

Honors PreCalc 22 -23
Fall Final – Part I (30 minutes)
Dr. Quattrin
CALCULATOR ALLOWED

Name Solution Key

Score _____

1. A particle moves along the x -axis so that at time $t \geq 0$ its position is given by

$x(t) = 2t^3 - 15t^2 + 24t - 60$. At what time t does the particle have no acceleration?

$$v(t) = 6t^2 - 30t + 24$$

$$a(t) = 12t - 30 = 0 \Rightarrow t = \frac{5}{2}$$

- a) $t = 1$ only b) $t = 4$ only c) $t = \frac{5}{2}$ only

d) $t = 1$ and $t = \frac{7}{2}$

e) $t = 1$ and $t = 4$

2. Find an equation of the line tangent to $f(x) = x^3 - 3x^2 + 15$ at the point

(-2, -5).

$$f' = 3x^2 - 6 \rightarrow m = 24$$

- a) $24x - y = -43$ b) $x + 24y = -122$
c) $y - 24x = -53$ d) $24y + x = -122$

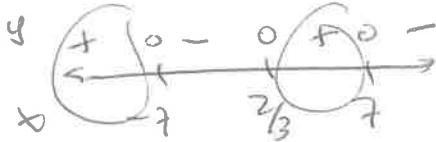
$$y + 5 = 24(x + 2)$$

3. Solve the inequality $-3x^3 + 2x^2 + 147x - 98 > 0$ +

a) $x \in (-\infty, -7)$

$$-x^2(3x+2) + 49(3x-2)$$

b) $x \in (-\infty, -7) \cup \left(\frac{2}{3}, 7\right)$



c) $x \in \left(-7, \frac{2}{3}\right) \cup (7, \infty)$

d) $x \in \left(-7, -\frac{2}{3}\right) \cup (7, \infty)$

e) $x \in (-\infty, -7) \cup \left(-\frac{2}{3}, 7\right)$

4. Given $g(x) = 3 - 2 \tan\left[\frac{\pi}{6}(x+1)\right]$, which of the following statements is not true?

a) The amplitude of $g(x)$ is 2. +

b) The period of $g(x)$ is 12. $P = \pi/B = 6$

c) The phase shift is -1.

d) The vertical shift is 3.

5. Which of the following is equivalent to $\sin(A + 30^\circ) + \cos(A + 60^\circ)$ for what values of A ?
- $\sin A \cos 30^\circ + \cos A \sin 30^\circ + \cos A \cos 60^\circ - \sin A \sin 60^\circ$
- a) $\sin A$ b) $\cos A$ c) $\frac{\sqrt{3} \sin A}{2} + \frac{1}{2} \cos A - \frac{\sqrt{3}}{2} \sin A$
 ~~$\frac{\sqrt{3} \sin A}{2}$~~ d) $\sqrt{3} \sin A$ e) $\sqrt{3} \cos A$
-

6. If $f(x)$ is a linear function $f(1) = 2$ and $f(-2) = 4$, then $f(x) =$

- a) $f(x) = 2x + 8$ b) $f(x) = \frac{2}{3}x + \frac{4}{3}$ c) $f(x) = -\frac{3}{2}x + \frac{7}{2}$
d) $f(x) = -\frac{2}{3}x + \frac{4}{3}$ e) $f(x) = -\frac{2}{3}x + \frac{8}{3}$

$$m = \frac{4-2}{-2-1} = -\frac{2}{3}$$

7. $\lim_{x \rightarrow -4} \frac{x^4 - 14x^2 + 3x - 20}{x^3 + 6x^2 - 2x - 40} = \lim_{x \rightarrow -4} \frac{(x+4)(x^3 - 4x^2 + 2x - 5)}{(x+4)(x^2 + 2x - 10)} =$

- a) 0 b) $\frac{141}{77}/2$ c) $\frac{2}{77}$ d) $-\frac{141}{77}/2$ e) dne

$$\begin{array}{r} 1 \ 0 \ -14 \ -3 -20 \\ \underline{-4} \ 16 \ -8 \ 20 \\ 1 \ -4 \ 2 \ -5 \ 0 \end{array}$$

$$\begin{array}{r} 1 \ 6 \ -2 \ -40 \\ \underline{-4} \ -4 \ -8 \ 40 \\ 1 \ 2 \ -6 \ 0 \end{array}$$

8. At the point where $x = 1$, the slope of the line normal to $f(x) = 2x^3 - 5x^2 - 4x + 10$ is

- a) -8 b) 8 c) $\frac{1}{8}$ d) $-\frac{1}{8}$ e) 9

$$f' = 6x^2 - 10x - 4 \quad m_{\text{tan}} = -8$$

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$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2\sin x}{\sin^2 x - \cos^2 x} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin x}{\sin 2x} = \frac{\sin \left(\frac{\pi}{6}\right)}{\sin \left(2 \cdot \frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

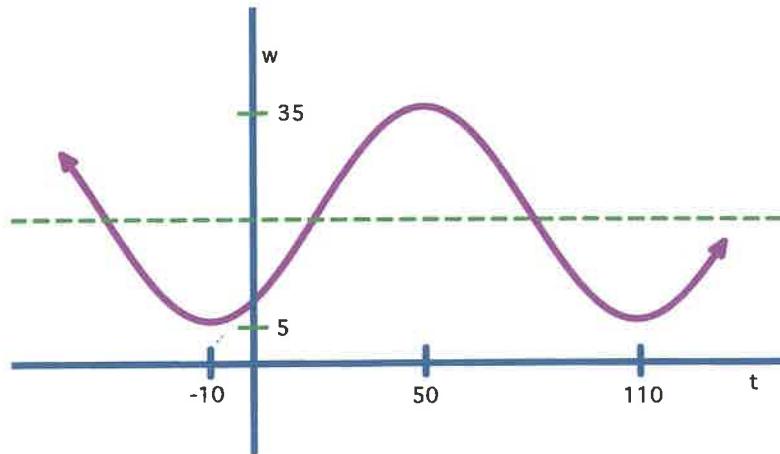
- a) ~~1~~ b) 0 c) ~~1~~ d) $\frac{2\sqrt{3}}{3}$ e) $-\frac{2\sqrt{3}}{3}$
-

10. Consider a particle moving such that its position is described by the function

$$x(t) = \frac{t^5}{5} - t^4. \text{ When is the particle at rest?} \quad v = t^4 - 4t^3$$

- a) $t = 0$ b) $t = 3$ c) $t = 4$
 d) $t = 0 \text{ and } 3$ e) $t = 0 \text{ and } 4$
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11. Which of the following is NOT an equation for this graph



- a) $y = 20 + 15\sin\left[\frac{\pi}{60}(x - 20)\right]$ b) $y = 20 + 15\cos\left[\frac{\pi}{60}(x - 50)\right]$
 c) $y = 20 - 15\cos\left[\frac{\pi}{60}(x + 10)\right]$ d) $y = 20 - 15\cos\left[\frac{\pi}{60}(x - 10)\right]$
-

12. Find the polynomial of degree 3 whose zeros are $(-3, 0)$, $\left(\frac{2}{3}, 0\right)$ and $(2, 0)$ and goes through $(1, -3)$.

a) $g(x) = (x + 3)(3x - 2)(x - 2)$ b) ~~$g(x) = -\frac{3}{4}(x + 3)(3x - 2)(x - 2)$~~

c) $g(x) = \frac{4}{3}(x + 3)(3x - 2)(x - 2)$ d) $g(x) = -\frac{4}{3}(x + 3)(3x - 2)(x - 2)$

e) $g(x) = \frac{3}{4}(x + 3)(3x - 2)(x - 2)$

$$a(x+3)(\cancel{3x-2})(\cancel{x-2}) = y$$

$$-3 = a(4)(+\cancel{2})(-1) \quad a = -3/4$$

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Directions: Round to 3 decimal places. Show all work.

1. Solve $\underbrace{\cos 2x \sin x + \cos x \sin 2x}_{\sin(3x)} - 2 \sin 3x = \frac{1}{2}$ for $x \in [-\pi, \pi]$.

$$\sin(3x) - 2 \sin 3x = \frac{1}{2}$$

$$-\sin 3x = \frac{1}{2}$$

$$\sin 3x = -\frac{1}{2}$$

$$3x = \begin{cases} -\frac{\pi}{6} \pm 2\pi n \\ \frac{7\pi}{6} \pm 2\pi n \end{cases}$$

$$x = \begin{cases} \frac{-\pi}{18} \pm \frac{2\pi}{3} n \\ \frac{7\pi}{18} \pm \frac{2\pi}{3} n \end{cases}$$

$$x = \left\{ \begin{array}{l} -\frac{\pi}{18}, \frac{11\pi}{18}, \frac{2\pi}{3}, \\ \frac{7\pi}{18}, -\frac{5\pi}{18}, -\frac{17\pi}{18} \end{array} \right\}$$

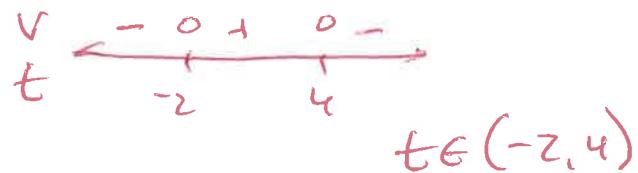
2. $D_x \left[3x^2 - 4x^3 + \frac{3}{x^2} - \frac{1}{\sqrt[3]{x}} + 8\pi^3 \right] = 6x - 12x^2 - 6x^{-3} - \frac{1}{3}x^{-4/3}$

$$= D_x \left[3x^2 - 4x^3 + 3x^{-2} - x^{-1/3} + 8\pi^3 \right]$$

3. The motion of a particle is described by $x(t) = -t^3 + 3t^2 + 24t - 5$.

- a) On what time intervals is the particle moving right?

$$v(t) = -3t^2 + 6t + 24 = -3(t^2 - 2t - 8) \\ = -3(t-4)(t+2)$$



- b) Find the position and acceleration when $v(t) = 0$.

$$t = -2, 4$$

$$a(t) = -6t + 6$$

$$x(-2) = -33$$

$$a(-2) = 18$$

$$x(4) = 75$$

$$a(4) = -18$$

- c) Find the position and velocity when $a(t) = 0$.

$$a(t) = 0 \Rightarrow t = 1$$

$$x(1) = 21$$

$$v(1) = 27$$

4a. Find the zeros algebraically of $y = 6x^4 + 17x^3 - 29x^2 - 2x + 8$. Show the synthetic division.

$$\begin{array}{r} -4 \\ \boxed{-4} \end{array} \left| \begin{array}{rrrrr} 6 & 17 & -29 & -2 & 8 \\ & -24 & 28 & 4 & -8 \\ \hline & 6 & -7 & -9 & 2 \end{array} \right.$$

Zeros: $(-4, 0)$ $(1, 0)$
 $(\frac{1}{3}, 0)$ $(-\frac{1}{2}, 0)$

$$\begin{array}{r} 1 \\ \boxed{1} \end{array} \left| \begin{array}{rrrr} 6 & -7 & -1 & 2 \\ & 6 & -1 & 2 \\ \hline & 6 & -1 & -2 \end{array} \right.$$

$$(x+4)(x+1)(6x^2 - x - 2) = (x+4)(x+1)(3x-2)(2x+1)$$

4b. Find the extreme points of $y = 6x^4 + 17x^3 - 29x^2 - 2x + 8$. Show the derivative.

$$\frac{dy}{dx} = 24x^3 + 51x^2 - 58x - 2 = 0$$

$$x = -2.937, -0.335109, 0.84643302$$

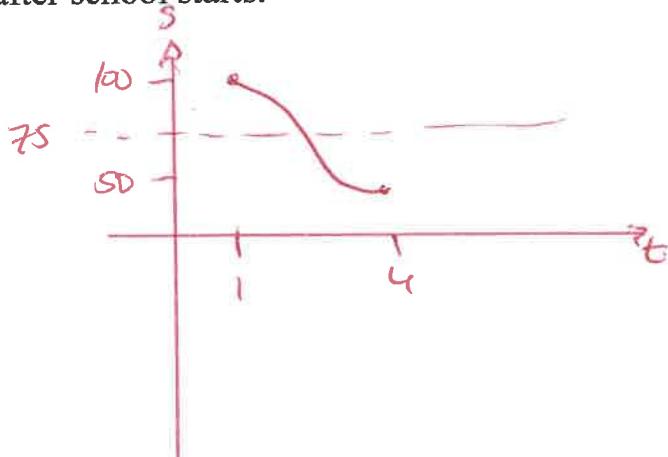
$$(-2.937, -220.52)$$

$$(-0.334, 8.034)$$

$$(0.846, -1.081)$$

5. Mr. Jackanich's teaching varies sinusoidally with time. One month after the start of school, in September, he is teaching at his best, reaching all 100 of his students. Three months later he reaches his lowest, only 50 of his students.

- a) Sketch a graph of the number of students reached as a function of months after school starts.



- b) Write a sinusoidal equation that describes the number of students reached in terms of months after school starts.

$$S = 75 + 25 \cos \frac{\pi}{3}(t-1)$$

- c) How many students were reached at the start of school?

$$S(0) = 87.5 \approx 88$$

- d) When are the first two times he will reach 85 students?

$$85 = 75 + 25 \cos \frac{\pi}{3}(t-1)$$

$$14 = \cos \frac{\pi}{3}(t-1)$$

$$\pm 1.159 \approx 2\pi n \pm \frac{\pi}{3}(t-1)$$

$$\pm 1.107 \pm 6n = t-1$$

$$t = \begin{cases} -0.107 \pm 6n \\ 2.107 \pm 6n \end{cases}$$

$$t = \boxed{-0.107, 2.107}$$

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Fall Final – Part II (25 minutes)

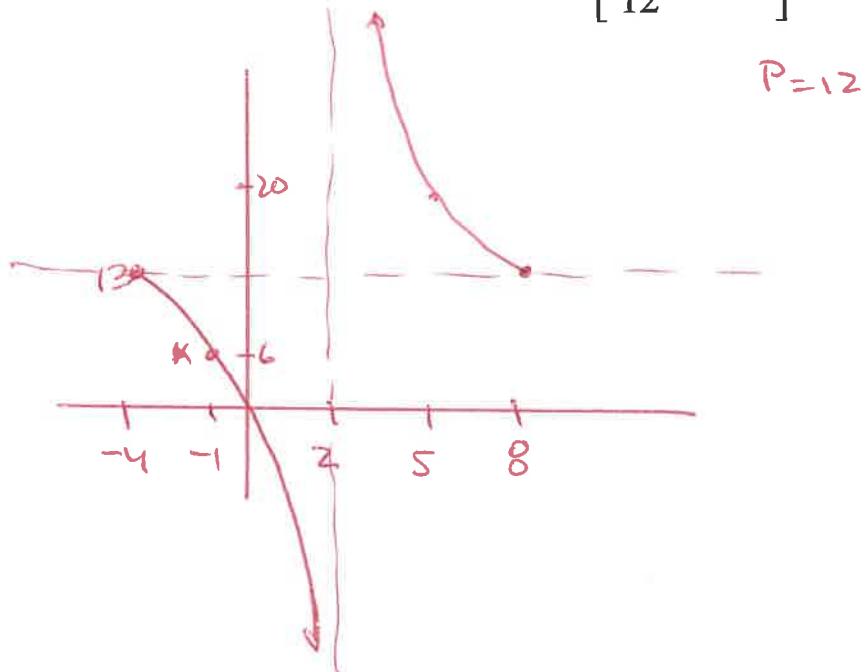
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6. Sketch the primary cycle of $y = 13 - 7\tan\left[\frac{\pi}{12}(x + 4)\right]$



7. Find an inequality that has this sign pattern and solution:

$$\begin{array}{ccccccc} y & - & 0 & + & 0 & + & 0 \\ \hline x & \leftarrow & -\frac{1}{3} & \frac{2}{3} & \frac{4}{3} & \rightarrow & \end{array} \quad \text{and} \quad x \in \left(-\frac{1}{3}, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \frac{4}{3}\right)$$

$$-(3x+1)(3x-2)^2(3x-4) > 0$$

$$8a. \lim_{x \rightarrow -2} \frac{x^3 + 8}{3x^3 + 6x^2 + x + 2} = \lim_{x \rightarrow -2} \frac{(x+2)(x^2 - 2x + 4)}{3x^2(x+2) + 1(x+2)}$$

$$= \lim_{x \rightarrow -2} \frac{x^2 - 2x + 4}{3x^2 + 1} = \frac{12}{13}$$

$$8b. \lim_{x \rightarrow 5} \frac{x^2 - 5x + 4}{x - 3} = \frac{4}{2} = 2$$

$$8c. \lim_{x \rightarrow -1} \frac{x^4 + x^2 - 2}{x^3 + 7x^2 - 6} = \lim_{x \rightarrow -1} \frac{(x^2 + 2)(x^2 - 1)}{(x - 1)(x^2 + 6x + 6)} = \frac{3(-2)}{-11}$$

$$\begin{array}{r} \boxed{-1} \ 1 \ 7 \ 0 \ -6 \\ \underline{-1 \ -6 \ 6} \\ 1 \ 6 \ -6 \ \cancel{0} \end{array} \quad = \text{deg } \frac{6}{11}$$