-1}{4x^2-1}$  No POE

~~(B)~~  $h(x) = \frac{x-1}{2x^2-3x+1} = \frac{x-1}{(2x-1)(x-1)}$  POE @  $(1, \neq 3)$

~~(C)~~  $f(x) = \frac{2x+1}{4x^2-1} = \frac{2x+1}{(2x+1)(2x-1)}$  VA @  $x = -\frac{1}{2}$  BUT POE @  $x = \frac{1}{2}$

~~(D)~~  $k(x) = \frac{3x-3}{2x^2-3x+1}$

---

12. Let a function be defined as  $f(x) = \begin{cases} cx^2 - 16c, & \text{if } x > 4 \\ \frac{x-4}{4c^2}, & \text{if } x \leq 4 \end{cases}$ , where  $c$  is a constant. For what value(s) of  $c$  will  $f(x)$  be continuous?

(A) 0 only

$$\frac{c(x^2-16)}{x-4} \approx 8c$$

(B) 2 only

(C) 4 only

~~(D)~~ 0 and 2 only

(E) No such value exists

$$\begin{aligned} 8c &= 4c^2 \\ 0 &= 4c^2 - 8c \\ &= 4c(c-2) \end{aligned}$$


---

13. Let  $f(x)$  be the function given by  $f(x) = \sqrt[3]{x+3}$ . What is the slope of the line tangent to  $f(x)$  at  $(5, 2)$ ?

$$f' = \frac{1}{3}(x+3)^{-\frac{2}{3}} \rightarrow m = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{8^2}} =$$

- a)  $\frac{1}{4}$     b)  $\frac{1}{2}$     c)  $\frac{1}{12}$     d)  $\frac{1}{4\sqrt{2}}$     e)  $\frac{7}{4\sqrt{2}}$
- 

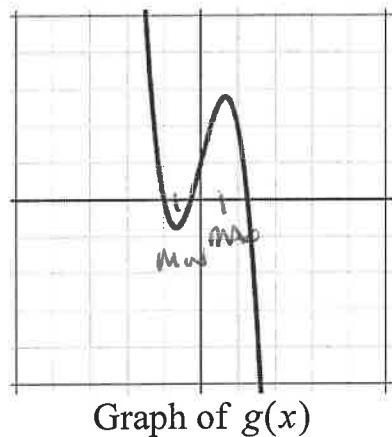
14. A particle's velocity is given by  $v(t) = \cos^2\left(\frac{\pi}{3}t\right)$ . The particle's acceleration at  $t = 1$  is:

- (A)  $\frac{-\pi\sqrt{3}}{6}$     (B)  $\frac{-\sqrt{3}}{4}$     (C)  $\frac{3}{4}$     (D)  $\frac{\pi}{4}$     (E) DNE
- 

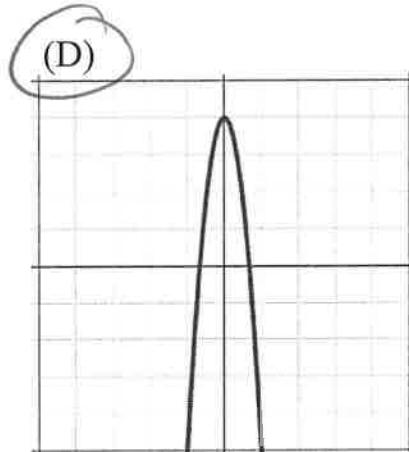
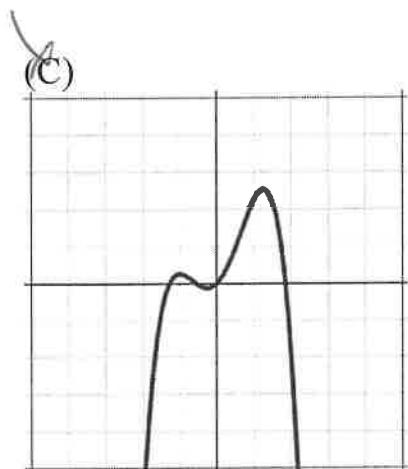
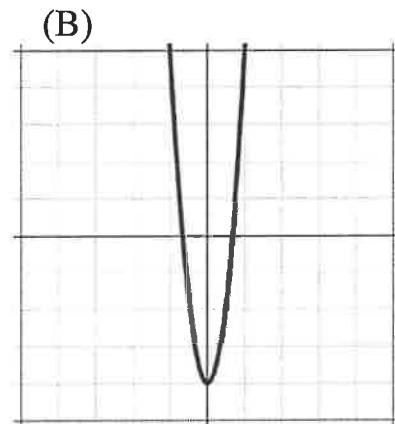
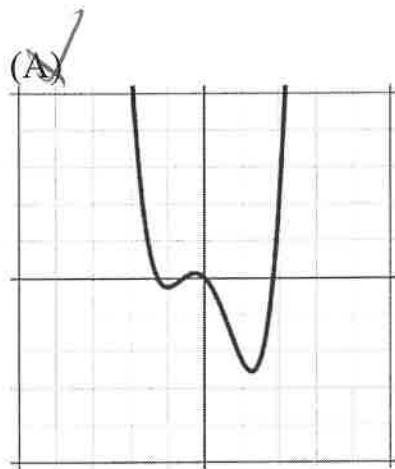
$$a = 2\cos\frac{\pi}{3}t \left(-\sin\frac{\pi}{3}t\right) \left(\frac{\pi}{3}\right)$$

$$\begin{aligned} a(1) &= 2\cos\frac{\pi}{3} \left(-\sin\frac{\pi}{3}\right) \frac{\pi}{3} \\ &= 2 \left(\frac{1}{2}\right) \left(-\frac{\sqrt{3}}{2}\right) \left(\frac{\pi}{3}\right) \end{aligned}$$

15. Part of the graph of  $g(x)$  is shown below. Which of these could be the graph of  $g'(x)$ , the derivative of  $g(x)$ ?



$g'$  is a PARABOLA  
which SWITCHES - TO +  
At  $x = -1/2$



Honors Precalc '22

Name: Solution Key

Spring Final – Part IIA; 40 Minutes

Dr. Quattrin

Calculator Allowed

score \_\_\_\_\_

1. Find the domain and extreme points of  $f(x) = -\sqrt{-x^2 - 10x - 16}$ . Show the supporting derivative work.

$$\frac{dy}{dx} = -\frac{1}{2}(-x^2 - 10x - 16)^{-\frac{1}{2}} (-2x - 10) = \frac{x+5}{(-x^2 - 10x - 16)^{\frac{1}{2}}}$$

Domain:  $x \in [-8, -2]$

Extreme Points:  $(-8, 0), (-2, 0), (-5, -3)$

$$\frac{dy}{dx} = -\frac{1}{2}(-x^2 - 10x - 16)^{-\frac{1}{2}} (-2x - 10) = \frac{x+5}{(-x^2 - 10x - 16)^{\frac{1}{2}}}$$

i)  $\frac{dy}{dx} = 0 \rightarrow x = -5 \quad (-5, -3)$

ii)  $\frac{dy}{dx} \text{ DNE} \rightarrow x = -8, -2 \quad (-8, 0), (-2, 0)$

iii) None New

2. Find the domain,  $y$ -intercept, and extreme points of  $g(x) = xe^{-x} + 2e^{-x}$ .  
Show the supporting Calculus work.

Domain: All Reals

$y$ -intercept: (0, 2)

Extreme Points: (-1, 2.718)

$$g(x) = e^{-x}(x+2)$$

$$g'(x) = e^{-x}(1) + e^{-x}(-1)(x+2) = e^{-x}(-x-1)$$

i)  $g' = 0 \Rightarrow x = -1$

$$(-1, e^1) = (-1, 2.718)$$

ii)  $g'$  DNE  $\Rightarrow$  none

iii) END POINTS  $\Rightarrow$  none

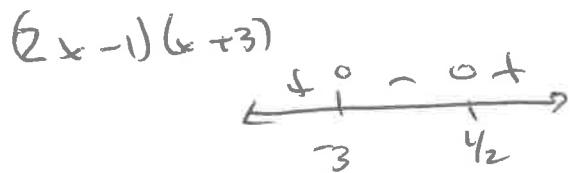
3. Find the domain, zeros, and extreme points of  $h(x) = \ln(2x^2 + 5x - 3)$ . Show the supporting work.

Domain:  $x \in (-\infty, -3) \cup (\frac{1}{2}, \infty)$

Zeros:  $(-3, 137, 0), (\frac{1}{2}, 37, 0)$

Extreme Points: No Extremes

$$\frac{dy}{dx} = \frac{4x+5}{2x^2+5x-3}$$



Zeros

$$2x^2 + 5x - 3 = 0$$

$$2x^2 + 5x - 4 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(-4)}}{4}$$

$\left. \begin{array}{l} x = -3.127 \\ x = 0.37 \end{array} \right\}$

i)  $\frac{dy}{dx} = 0 \rightarrow 4x+5=0 \rightarrow x = -\frac{5}{4}$

ii)  $\frac{dy}{dx} \text{ DNE} \rightarrow x = -3, \frac{1}{2}$

iii) NONE GIVEN

Honors Precalc '22

Spring Final – Part IIB; 20 Minutes

Dr. Quattrin

No Calculator Allowed

Name: Solution Key

score \_\_\_\_\_

4a.  $\frac{d}{dx}(\sin(x^2 + 4))$

$$= \cos(x^2 + 4) (2x)$$

$$= 2x \cos(x^2 + 4)$$

4b.  $\frac{d}{dx}(e^{2x} \sqrt{\tan 3x})$

$$= e^{2x} \sec^2 3x (3) + \tan 3x e^{2x} (2)$$

$$= e^{2x} (3 \sec^2 3x + 2 \tan 3x)$$

4c.  $\frac{d}{dx}(\sqrt{\csc(1-x^2)}) = \frac{1}{2} (\csc(1-x^2))^{-\frac{1}{2}} (-\csc(1-x^2) \cot(1-x^2)) (-2x)$

$$= -x \csc^{\frac{1}{2}}(1-x^2) \cot(1-x^2)$$

4d.  $\frac{d}{dx}(\cos^{-1}(e^{5x})) = \frac{-1}{\sqrt{1-(e^{5x})^2}} (e^{5x})(5)$

$$= \frac{-5e^{5x}}{\sqrt{1-e^{10x}}}$$

5. Using the functions from #1 and #2, find the traits and sketch

$$K(x) = \begin{cases} f(x) & \text{if } x \leq -2 \\ g(x) & \text{if } x > -2 \end{cases}$$

$$K'(x) = \begin{cases} \frac{x+5}{(x^2-12-16)^{1/2}} & \text{if } x \leq -2 \\ e^{-2x} - (8-x-1) & \text{for } x > -2 \end{cases}$$

Domain:  $x \in [-8, \infty)$

Zeros:  $(-2, 0), (-8, 0)$

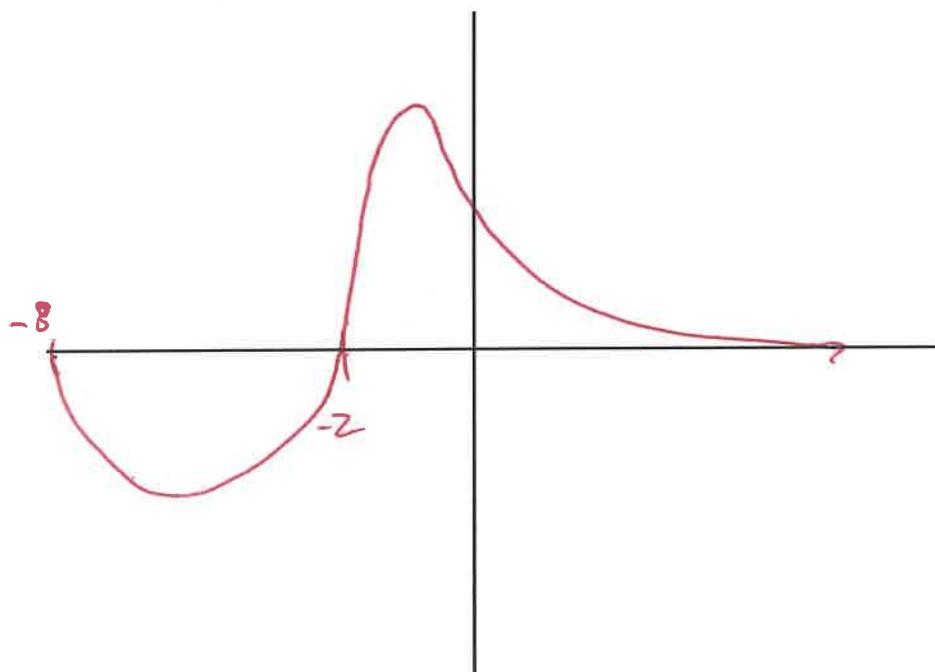
EB (Left): DNE

Range:  $y \in [-3, 2.718]$

Discontinuities: DNE

Points of non-differentiability:  $x = -2$

Extreme Points:  $(-8, -3), (-1, 2.718), (-2, 0)$



6. Find the traits and sketch  $h(x) = \ln(2x^2 + 5x - 3)$ .

Domain:  $x \in (-\infty, -3) \cup (1, \infty)$

Zeros:  $(-\frac{5}{2}, 0)$ ,  $(\frac{1}{2}, 0)$   $y$ -intercept:  $\text{NONE}$

VAs:  $x = -3, \frac{1}{2}$

EB (Left):  $\text{UP}$

EB (Left):  $\text{UP}$

Range:  $\text{All Reals}$

Discontinuities:  $\text{NONE}$

Points of non-differentiability:  $\text{NONE}$

Extreme Points:  $\text{NONE}$

