

Honors PreCalculus '23

Dr. Quattrin

Limits and Derivatives Test

CALCULATOR ALLOWED

Round to 3 decimal places. Show all work.

Name: SOLUTION KEY

Score _____

$$1. \lim_{x \rightarrow 8} \frac{x^2 - 64}{x - 8} = \lim_{x \rightarrow 8} \frac{(x-8)(x+8)}{x-8}$$

- a) 0 b) 8 c) 16 d) -8 e) DNE
-

2. Consider a particle moving such that its position is described by the function

$$x(t) = \frac{t^5}{5} - t^4. \text{ When is the acceleration of the particle equal to zero?}$$

- a) $t = 0$ b) $t = 3$ c) $t = 4$
d) $t = 0 \text{ and } 3$ e) $t = 0 \text{ and } 4$
-

$$v(t) = t^4 - 4t^3$$

$$a(t) = 4t^3 - 12t^2$$

$$= 4t^2(t-3) = 0$$

3. Suppose f is a differentiable function such that $f(3) = 5$ and $f'(3) = -4$. Using the line tangent to the graph of $f(x)$ at $x = -1$, find the approximation of $f(2.9)$

$$y - 5 \approx -4(x - 3)$$

- a) -3.5 b) 11.4 c) 4.6 d) -4.6 e) 5.4

$$f(2.9) \approx -4(2.9 - 3) + 5$$

4. Find an equation of the normal line to the curve $f(x) = \sqrt[3]{x^4} - 2x^2$ at $x = 1$.

- a) $y + 1 = -\frac{8}{3}(x - 1)$ b) $y + 1 = \frac{8}{3}(x - 1)$ $f' = \frac{4}{3}x^{1/3} - 4x$
 c) $y + 1 = \frac{3}{8}(x - 1)$ d) $y + 1 = -\frac{3}{8}(x - 1)$ $M = \frac{4}{3} - 4 = -\frac{8}{3}$

5. A particle moving in a straight line such that $x(t) = -90t + 33t^2 - 4t^3$. When is the particle at rest?

- a) $t = 0$ b) $t = 3$ c) $t = \frac{5}{2}$

- d) $t = \frac{5}{2}, 3$ e) $t = 0, \frac{5}{2}, 3$

$$V(t) = -12t^2 + 66t - 90 = 0$$

$$\begin{aligned} &= 2t^2 - 11t + 15 \\ &(2t - 5)(t - 3) = 0 \end{aligned}$$

$$6. \quad \frac{d}{dx} \left[-3x^7 - \frac{7}{5} \sqrt[4]{x^7} - \frac{3}{\sqrt[5]{x^6}} - \frac{1}{4x} \right] =$$

a) $-3x^7 - \frac{7}{5}x^{7/4} - 3x^{-6/5} - 4x^{-1}$

b) $-21x^6 - \frac{49}{20}x^{3/4} + \frac{18}{5}x^{-11/5} + 4x^{-2}$

c) $-21x^6 - \frac{49}{20}x^{3/4} + \frac{18}{5}x^{-11/5} + \frac{1}{4}x^{-2}$

d) $-21x^6 - \frac{4}{5}x^{-1/7} + \frac{18}{5}x^{-11/5} + 4x^{-2}$

e) $-21x^6 - \frac{4}{5}x^{-1/7} + \frac{18}{5}x^{-11/5} + \frac{1}{4}x^{-2}$

$$7. \quad \lim_{\substack{x \rightarrow 5 \\ x \rightarrow \frac{5}{2}}} \frac{2x^2 - 7x + 5}{6x^2 - 11x - 10} = \underset{x \rightarrow \frac{5}{2}}{\text{LIM}} \frac{(2x-5)(x-1)}{(2x+5)(3x+2)} = \frac{3/2}{19/2}$$

a) $\frac{3}{19}$ b) $\frac{19}{3}$ c) $\frac{11}{3}$ d) $\frac{3}{11}$

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1. Use the Power Rule to find:

a) Find $\frac{dy}{dx}$ if $y = 5x^3 - 3x^2 + 2x + 73$

$$\frac{dy}{dx} = 15x^2 - 6x + 2$$

b) Find $f'(x)$ if $f(x) = 13x^{12} - 2x^3 + 7x + 3 + \frac{1}{5x}$

$$f'(x) = 156x^{11} - 6x^2 + 7 - \frac{1}{5x^2}$$

c) $D_x \left[\sqrt[3]{x^5} - \frac{2}{\sqrt{x^3}} - \sqrt[5]{x} + \pi^2 + 2x \right] = D_x \left[x^{5/3} - 2x^{-3/2} - x^{1/5} + \pi^2 + 2x \right]$
 $= \frac{5}{3}x^{2/3} + 3x^{-5/2} - \frac{1}{5}x^{-4/5} + 2$

2. The motion of a particle is described by $y = 2t^3 - t^2 - 4t + 2$

$$a(t) = 12t - 2$$

- a) Find the position and acceleration when $v(t) = 0$.

$$\begin{aligned} V &= 6t^2 - 2t - 4 = 0 & t &= 1, -\frac{2}{3} & x(1) &= -1 \\ &= 3t^2 - t - 2 = 0 & & & a(1) &= 10 \\ &= (3t + 2)(t - 1) = 0 & & & x\left(-\frac{2}{3}\right) &= 3.630 \\ & & & & a\left(-\frac{2}{3}\right) &= -10 \end{aligned}$$

- b) Find the position and velocity when $a(t) = 0$.

$$a = 0 \rightarrow t = \frac{1}{6}$$

$$x(\frac{1}{6}) = 1.315$$

$$v(\frac{1}{6}) = -4.167$$

3. Set up, but do not solve, the limit definition of the derivative for

$$f(x) = 3x - \sqrt{x} + \frac{3}{\sqrt{x}} - 4$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\sqrt{x} (3(x+h) - \sqrt{x+h} + \frac{3}{\sqrt{x+h}} - 4) - (\sqrt{x} (3x - \sqrt{x} + \frac{3}{\sqrt{x}} - 4))}{h}$$

4. Use the equation of the line tangent to $f(x) = 3x - \sqrt{x} + \frac{3}{\sqrt{x}} - 4$ at $x = 1$

to approximate $f(0.9)$

$$f'(x) = 3 - \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{-3/2}$$

$$f(1) = 1$$

$$m = f'(1) = 3 - \frac{1}{2} - \frac{3}{2} = 1$$

$$y - 1 = 1(x - 1)$$

$$f(0.9) \approx 1 + (-.1)$$

$$= 0.9$$

5. The motion of a particle is described by $x(t) = 2t^3 - 5t^2 - 56t + 8$.

a) When is the particle stopped?

$$v(t) = 6t^2 - 10t - 56 = 0$$

$$t = 4, -7/3$$

$$3t^2 - 5t - 28 = 0$$

$$(t-4)(3t+7) = 0$$

b) Which direction it is moving at $t = 2$?

$$v(2) = -26 \therefore \text{LEFT}$$

c) Where is it when $t = 2$?

$$x(2) = -108$$

d) Find $a(2)$. $a(t) = 12t - 10$

$$a(2) = 14$$

6. Evaluate the following limits:

$$a) \lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 - x - 12} = \lim_{x \rightarrow 4} \frac{(x-4)(x+4)}{(x-4)(x+3)} = \frac{8}{7}$$

$$b) \lim_{x \rightarrow 3} \frac{x^3 - 27}{4x^3 - 12x^2 - x + 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{(x-3)(4x^2 - 1)} = \frac{27}{35}$$

$$c) \lim_{x \rightarrow -5} \frac{x^3 + 5x^2 - 9x - 45}{2x^3 + 11x^2 - x - 30} =$$

$$= \lim_{x \rightarrow -5} \frac{(x+5)(x^2 - 9)}{(x+5)(2x^2 + x - 6)} =$$

$$= \frac{16}{39}$$

$$\begin{array}{r} -5 | \\ \hline 1 & 5 & -9 & -45 \\ -5 & & 0 & 45 \\ \hline 1 & 0 & -9 & 0 \end{array}$$

$$\begin{array}{r} -5 | \\ \hline 2 & 11 & -1 & -30 \\ -10 & & -5 & 30 \\ \hline 2 & 1 & -6 & 0 \end{array}$$